On some interesting connections between deterministic and stochastic differential equations

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Abstract

Relations will be discussed between the ordinary differential equation

$$x'(t) = B(x(t))$$

and the stochastic differential equation

$$dx_t = dw_t + B(x_t)dt,$$

corresponding to the same continuous vector field B on a finite or infinite dimensional space, where in the stochastic case w_t is a random process (typically, a Wiener process). The discussion will mainly concern existence and uniqueness theorems. Examples will be considered when one of the two equations is uniquely solvable whereas the other one has no solutions or a solution is not unique. Principal positive results in finite and infinite dimensions will be presented. An interesting recent direction in the study of equations of both types deals with vector fields B which possess low regularity such as the membership in the first Sobolev class (which guarantees continuity, but not Lipschitzness). In addition, we shall consider the solvability of the stochastic equations

$$dx_t^\varepsilon = \varepsilon dw_t + B(x_t^\varepsilon)dt$$

with a small parameter ε and the behaviour of solutions as $\varepsilon \to 0$. In the infinite dimensional case, even linear equations exhibit nontrivial effects. Several difficult

recent positive results on linear equations will be mentioned and interesting examples will be described. Finally, some open problems in this area will be discussed. Prerequisites from the theory of random processes will be presented.