RATIONAL CURVES OF MINIMUM DEGREE ON PROJECTIVE VARIETIES AND APPLICATIONS

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In classification theory of higher dimensional varieties, the analysis of rational curves on a given projective variety plays a very crucial rôle. In this talk, I will stress the importance of rational curves of minimum degree in the analysis of uniruled varieties, Fano varieties in particular.

Our main result is the following very natural one:

Theorem. *Let X be a normal complex projective variety, $x \in X$ a point, and $\mathcal{C} = \{C_s \subset X\}_{s \in S}$ a family of irreducible rational curves parametrized by an irreducible variety S. Assume that the C_s have minimum degree among all the rational curves on X and that every C_s passes through x. Then every member C_s is smooth at x provided x is general on X.*

The minimality of the degree is essential in this theorem. For instance let X be $\mathbb{P}^n$ and $\{C_s\}$ be the set rational curves of degree d passing through x. Then every $\{C_s\}$ is smooth at x if and only if $d = 1$.

One of the implications of this theormem is this:

Theorem. *Let $\mathcal{C} = \{C_s \subset X\}$ be a family of rational curves of minimum degree through x as in the previous theorem. Assume that $\{C_s\}$ is effectively parametrized by S (in other words, S is a subset of the Chow scheme of X). Then $\mathcal{C} \subset S \times X$ is birational to its natural image in X.*

This, in turn, implies a new characterization of projective spaces.

Theorem. *An n-dimensional normal projective variety X over $\mathbb{C}$ is isomorphic to $\mathbb{P}^n$ if and only if any two points on X can be joined by a single rational curve of minimum degree on X.*

All the characterizations of projective spaces known so far follows from ours.

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Two applications to complex symplectic geometry would be worth noting: namely the structure of Lagrangian fibrations and complex symplectic resolutions.