A Conjecture in the Geometry of Polynomials

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Abstract

This lecture is about a conjecture I formulated almost half a century ago. It is strikingly simple to state and can be explained to high-school students: Given a polynomial p of degree $n \geq 2$ with all zeros $z_1, z_2, \ldots, z_n$ in a closed disk with a radius r, for each zero z_k at least one zero of the derivative p' is in the closed disk with radius r and center z_k. Since then a number of mathematicians have been interested in this problem, se [2] p. 224 -237. As for now, the conjecture is proved by Brown and Xiang [1] for $n \leq 8$. During the years 2002 and 2003, G. Schmieder presented in arXiv.math 8 consecutive proofs of this conjecture, which are not correct. In a forthcoming paper, J. Borcea shows the weak points in the proofs of G. Schmieder and demonstrate that the variational method used by Schmieder is not applicable to this conjecture. My aim in the present lecture is to give a short overview to the work in more than 100 related papers that are known to me, see [3].

References

[1] Brown, J. E. and G. Xiang: *Proof of the Sendov conjecture for polynomials of degree at most eight*, J. Math. Anal. Appl. **232**, # 2 (1999), 272–292.

[2] Rahman , Q. I. and G. Schmeisser: Analytic Theory of Polynomials, Oxford Univ. Press Inc., New York, 2002.

[3] Sendov, Bl.: *Hausdorff Geometry of Polynomials*, East Journal on Approximation, **7** # 2 (2001), 1 - 56.