Loop-soup cluster decompositions and Gaussian fields

Wendelin Werner (ETH Zürich)

We will describe some new results about the decomposition of critical two-dimensional Brownian loop-soup clusters. These rather surprising decompositions of clusters shed some new light on the relation between these random collection of overlapping Brownian loops in a domain with Gaussian fields and their squares, and provides a direct link with the CLE(4) (i.e., via SLE(4) loops) decomposition of these Gaussian fields pointed out by Miller and Sheffield. We will also point out the general Markovian property of such critical loop-soups, its significance and some field-theoretical consequences.

A number of ideas in this approach to two-dimensional fields are inspired from Itô's excursion measure for one-dimensional Brownian motion.

This is partly joint work with Wei Qian (Zürich).

Scaling limits for random fields with a pinning effect

Tadahisa Funaki (University of Tokyo)

We discuss scaling limits for Gaussian (or non-Gaussian) random fields with a pinning effect under the critical situation that two possible candidates of the limits exist at the level of large deviation principle. We show the law of large numbers under the assumption that the strength of pinning is sufficiently large and the dimension of the fields satisfies $d \geq 3$. The talk is based on a joint paper with Erwin Bolthausen and Taizo Chiyonobu: *Scaling limits for weakly pinned Gaussian random fields under the presence of two possible candidates*, to appear in the special issue for Kiyosi Itô, J. Math. Soc. Japan, 2015.

Stochastic approach to the derivation of macroscopic energy diffusion in Hamiltonian systems

Makiko Sasada (University of Tokyo)

One of the most important problems in the study of non-equilibrium statistical mechanics is the derivation of Fourier's law or transport equations for conserved quantity, in particular energy transport, from a microscopic Hamilton dynamics through a diffusive space-time scaling limit. In this talk, we consider two approaches to this problem using the techniques of stochastic analysis.

Firstly, we consider a chain of an-harmonic oscillators whose Hamiltonian dynamics is perturbed by a local energy conserving noise. Under a diffusive space-time rescaling, we prove that the fluctuations of the energy evolve following a linear SPDE, with diffusion coefficient given by the Green-Kubo formula. We use Varadhan's approach for the hydrodynamic limit of non-gradient systems, and this is the first application of the method to a mechanical model with noise. We also consider a chain of harmonic oscillators perturbed by a different type of stochastic noise which conserves the energy and a second quantity called the volume, and destroys all the other ones. We then add to this model a second energy conserving noise whose intensity is given by a parameter γ, that annihilates the volume conservation. When γ is of order one, the energy diffuses according to the standard heat equation after a space-time diffusive scaling. On the other hand, when $\gamma = 0$ the energy superdiffuses according to a 3/4-fractional heat equation after a subdiffusive space-time scaling. We study the existence of a crossover between these two regimes as a function of γ.

Secondly, we report recent studies on the derivation of heat transport from mechanical dynamics via a so-called two-step approach. In the context of the derivation of heat transport, the two-step approach is the strategy consists of (i) a derivation of a mesoscopic stochastic dynamics for energies from a microscopic mechanical dynamics via a rare (or weak) interaction limit and (ii) a derivation of a time evolution equation describing the macroscopic energy transport for the mesoscopic stochastic dynamics via a proper space-time scaling limit (more precisely, the hydrodynamic limit). There are two typical examples of the dynamics studied by this approach: these are "localized hard balls with elastic collisions" studied by Gaspard and Gilbert and "energy transfer in a fast-slow Hamiltonian system" studied by Dolgopyat and Liverani. From these dynamics, they derived respective stochastic dynamics for energies. In this talk, we consider generalizations of these stochastic processes, called stochastic energy exchange models (SEE) and energy conserving stochastic Ginzburg-Landau models (ECGL).

SSE and ECGL are both Markov processes on the state space $\mathbb{R}_+^N := (0,\infty)^N = \{(x_i)_{i=1}^N; x_i > 0\}$ where N represents the number of particles and x_i represents the energy of i-the particle.

SSE is a pure jump process with model parameters (Λ, P) where a continuous function $\Lambda : \mathbb{R}_+^2 \to \mathbb{R}_+$ represents the collision rate between particles and a probability measure-valued continuous function $P : \mathbb{R}_+^2 \to \mathcal{P}((0,1))$ represents the collision kernel. The infinitesimal generator $\mathcal{L}$ of SSE acting on bounded functions $f : \mathbb{R}_+^N \to \mathbb{R}$ is

$$\mathcal{L}f(x) = \sum_{i=1}^{N-1} \Lambda(x_i, x_{i+1}) \int P(x_i, x_{i+1}, d\alpha)[f(T_{i,i+1,\alpha}x) - f(x)]$$

where

$$(T_{i,i+1,\alpha}x)_k = \begin{cases} \alpha(x_i + x_{i+1}) & \text{if } k = i \\ (1-\alpha)(x_i + x_{i+1}) & \text{if } k = i+1 \\ x_k & \text{if } k \neq i, i+1. \end{cases}$$

ECGL is a multi-dimensional diffusion process with model parameters (κ, σ) where $\kappa : \mathbb{R}_+^2 \to \mathbb{R}$ and $\sigma : \mathbb{R}_+^2 \to \mathbb{R}_+$ are smooth functions satisfying a condition to guarantee the process remains in the state space $\mathbb{R}_+^N$ forever. The infinitesimal generator $\mathcal{L}$ of ECGL acting on smooth bounded functions $f : \mathbb{R}_+^N \to \mathbb{R}$ is

$$\mathcal{L}f(x) = \sum_{i=1}^{N-1} \kappa(x_i, x_{i+1})(\partial_{x_{i+1}} - \partial_{x_i})f + \frac{1}{2}\sigma^2(x_i, x_{i+1})(\partial_{x_{i+1}} - \partial_{x_i})^2 f.$$

The process is also defined by the following SDEs:

$$\begin{cases} dx_i &= dJ_{i-1,i} - dJ_{i,i+1} \\ dJ_{i,i+1} &= \kappa(x_i, x_{i+1})dt + \sigma(x_i, x_{i+1})dB_{i,i+1}. \end{cases}$$

Our goal is to derive the hydrodynamic limit for SSE or ECGL under "good" condition on model parameters (Λ, P) or (κ, σ). Here, "good " means the class satisfying the condition is general enough as it includes all the examples mentioned above, but not so general as we can show the hydrodynamic limit rigorously for the models in the class. Unfortunately, we do not achieve the goal so far since both of the models are of non-gradient type in general, but we have some results on the keys of the proof of hydrodynamic limit for non-gradient systems. Precisely, we have the characterization of reversible measures and the spectral gap estimate. In this talk, we also discuss a formal description of the macroscopic diffusion coefficient for the hydrodynamics equation. We show that under some natural assumptions, the diffusion coefficient $D(E)$ should be $D(E) = cE^m$ with constant c. Surprisingly, it is true even if the process is of non-gradient type.

Professor Kiyosi Itô and some aspects of his mathematical legacy

David Elworthy (University of Warwick)

I will combine reminiscences concerning Professor Itô with comments on a part of his mathematical legacy.

Another look at Kolmogorov's and related hypoelliptic operators

Daniel W. Stroock (MIT)

Kolmogorov considered the parabolic equation

$$\partial_t u = \partial_{x_1}^2 u + x_1^k \partial_{x_2} u$$

and, using the fact, which is obvious from Itô's representation, that the corresponding diffusion is Gaussian, wrote down the fundamental solution for the associated Cauchy initial value problem. Until L. Hörmander came up in 1967 with a theory that explained why such a degenerate equation admits a smooth fundamental solution, Kolmogorov's example remained a thorn in the side of experts in the theory of partial differential equations. Unfortunately, at least from a probabilistic perspective, Hörmander's techniques ignore the intuition that underlies Kolmogorov's result. In this lecture, I use ideas derived from the work of P. Malliavin to re-examine Kolmogorov's and related examples in a way that restores the probabilistic intuition.

Mathematical legacy of Itô sensei

Masatoshi FUKUSHIMA (Osaka University)

Mathematical legacy of Kiyosi Itô on Lévy process, stochastic differential equation, one-dimensional diffusion and excursion theory will be recalled by quoting those passages from his own memoirs. Some of later developements of the last two subjects will then be reviewed.

The signature of a rough path – uniqueness

Terry LYONS (University of Oxford)

The signature operator is a fundamental transformation from paths into algebraic sequences. Ben Hambly and L. characterised the sense in which this transformation was injective on paths of finite variation. Until very recently, the extension of this result to general rough paths seemed out of reach. It is now proven.

We will explain why the result is interesting.

The Signature of a Rough Path: Uniqueness. Horatio Boedihardjo, Xi Geng, Terry Lyons and Danyu Yang.

Malliavin calculus and diffusion equation with Dirichlet boudary condition

Shigeo KUSUOKA (University of Tokyo)

Let $(\Omega, \mathcal{F}, P)$ be a probability space and $B(t) = (B^1(t), \ldots, B^d(t))$, $t \geq 0$, be a $d-$dimensional standard Brownian motion. Let $V_i \in C_b^\infty(\mathbf{R}^N; \mathbf{R}^N)$, $i = 0, 1, \ldots, d$, and think of a Stratonovich type SDE on $\mathbf{R}^N$

$$X(t,x) = x + \sum_{k=0}^{d} \int_0^t V_k(X(t,x)) \circ dB^k(t), \qquad t \geqq 0.$$

Here we use the convention that $B^0(t) = t$.

Let $\mathcal{A} = \bigcup_{n=1}^\infty \{0, 1, \ldots, d\}^n$, and for for $\alpha = (\alpha^1, \ldots, \alpha^n) \in \{0, 1, \ldots, d\}^n \subset \mathcal{A}$, let $||\alpha|| = n + \#\{k = 1, \ldots, n;\ \alpha_i = 0\}$ and let $V_\alpha \in C_b^\infty(\mathbf{R}^N; \mathbf{R}^N)$ be given by $V_\alpha = [V_{\alpha_1}, [V_{\alpha_2}, \cdots [V_{\alpha_{n-1}}, V_{\alpha_n}] \cdots]$. Let $\mathcal{A}_0 = \mathcal{A} \setminus \{(0)\}$.

We say that a system of vector fields $V_0, V_1, \ldots, V_d$ satisfies UFG condition, if there exists a finite subset $L \subset \mathcal{A}_0$, and $\phi_{\alpha,\beta} \in C_b^\infty$, $\alpha \in \mathcal{A}_0$, $\beta \in L$ such that

$$V_\alpha = \sum_{\beta \in L} \phi_{\alpha,\beta} V_\beta \qquad \alpha \in \mathcal{A}_0.$$

We consider the follwing special case in this paper as follows:

$$V_1^i(x) = \begin{cases} 1, & i = 1 \\ 0, & i = 2, \ldots, N \end{cases}$$

and

$$V_k^1(x) = 0, \qquad k = 2, \ldots, d.$$

Let

$$\hat{P}_t f(x) = E[M(t,x) f(X(t,x)), \min_{s\in[0,t]} X^1(s,x) > 0]$$

Here $M(t,x)$ is a 'multiplicative' functional.

Our main goal is the following.

Theorem 1. *Assume that UFG condition is satisfied. Let* $\mathcal{A}_{00} = \mathcal{A}_0 \setminus \{(1)\}$*. For any* $k, k' \geqq 0$ *with* $k + k' \leqq m$, $\alpha^1, \ldots, \alpha^{k+k'} \in \mathcal{A}_{00}$*, there is a* $C > 0$ *such that*

$$\sup_{x\in(0,\infty)\times\mathbf{R}^{N-1}} |V_{\alpha^1} \cdots V_{\alpha^k} \hat{P}_t V_{\alpha^{k+1}} \cdots V_{\alpha_{k+k'}} f(x)|$$

$$\leq C t^{-(||\alpha^1||+\cdots||\alpha^{k+k'}||/2)} \sup_{x\in(0,\infty)\times\mathbf{R}^{N-1}} |f(x)| \qquad t \in (0,1],\ f \in C_b^\infty(\mathbf{R}^N).$$

A related result has been shown by Ben Arous-K.-Stroock. However, we do not use a partial Malliavin calculus here, because if we do it we cannot handle $V_{[01]} = [V_0, V_1]$ for example.

On solutions to reflected rough differential equations

Shigeki Aida (Tohoku University)

Let X be a p-rough path $(2 \leq p < 3)$ and $p < \gamma < 3$. Let us consider rough differential equations (=RDEs), $dY_t = \sigma(Y_t)dX_t, Y_0 = y_0$. The existence of the solutions were proved by A.M. Davie (2008) under the assumption that $\sigma \in \mathrm{Lip}^{\gamma-1}$. Also he proved that there exist infinitely many solutions to the RDE for a certain $\sigma \in \mathrm{Lip}^{2-\varepsilon}$ and for almost all Brownian rough paths. On the other hand, if the state space of stochastic processes is a domain of a Euclidean space, we need to study stochastic differential equations with normal reflection at the boundary. The equation contains the bounded variation term $\Phi(t)$ corresponding to the local time term. By using the Skorohod map, the SDE is transformed to a path-dependent SDE without reflection term. However the Skorohod map is at most Lipschitz continuous on the continuous path space and is not Lipschitz continuous generally. Therefore previous studies on RDEs cannot be applied directly to RDEs with normal reflection.

Let us explain what reflected rough differential equations(=RRDEs) are. First we recall the definition of the set of inward unit normal vectors $\mathcal{N}_x$ at the boundary point $x \in \partial D$. Let $B(z,r)$ be the ball centered at $z \in \mathbb{R}^d$ with radius r and define

$$\mathcal{N}_{x,r} = \left\{ \boldsymbol{n} \in \mathbb{R}^d \mid |\boldsymbol{n}| = 1, B(x - r\boldsymbol{n}, r) \cap D = \emptyset \right\}, \quad \mathcal{N}_x = \cup_{r>0}\mathcal{N}_{x,r}.$$

Let $\sigma \in \mathrm{Lip}^{\gamma-1}(\mathbb{R}^d, L(\mathbb{R}^n, \mathbb{R}^d))$ and $X_{s,t}$ be a p-rough path on $\mathbb{R}^n$. RDE

$$dY_t = \sigma(Y_t)dX_t + d\Phi_t \quad 0 \leq t \leq T, \quad Y_0 = y_0 \in \mathbb{R}^d \tag{1}$$

is called a RRDE defined on D if the following hold. The mapping $t \mapsto \Phi(t)$ is bounded variation. It holds that $Y_t(:= y_0 + Y^1_{0,t}) \in \bar{D}$ $(0 \le t \le T)$ and there exists a Borel measurable map $s(\in [0,T]) \mapsto \boldsymbol{n}(s) \in \mathbb{R}^d$ such that $\boldsymbol{n}(s) \in \mathcal{N}_{Y_s}$ if $Y_s \in \partial D$ and

$$\Phi(t) = \int_0^t 1_{\partial D}(Y_s)\boldsymbol{n}(s)d\|\Phi\|_{[0,s]} \quad 0 \le t \le T,$$

where $\|\Phi\|_{[0,s]}$ denotes the total variation of $\Phi(u)$ $(0 \le u \le s)$. Note that the pair (Y, Φ) is a solution to the RRDE.

To explain our results, we introduce conditions on the boundary.

(A) There exists $r_0 > 0$ such that $\mathcal{N}_x = \mathcal{N}_{x,r_0} \neq \emptyset$ for any $x \in \partial D$.

(H1) The Skorohod problem

$$\xi_t = w_t + \phi(t), \quad \xi_t \in \bar{D} \quad 0 \le t \le T,$$
$$\phi(t) = \int_0^t 1_{\partial D}(\xi_s)\boldsymbol{n}(s)d\|\phi\|_{[0,s]}, \qquad \boldsymbol{n}(s) \in \mathcal{N}_{\xi_s} \text{ if } \xi_s \in \partial D$$

is uniquely solved for any $w \in C([0,T] \to \mathbb{R}^d)$. Moreover there exists a positive constant C_D such that for all continuous paths w on $\mathbb{R}^d$ we have

$$\|\phi\|_{[s,t]} \le C_D \max_{s \le u,v \le t} |w(v) - w(u)| \quad 0 \le s \le t \le T,$$

where $\|\phi\|_{[s,t]}$ denotes the total variation of ϕ_u $(s \le u \le t)$.

Tanaka (1979) proved that (H1) holds if D is convex and there exists a unit vector $l \in \mathbb{R}^d$ such that

$$\inf\{(l, \boldsymbol{n}(x)) \mid \boldsymbol{n}(x) \in \mathcal{N}_x, x \in \partial D\} > 0.$$

The following is the first main result in this talk.

Theorem 2. *Assume* (A), (H1) *and* $\sigma \in \mathrm{Lip}^{\gamma-1}$.
(1) *Let ω be the control function of $X_{s,t}$. Then there exists a solution (Y, Φ) to the reflected rough differential equation* (1) *such that for all* $0 \le s \le t \le T$,

$$|Y^i_{s,t}| \le C(1 + \omega(0,T))^3 \omega(s,t)^{i/p}, \qquad i = 1, 2,$$
$$\|\Phi\|_{[s,t]} \le C(1 + \omega(0,T))^3 \omega(s,t)^{1/p},$$

where the positive constant C depends only on σ, C_D, p.
(2) *We consider the set of geometric p-rough path $G\Omega_p(\mathbb{R}^n)$. Then there exists a universally measurable solution map $I : X \in G\Omega_p(\mathbb{R}^n) \to (Y, \Phi) \in G\Omega_p(\mathbb{R}^d) \times V_p(\mathbb{R}^n)$. Here $V_p(\mathbb{R}^n)$ denotes the set of continuous paths of finite p-variations on $\mathbb{R}^n$. This map satisfies that for any $X \in G\Omega_p(\mathbb{R}^n)$, there exist smooth rough paths such that X_N and $I(X_N)$ converge to X and $I(X)$ in p-variation distances respectively.*

By the example of Davie, we cannot expect the uniqueness of the solutions. However, any solutions in Theorem 2 satisfy the following continuity property at the smooth rough path.

Theorem 3. *Assume* $\sigma \in \mathrm{Lip}^{\gamma-1}$ *and* D *is a half space of* $\mathbb{R}^d$ *or a bounded* C^3 *domain. Let* h *be a bounded variation path on* $\mathbb{R}^n$ *and* $\bar{h}$ *denotes the associated smooth rough path. Assume*

$$|X^i_{s,t} - \bar{h}^i_{s,t}| \le \varepsilon\omega(s,t)^{i/p}, \quad |h_t - h_s| \le C\omega(s,t).$$

Let Y_t *be a solution constructed in Theorem 2 and* $Y(h)_t$ *be the solution to*

$$dY(h)_t = \sigma(Y(h)_t)dh_t + d\Phi(h)_t, Y(h)_0 = y_0.$$

Then there exists a positive constant C' *which depends only on* $\sigma, C, \omega(0,T)$ *such that*

$$\sup_{0\le t\le T} |Y_t - Y(h)_t| \le C'\varepsilon.$$

Note that the above theorems hold for $D = \mathbb{R}^d$ too. These results can be used to establish support theorems for the above universally measurable solutions of a certain class of RRDEs.

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Expectation weighted Sobolev spaces, BSDE and path dependent PDE

Shige Peng (Shandong University)

Beginning from a space of smooth, cylindrical and non-anticipative processes defined on a Wiener probability space $(\Omega, \mathcal{F}, P)$, we introduce a P-weighted Sobolev space of non-anticipative path-dependent processes such that the corresponding Sobolev derivatives are well defined. We identify each element of this Sobolev space with the one in the space of classical Itô's process. Consequently, a new path-dependent Itô's formula is applied to all such Itô processes.

We also replace the above linear Wiener expectation by a sublinear G-expectation and thus introduce the corresponding G-expectation weighted Sobolev space, or "G-Sobolev space", in which the derivatives of time and the 1st and second order derivatives of space are all well defined separately. We then formulate a type of fully nonlinear PPDEs in the G-Sobolev space and then identify them to a type of backward SDEs driven by G-Brownian motion.

Scaling limits of the uniform spanning tree

Martin T. Barlow (University of British Columbia)

The uniform spanning tree (UST) has played a major role in recent developments in probability. In particular the study of its scaling limit led to the discovery of SLE by Oded Schramm.

In this talk I will discuss the geometry of the UST in 2 dimensions, and what we can say about its scaling limit.

This is a joint work with David Croydon (University of Warwick), Takashi Kumagai (RIMS, Kyoto).

Fluctuation of spectra in random media revisited

Ryoki Fukushima (Kyoto University)

In this talk, we will discuss a homogenization problem for the so-called Anderson Hamiltonian, that is, the Schrödinger operator with a random potential. This type of problem has been well-studied in a similar but more singular setting called "the crushed ice problem", that is, the Lapcian in a randomly perforated domain. Kac [3] and Rauch-Taylor [5] established the convergence (homogenization) of eigenvalues in a certain limiting regime. Later Figari-Orlandi-Teta [2] and Ozawa [4] found a Gaussian fluctuation of the eigenvalues around the limits in the three dimensional case. The proof of homogenization is based on the analysis of the Wiener sausage whereas the fluctuation result is proved by a rather heavy perturbation method.

We propose a probabilistic approach to the fluctuation result based on a martingale central limit theorem. It is carried out in the Anderson Hamiltonian setting, yielding a central limit theorem in general dimensions. Our results partially extend a previous work by Bal [1], based on the perturbation method, which covers the dimensions less than or equal to three.

Based on a joint work with Marek Biskup (UCLA) and Wolfgang König (WIAS).

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On some disconnection problems

Alain-Sol Sznitman (ETH Zürich)

In this talk we will discuss some large scale asymptotics related to disconnection problems. One typical question is for instance to understand how the trajectory of a simple random walk in dimension bigger or equal to 3 can disconnect a macroscopic body from infinity. We will relate this question to similar disconnection problems for random interlacements and the level sets of the Gaussian free field. Some of the results discussed in this talk have been obtained in collaboration with Xinyi Li.

Lifetime of persistent homology and minimum spanning acycle in random topology

Tomoyuki Shirai (Kyushu University)

Persistent homology theory appeared in the beginning of this century as a tool of Topological Data Analysis for point cloud data, protein data, image data, material sciences, and so on. It describes "birth and death" of homology classes as persistence diagram by providing an increasing sequence of simplicial complexes as input. We are interested in random topology of random object, here in particular, random persistence diagram obtained from random input. The Erdős-Rényi graph process is such a typical example of increasing stochastic process and we can see its random persistence diagram (point process) as an output. In this talk, we mainly focus on simplicial complex versions of the Erdős-Rényi graph process and discuss the mean lifetime of its homology classes by emphasizing the relationship between mean lifetime of persistent homology and minimum spanning acycle.

This talk is based on a joint work with Yasuaki Hiraoka (AIMR, Tohoku University).

The ant in a labyrinth, recent progress

Gérard Ben Arous (Courant Institute)

In a famous paper of the 70s, Pierre-Gilles de Gennes called "the ant in the labyrinth" the random walker on percolation clusters. It is probably the central example of the possibly anomalous behavior of diffusion (and transport). Rigorous mathematical progress has been initially rather slow, but recent progress has been steady (due mainly to Martin Barlow, Takashi Kumagai, Remco van der Hofstad, Gady Kozma, Asaf Nachmias among others). We will survey some of the recent results on this central topic of statistical mechanics in disordered media, as well as very recent joint work with Alexander Fribergh and Manuel Cabezas.

In this recent work, we give a scheme of proof to get the natural scaling limit for the Random Walk on critical percolation structures, and apply this scheme to a simple case, i.e. large critical branching random walks, in high enough dimension. If time permits, I will also describe the scaling limit for the walk projected on the backbone for the incipient infinite cluster and shows how it relates to a specific exemple of the class of SSBMs, or Spatially Subordinated Brownian Motions, recently introduced in a joint work with Cabezas, Černý and Royfman.

Regularity structures

Martin Hairer (University of Warwick)

Over the last few years, the theory of regularity structures has enabled us to give a rigorous mathematical meaning to a number of ill-posed stochastic PDEs and to unify various existing approaches to these equations. I will give a short survey of the main concepts of the theory and some of the results one can obtain with it, focusing on an approcimation result for one-dimensional parabolic stochastic PDEs. This provides the first generalisation of the classical Wong-Zakai theory to a situation where the Itô-Stratonovich correction term is infinite.

Toward stochastic differential calculus based on measurable Riemannian structures

Masanori Hino (Osaka University)

Differential-like structures on nonsmooth spaces have been studied in various contexts thus far. In the field of analysis on metric measure spaces, the concept of upper gradient is defined on metric measure spaces to study deep analysis on a variety of functional inequalities, function spaces and some geometric concepts (see, e.g., recent monographs [4, 1]).

On the other hand, in the theory of Dirichlet forms, it has been recognized that strongly local (quasi-) regular Dirichlet forms provide measurable Riemannian metrics on Hilbert spaces that play the role of abstract tangent spaces (see, e.g., [2, 3]). In [5], the effective dimension of such a Riemannian metric at each point was named *pointwise index* and its essential supremum, called *index*, was identified with the martingale dimension of the diffusions associated with the Dirichlet form. By using these concepts, natural gradient operators are defined, which admits stochastic analysis of martingale additive functionals for the diffusion [6]. Typical nontrivial examples in mind are diffusions on fractal sets. It is expected that this kind of study will lead us to a theory of (stochastic) differential calculus/geometry on singular spaces. A preliminary study is found in [7].

In this talk, we discuss further progress of stochastic analysis based on such type of Riemannian structures. In particular, we propose the concept of BV functions, that is, functions of bounded variation, associated with strongly local regular Dirichlet forms. We also discuss the existence of nice coordinate-like functions, the coarea formula, and their applications to stochastic analysis.

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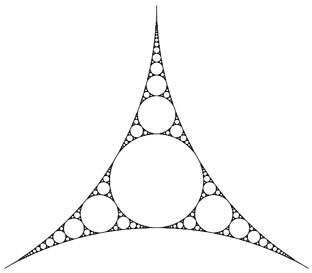
The Laplacian on the Apollonian gasket and its Weyl type eigenvalue asymptotics

Naotaka Kajino (Kobe University)



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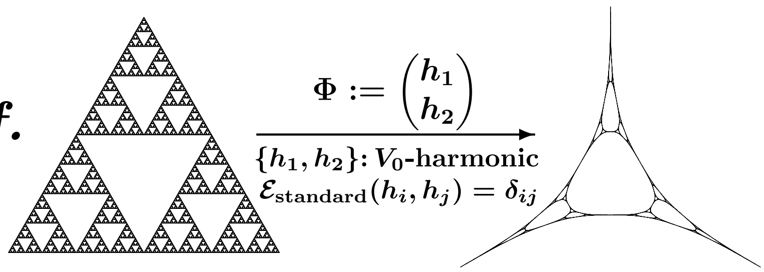

Fig. 1. Apollonian gasket

Fig. 2. Sierpiński gasket & harmonic Sierpiński gasket

1. Introduction: Apollonian gasket

Since Apollonius of Perga (262–190 BC) in ancient Greece it is well-known that an *ideal triangle*, i.e., the closed subset of $\mathbb{R}^2$ enclosed by three circles each of which is tangent to the other two, has a unique inner tangent circle. The *Apollonian gasket* associated with an ideal triangle (Fig. 1)[1] is a compact fractal subset of $\mathbb{R}^2$ obtained from the given ideal triangle by repeating indefinitely the process of removing the interior of the inner tangent circles of the ideal triangles. The Apollonian gasket is homeomorphic to the (usual) Sierpiński gasket (Fig. 2, left) as can be easily seen from its construction, and has been extensively studied in relation to various fields of mathematics such as fractal geometry, dynamical systems and theory of Kleinian groups.

Teplyaev [10] observed that the notion of a canonical energy form can be introduced for functions on the Apollonian gasket, but no further analysis of this form has been

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Keywords: Apollonian gasket, Laplacian, Weyl type eigenvalue asymptotics, ergodic theory on Kleinian groups

[1] From the viewpoints of hyperbolic geometry and theory of Kleinian groups it is more natural to consider also the cases where two of the first three circles are inside the other one and where at least one of the first three circles is a straight line. These cases, however, seem to require additional non-trivial arguments and hence are excluded from the framework of the main results of this talk.

done so far. The purpose of this talk is to present the author's recent results on a concrete description of this energy form, its closedness in an appropriate L^2-space, and the Weyl type eigenvalue asymptotics of the associated Laplacian, where the growth order of the large eigenvalues is half of the Hausdorff dimension d of the gasket and the limit is proportional to its d-dimensional Hausdorff measure.

2. Construction of the "canonical energy form": the idea

The idea of the construction of the "canonical energy form" derives from the study of the measurable Riemannian structure on the Sierpiński gasket given by a "harmonic embedding" into the plane. It is well-known that on the Sierpiński gasket K we can define a canonical self-similar Dirichlet form (energy form) $(\mathcal{E}_{\mathrm{st}}, \mathcal{F}_{\mathrm{st}})$, which corresponds to the Brownian motion on K constructed in [2, 7] and studied intensively in [1]. Kigami [5] observed that a pair (h_1, h_2) of linearly independent non-constant V_0-harmonic functions[2] with respect to $(\mathcal{E}_{\mathrm{st}}, \mathcal{F}_{\mathrm{st}})$ yields an embedding $\Phi := (h_1, h_2) : K \to \mathbb{R}^2$ of K into $\mathbb{R}^2$ (Fig. 2) and that we can regard K as equipped with a measure-theoretic "Riemannian structure" inherited from $\mathbb{R}^2$ through the embedding Φ. Analytically, this amounts to considering the Dirichlet space $(K, \mu, \mathcal{E}_{\mathrm{st}}, \mathcal{F}_{\mathrm{st}})$, where $\mu := \mu_{\langle h_1 \rangle} + \mu_{\langle h_2 \rangle}$ plays the role of the "Riemannian volume measure".[3] More recently, Kigami [6] has proved a two-sided Gaussian bound for the heat kernel of $(K, \mu, \mathcal{E}_{\mathrm{st}}, \mathcal{F}_{\mathrm{st}})$, and further detailed studies on its analytic and geometric properties have been done in [3, 8, 4].

As an analogy with the harmonic embedding Φ of the Sierpiński gasket described above, Teplyaev [10] proved that

each Apollonian gasket can be realized as the image of a harmonic embedding with respect to SOME energy form on the Sierpiński gasket.

Moreover, it is not difficult to see the uniqueness of such an energy form. Then it is natural to expect that we can obtain a canonical Laplacian on the Apollonian gasket which exhibits properties similar to the Laplacian on Riemannian manifolds, by using the sum of the energy measures of the (harmonic) coordinate functions as the "Riemannian volume measure" as was done in [5, 6, 3, 8, 4]. Teplyaev [10] did not give any explicit description of this energy form, and even its closedness as a non-negative definite symmetric bilinear form was not known. The author has recently clarified these fundamental issues and furthermore proved Weyl type eigenvalue asymptotics of the associated Laplacian, which are the main results of this talk and are described briefly in the next section.

3. Results: the energy form & Laplacian eigenvalue asymptotics

Let $\alpha, \beta, \gamma \in (0, \infty)$ and consider an ideal triangle formed by three circles of radii $\alpha^{-1}, \beta^{-1}, \gamma^{-1}$. Let σ^{-1} and r^{-1} be the radii of its outer and inner tangent circles, respectively, and let $K_{\alpha,\beta,\gamma}$ denote the Apollonian gasket obtained from this ideal triangle. It is well-known, and elementary to see, that

$$\sigma = (\alpha\beta + \beta\gamma + \gamma\alpha)^{1/2} \qquad \text{and} \qquad r = \alpha + \beta + \gamma + 2\sigma. \tag{3.1}$$

[2] $V_0 := \{q_1, q_2, q_3\}$ denotes the set of the three outmost vertices of K.
A function $h \in \mathcal{F}_{\mathrm{st}}$ is called V_0*-harmonic with respect to* $(\mathcal{E}_{\mathrm{st}}, \mathcal{F}_{\mathrm{st}})$ if and only if $\mathcal{E}_{\mathrm{st}}(h, u) = 0$ for any $u \in \mathcal{F}_{\mathrm{st}}$ with $u|_{V_0} = 0$, or equivalently, $\mathcal{E}_{\mathrm{st}}(h, h) = \min\{\mathcal{E}_{\mathrm{st}}(u, u) \mid u \in \mathcal{F}, u|_{V_0} = h|_{V_0}\}$.

[3] $\mu_{\langle u \rangle}$ denotes the $\mathcal{E}_{\mathrm{st}}$-energy measure of $u \in \mathcal{F}_{\mathrm{st}}$ (the energy of a function u regarded as a Borel measure on K), characterized by the equality $\int_K f \, d\mu_{\langle u \rangle} = \mathcal{E}_{\mathrm{st}}(fu, u) - \mathcal{E}_{\mathrm{st}}(f, u^2)/2$, $f \in \mathcal{F}_{\mathrm{st}}$.

An easy (but important) consequence of (3.1) is that, for each of the three ideal triangles of next level, the quadruple $(\alpha_k, \beta_k, \gamma_k, \sigma_k)$ of (the reciprocals of) the radii of its three edges and outer tangent circle is given by

$$\begin{pmatrix}\alpha_k & \beta_k & \gamma_k & \sigma_k\end{pmatrix} = \begin{pmatrix}\alpha & \beta & \gamma & \sigma\end{pmatrix} M_k \tag{3.2}$$

where $k \in \{1, 2, 3\}$ corresponds to each of the three ideal triangles and

$$M_1 := \begin{pmatrix}1 & 0 & 0 & 0\\ 1 & 1 & 0 & 1\\ 1 & 0 & 1 & 1\\ 2 & 0 & 0 & 1\end{pmatrix}, \quad M_2 := \begin{pmatrix}1 & 1 & 0 & 1\\ 0 & 1 & 0 & 0\\ 0 & 1 & 1 & 1\\ 0 & 2 & 0 & 1\end{pmatrix}, \quad M_3 := \begin{pmatrix}1 & 0 & 1 & 1\\ 0 & 1 & 1 & 1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 2 & 1\end{pmatrix}. \tag{3.3}$$

We equip (the set V_0 of the three vertices of) the initial ideal triangle with an electrical network structure with resistances $\frac{2\alpha\sigma}{\alpha^2+\sigma^2}, \frac{2\beta\sigma}{\beta^2+\sigma^2}, \frac{2\gamma\sigma}{\gamma^2+\sigma^2}$. Similarly, the set V_m of the vertices of the ideal triangles of mth level is equipped with an electrical network structure whose resistances on each ideal triangle is determined in the same way. (Note that here *we do NOT need any scaling factor for the resistances.*) Let $\mathcal{E}^{\alpha,\beta,\gamma}_m$ be the canonical Dirichlet form on V_m associated with this electrical network structure. Then some elementary computations making (careful) use of the relation (3.2)-(3.3) show:

Proposition 4. *Let $m \in \mathbb{N} \cup \{0\}$. Then*

$$\mathcal{E}^{\alpha,\beta,\gamma}_m(u, u) = \min\{\mathcal{E}^{\alpha,\beta,\gamma}_{m+1}(v, v) \mid v \in \mathbb{R}^{m+1},\ v|_{V_m} = u\} \quad \textit{for any } u \in \mathbb{R}^{V_m}, \tag{3.4}$$

$$\mathcal{E}^{\alpha,\beta,\gamma}_m(h|_{V_m}, h|_{V_m}) = \mathcal{E}^{\alpha,\beta,\gamma}_0(h|_{V_0}, h|_{V_0}) \quad \textit{for any affine function } h : \mathbb{R}^2 \to \mathbb{R}. \tag{3.5}$$

Definition 5. We set $V_* := \bigcup_{m=0}^{\infty} V_m$ and define $\mathcal{F} \subset \mathbb{R}^{V_*}$ and $\mathcal{E}^{\alpha,\beta,\gamma} : \mathcal{F} \times \mathcal{F} \to \mathbb{R}$ by[4]

$$\begin{aligned}\mathcal{F} &:= \{u \in \mathbb{R}^{V_*} \mid \lim\nolimits_{m\to\infty} \mathcal{E}^{\alpha,\beta,\gamma}_m(u|_{V_m}, u|_{V_m}) < \infty\},\\ \mathcal{E}^{\alpha,\beta,\gamma}(u, v) &:= \lim\nolimits_{m\to\infty} \mathcal{E}^{\alpha,\beta,\gamma}_m(u|_{V_m}, v|_{V_m}) \in \mathbb{R}, \qquad u, v \in \mathcal{F}.\end{aligned} \tag{3.6}$$

(3.5) implies that for any affine function $h : \mathbb{R}^2 \to \mathbb{R}$, $h|_{V_*} \in \mathcal{F}$ and $h|_{V_*}$ is V_0-harmonic with respect to $(\mathcal{E}^{\alpha,\beta,\gamma}, \mathcal{F})$, in which sense the geometry of $K_{\alpha,\beta,\gamma}$ is realized as a harmonic embedding with respect to $(\mathcal{E}^{\alpha,\beta,\gamma}, \mathcal{F})$. Moreover, we can define for each $u \in C(K_{\alpha,\beta,\gamma})$ with $u|_{V_*} \in \mathcal{F}$ its $\mathcal{E}^{\alpha,\beta,\gamma}$-energy measure $\mu^{\alpha,\beta,\gamma}_{\langle u\rangle}$, and in particular the "Riemannian volume measure" $\mu^{\alpha,\beta,\gamma} := \mu^{\alpha,\beta,\gamma}_{\langle h_x\rangle} + \mu^{\alpha,\beta,\gamma}_{\langle h_y\rangle}$, where $h_x, h_y : K_{\alpha,\beta,\gamma} \to \mathbb{R}$ denote the projections onto the two coordinate axes.

Proposition 6 (K.). *There exists a unique linear injection $\iota : \mathcal{F} \to L^2(K_{\alpha,\beta,\gamma}, \mu_{\alpha,\beta,\gamma})$ such that $\iota(u|_{V_*}) = u$ for any $u \in C(K)$ with $u|_{V_*} \in \mathcal{F}$ and*

$$\|\iota(u)\|_{L^2(K_{\alpha,\beta,\gamma},\mu_{\alpha,\beta,\gamma})}{}^2 \le c_1\sigma^{-2}\mathcal{E}^{\alpha,\beta,\gamma}(u, u) \qquad \textit{for any } u \in \mathcal{F} \textit{ with } u|_{V_0} = 0 \tag{3.7}$$

for some $c_1 \in (0, \infty)$.[5] *Under the identification of $\mathcal{F}$ with $\iota(\mathcal{F}) \subset L^2(K_{\alpha,\beta,\gamma}, \mu_{\alpha,\beta,\gamma})$, $(\mathcal{E}^{\alpha,\beta,\gamma}, \mathcal{F})$ is a strongly local regular Dirichlet form on $L^2(K_{\alpha,\beta,\gamma}, \mu^{\alpha,\beta,\gamma})$.*

Let d be the Hausdorff dimension of $K_{\alpha,\beta,\gamma}$ with respect to the Euclidean metric and let $\mathcal{H}^d$ be the d-dimensional Hausdorff measure on $\mathbb{R}^2$ with respect to the Euclidean metric. It is easy to see that d is independent of α, β, γ, and it is known (see, e.g., [9]) that $d \in (1, 2)$ and $\mathcal{H}^d(K_{\alpha,\beta,\gamma}) \in (0, \infty)$. The following is our main theorem.

[4] Identifying $K_{\alpha,\beta,\gamma}$ with the Sierpiński gasket K as topological spaces in the natural manner, we can easily verify that the domain $\mathcal{F}$ of the energy form $\mathcal{E}^{\alpha,\beta,\gamma}$ is independent of α, β, γ.

[5] Again, ι is independent of α, β, γ, and c_1 can be chosen to be independent of α, β, γ.

Theorem 7 (K.). *The non-negative self-adjoint operator on* $L^2(K_{\alpha,\beta,\gamma}, \mu^{\alpha,\beta,\gamma})$ *associated with* $(K_{\alpha,\beta,\gamma}, \mu^{\alpha,\beta,\gamma}, \mathcal{E}^{\alpha,\beta,\gamma}, \mathcal{F})$ *has discrete spectrum, and its eigenvalues* $\{\lambda_n^{\alpha,\beta,\gamma}\}_{n\in\mathbb{N}}$ *(each eigenvalue is repeated according to its multiplicity) satisfies the following asymptotics: there exists a constant* $c_2 \in (0,\infty)$ *which is independent of* α, β, γ*, such that*

$$\lim_{\lambda\to\infty} \lambda^{-d/2} \#\{n \in \mathbb{N} \mid \lambda_n^{\alpha,\beta,\gamma} \leq \lambda\} = c_2 \mathcal{H}^d(K_{\alpha,\beta,\gamma}). \tag{3.8}$$

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Obliquely reflected Brownian motion

Zhen-Qing Chen (University of Washington)

Boundary theory for one-dimensional diffusions is now well understood. Boundary theory for multi-dimensional diffusions is much richer and remains to be better understood. In this talk, we will focus on extensions of absorbing Brownian motion in simply connected planar domains. We show that the family of all obliquely reflected Brownian motions in a given domain can be characterized in two different ways, either by the field of angles of oblique reflection on the boundary or by the stationary distribution and the rate of rotation of the process about a reference point in the domain. We further show that Brownian motion with darning and excursion reflected Brownian motion can be obtained as a limit of obliquely reflected Brownian motions.

Based on joint works with M. Fukushima and with K. Burdzy, D. Marshall and K. Ramanan.

Harnack inequalities and local CLT for the polynomial lower tail random conductance model

Takashi Kumagai (Kyoto University)

We study on-diagonal heat kernel estimates and exit time estimates for continuous time random walks (CTRWs) among i.i.d. random conductances with a power-law tail near zero. For two types of natural CTRWs, we give optimal exponents of the tail such that the behaviors are 'standard' (i.e. similar to the random walk on the Euclidean space) above the exponents. We then establish the local CLT, parabolic Harnack inequalities and Gaussian bounds for the heat kernel of the CTRWs. Some of the arguments are robust and applicable for random walks on general graphs.

The talk is based on a joint paper with O. Boukhadra (Constantine) and P. Mathieu (Marseille), to appear in the special issue for Kiyosi Itô, J. Math. Soc. Japan, 2015.

Several limit theorems for Markov processes via convergence of excursions

Kouji Yano (Kyoto University)

Itô's excursion theory ([2], [1]) is a powerful tool for dealing with a sample path of a Markov process. If we decompose a sample path into excursions, then we obtain the *excursion measure* as the characteristic measure of the Poisson point process formed by the excursions. Conversely, if we run a Poisson point process with a given characteristic measure, then we can piece a sample path together from the excursions.

We may expect that convergence of a Markov process should be implied by convergence of its excursions, which should be implied by convergence of its excursion measure. In order to realize this idea, we must make clear in what sense the excursions converge or does the excursion measure.

In this talk, we explain several limit theorems for Markov processes ([3], [4]), which can be proved via convergence of excursions.

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Limit theorems for sums of polynomials of random variables with fat tails

S. R. Srinivasa Varadhan (Courant Institute)

We consider limit theorems for sums of the form

$$S_n = \sum_{i=0}^{n-1} f(X_{i+1}, X_{i+2}, \dots, X_{i+k})$$

where $f = f(x_1, x_2, \dots, x_k)$ is a polynomial in k variables and $\{X_i\}$ is a sequence of independent identically distributed random variables with fat tails. In particular we assume that for $\ell >> 1$

$$P[\pm X_i \geq \ell] \simeq r_\pm \ell^{-\alpha} (\log \ell)^\beta$$

with $0 < \alpha < 2$, $\beta \geq 0$, $r_\pm \geq 0$ and $r_+ + r_- > 0$. We show, in this joint work with Yuri Kifer, that for suitable normalizing constants a_n and $b_n > 0$, the distribution of the process $Z_n(t) = \frac{S[nt] - a_n t}{b_n}$ converges in Skorohod J_1 topology to that of a stable process.

Coupling by reflection of Brownian motions on metric measure spaces with a lower Ricci curvature bound

Kazumasa Kuwada (Tokyo Institute of Technology)

In this talk, I will construct a certain coupling of two Brownian motions on "Riemannian" metric measure spaces with a lower Ricci curvature bound by $K \in \mathbb{R}$ (more precisely, $\mathsf{RCD}(K, \infty)$ spaces) satisfying the following property: The distribution of the coupling time of this coupling is estimated by that of a 1-dimensional Ornstein-Uhlenbeck process. As a result, when $K \geq 0$, we can show that the coupled particles meet in a finite time with probability one.

$\mathsf{RCD}(K, \infty)$ metric measure spaces are defined in terms of a (displacement) convexity of the relative entropy functional on the space of probability measures equipped with L^2-Wasserstein distance, together with the so-called "infinitesimally Hilbertian" structure [1, 2]. This is an extension of Riemannian manifolds and in fact a measured Gromov-Hausdorff limit of Riemannian manifolds with a uniform lower Ricci curvature bounds belongs to this class of spaces. With the aid of the infinitesimally Hilbertian structure, the heat flow associated with a canonically defined energy functional on these spaces is linear and there is a "Brownian motion" corresponding to the heat flow. On these spaces, properties of heat flow are intensively studied and especially the so-called "Bakry-Émery's curvature-dimension condition" holds. This is indeed an alternative formulation of a lower Ricci curvature bound in abstract spaces. Nevertheless, properties of sample paths of the Brownian motion in connection with the curvature bound were not studied very much. As far as we know, the only result in this direction is a construction of "a coupling by parallel transport" from the corresponding estimate of the L^∞-Wasserstein distance for heat distributions by K.-Th. Sturm [7]. On Riemannian manifolds, we can make a connection between the behavior of (coupled) Brownian motions and Ricci curvature by (stochastic) differential calculus, but there is no differential structure on our space in the usual sense and hence there seems no direct way to apply the same approach on our spaces.

For the proof of the construction of our coupling, we show a monotonicity of an optimal transportation cost for heat distributions whose cost function is a time-dependent and concave function of the distance. Once we obtain it, the construction of the corresponding coupling of Brownian motions will be done in a similar way as the one by Sturm. We have shown this monotonicity on Riemannian manifolds by employing a coupling by reflection constructed by means of stochastic differential geometry [6]. Though the same method does not seem to work on our spaces, we are able to provide an alternative proof based on functional inequalities including the (reverse) Gaussian isoperimetric inequalities for the heat semigroup [3, 4]. Here the essential idea comes from a method to show an L^p-Wasserstein control for heat distributions from an L^q-gradient estimate for the heat semigroup [5].

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Optimal transport, Brownian motion, and super-Ricci flow for metric measure spaces

Karl-Theodor Sturm (University of Bonn)

We study heat equation and Brownian motion on time-dependent metric measure spaces with particular emphasis on mm-spaces which evolve as super-Ricci flows. A time-dependent family of Riemannian manifolds is a super-Ricci flow if $2\mathrm{Ric} + \partial_t g \geq 0$. This includes all static manifolds of nonnegative Ricci curvature as well as all solutions to the Ricci flow equation.

We characterize super-Ricci flows of metric measure spaces in terms of coupling properties of (backward) Brownian motions, gradient estimates for the (forward) heat equation, as well as dynamical convexity of the Boltzmann entropy on the Wasserstein space. And we prove stability and compactness of super-Ricci flows under mGH-limits.

Schemes and tails of infinite-dimensional stochastic equations with symmetry

Hirofumi Osada (Kyushu University)

I talk about a new method to prove the existence and pathwise uniqueness of strong solutions of stochastic equations for infinite particle systems, which I call "schemes and tails method". The typical example that the method can apply to is interacting Brownian motions in infinite dimensions related to random matrix theory such as sine, Airy, Bessel, tacnode, and Ginibre interacting Brownian motions.

The classic Itô theory on stochastic differential equations (SDEs) gives pathwise unique strong solutions under the Lipschitz continuity of coefficients. Itô scheme uses Picard approximation, hence a kind of Lipschitz continuity of coefficients is indispensable. This yields the difficulty of the direct application of Itô theory to infinite dimensions because we can no longer expect the Lipschitz continuity of coefficients for infinite-dimensional SDEs with symmetry.

We introduce plural schemes of Dirichlet forms and SDEs made from given infinitely dimensional SDEs, and various kinds of tails of objects. Tails are regarded as boundary conditions of the equations. We give new formulations of objects concerning on infinite-dimensional SDEs based on schemes. These are essentially equivalent to and much more tamed than the original ones.

Our method can be applied to various infinite-dimensional stochastic equations with symmetry. Moreover, applying to interacting Brownian motions, we have many spin-offs of our method. I will also explain about this if I would have a time.

The talk is based on the joint works with Hideki Tanemura and Yosuke Kawamoto.