Working Seminar on Integral Geometry

Date:	November 5 (Fri) 10:30–12:00
Place:	RIMS Room 402
Speaker:	Bernhard Krötz (RIMS)
Title:	Working Seminar on Integral Geometry 5

Abstract:

For a real symmetric space $X_{\mathbb{R}}$ the kernel of the real horospherical transform consist of all series of representations except the most-continuous one. In case the real horospherical transform is faithful (for example if $X_{\mathbb{R}}$ is a complex group or $X_{\mathbb{R}}$ is non-compact Riemannian) its inversion is essentially equivalent to the Plancherel formula.

To obtain a faithful horospherical transform Gindikin suggested to use complex horospheres in $X_{\mathbb{C}}$ without real points (they do not intersect $X_{\mathbb{R}}$) and to replace the real horospherical transform (i.e. integration over real horospheres) with a Cauchy type transform (with singularities of the Cauchy kernel on the horospheres without real points). In our first four lectures we have seen that this method works well for compact symmetric spaces.

The objective of the next two (or three) talks is to discuss affine symmetric spaces of Hermitian type. We will define a complex horospherical transform and show that it has no kernel on the holomorphic discrete series. In addition, on this part of the spectrum (i.e. hol. disc. series) there is an inversion formula in complete analogy to the compact case.

Instead of giving an account of the theory in full generality we decided to discuss the most basic case $X_{\mathbb{R}} = SL(2,\mathbb{R})/SO(1,1)$ where the results are already new and interesting. We intend to provide all details for this case; of particular interest might be our approach to spherical harmonic analysis for (unitary) lowest weight modules of $G = SL(2,\mathbb{R})$.

The results are taken from the recently finished paper Horospherical model for the holomorphic discrete series and the horospherical Cauchy transform (authored by Gindikin, Ólafsson and myself) and my article Formal dimension for semisimple symmetric spaces, Compositio Math. 125 (2001), no. 2, 155–191.