

Working Seminar on Integral Geometry

Date: October 8 (Fri), 2004, 10:30–12:00

Place: RIMS Room 402

Speaker: Bernhard Krötz (RIMS)

Title: Working Seminar on Integral Geometry I

Abstract:

The objective of this seminar is to provide an introduction to some recent trends in integral geometry on real Lie groups $G_{\mathbb{R}}$ and, more generally, on symmetric spaces $X_{\mathbb{R}} = G_{\mathbb{R}}/H_{\mathbb{R}}$. To be more specific, we want to discuss the possibilities for a suitable horospherical Radon transform on the aforementioned spaces.

The first obstacle one encounters in this context is the right notion of horospheres on $X_{\mathbb{R}}$. The naive approach, namely to define horospheres as orbits of certain unipotent subgroups of $G_{\mathbb{R}}$ is not fruitful – for example they do not exist for compact spaces. It was suggested by S. Gindikin that one should consider *complex horospheres without real points*, i.e. orbits of appropriate complex unipotent subgroups of the complexification $G_{\mathbb{C}}$ of $G_{\mathbb{R}}$ on $X_{\mathbb{C}} = G_{\mathbb{C}}/H_{\mathbb{C}}$ which do not intersect $X_{\mathbb{R}}$. Having the right notion of horospheres one then should be able to define a *horospherical Cauchy-Radon transform* for symmetric spaces. The final upshot would lie a new understanding of the L^2 -spectrum of $X_{\mathbb{R}}$, in particular the wave front sets of the arising series of representations should become much clearer.

Literature on the subject is rather scarce. There are some notes of Gindikin on compact symmetric spaces and several articles by him for the example $G_{\mathbb{R}} = \mathrm{Sl}(2, \mathbb{R})$. One of our main goals is to make these articles, often written in classical style, available to a wider audience. For us this will mean that after a careful and thorough discussion of the initial approach, we will comment on alternative points of view. So it will be a “hands on” seminar with no hiding of details and the possibility of unraveling hidden truths.

We shall start with a discussion of the compact case where the picture is complete and very beautiful.

References

- [1] S. Gindikin, *Integral Geometry on $\mathrm{Sl}(2, \mathbb{R})$* , Math. Res. Letters (2000), 417–432.
- [2] —, An analytic separation of series of representations for $\mathrm{Sl}(2, \mathbb{R})$, Moscow Math. J. **2** (2002), no 4., 1–11.
- [3] —, *Horospherical Cauchy-Radon transform on compact symmetric spaces*, preprint 2004.
- [4] S. Gindikin, B. Krötz and G. Ólafsson, *Horospherical model for the holomorphic discrete series*, in preparation.
- [5] —, *Holomorphic aspects of the Radon transform on Riemannian symmetric spaces*, in preparation.