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タイトル TITLE	**q-characters of tensor products and related questions from representation theory**		
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In my talk I will discuss q-characters of tensor products of representations of Lie algebras and some similar problems.

Let me describe situation in the simplest case. Consider abelian Lie algebra with basis $e(i)$, $i = 0, 1, 2, \ldots$. Such algebra has natural "evaluation" representation of dimension 2, $V(k)$ with basis $v(0)$ and $v(1)$ and the action is given by :

$$e(j)v(0) = (k^j)v(1), \quad e(j)v(1) = 0.$$

For any set $k^1, k^2, \ldots, k^n$ we have representation $W(k^1, \ldots, k^n)$ which is tensor product of $V(k^j)$. Note that if all k^i are distinct, representation W has a natural cyclic vector — product of $v(0)$. But if at least two k^j coincide, such vector is not cyclic. But it is easy to see that there exists another family of representations of the same Lie algebra — $\widehat{W}(k^1, \ldots, k^n)$ with properties : (a) if all k^i are pairwise distinct, then $\widehat{W}$ is isomorphic to W, (b) $\widehat{W}$ is cyclic for any point of configuration space.

Representation $\widehat{W}(0, \ldots, 0)$ is graded module where operator $e(i)$ is homogeneous of degree i.

This simple constuction is connected with representation theory of affine Lie algebra $\widehat{\mathfrak{sl}}_2$. Namely, in the integrable representations of Kac-Moody algebras there are some remarkable subspaces — so called Demazure subspaces, or Demazure modules. Actually our representation $\widehat{W}(0, \ldots, 0)$ coincide with some Demazure module for

$\widehat{\mathfrak{s}l}_2$. It means that starting with rather simple constructions for abelian Lie algebra with basis $e(i)$ we can recover $\widehat{\mathfrak{s}l}_2$ and $\widehat{\mathfrak{s}l}_2$ integrable representations.

Representations of any affine Lie algebra can be constructed by similar way. What is more interesting, that in some cases it is possible to recover new vertex operator algebras starting with very simple things.

In my talk I will present some examples and try to explain that we have in such constructions interesting geometry and rich combinatorics.