Topology from the Differentiable Viewpoint

Exercise 1.
Show that there are smooth maps from the open ball

$$\left\{ (x_1, \ldots, x_n) \in \mathbb{R}^n \;\middle|\; x_1^2 + \cdots + x_n^2 < 1 \right\}$$

to itself without any fixed points. Where does Hirsch's proof of the Brouwer Fixed Point Theorem fail?

Exercise 2.
Use the Intermediate Value Theorem to prove the Brouwer Fixed Point Theorem for continuous maps from the closed interval $[-1, 1]$ to itself.

Exercise 3.
Let A be a real $n \times n$ matrix with only non-negative entries. Show that A has a real non-negative eigenvalue $\lambda \geq 0$, i.e. there is a non-zero vector $x \in \mathbb{R}^n$ with $Ax = \lambda x$.

Hint. You may assume that 0 is not an eigenvalue of A, i.e. A is invertible. Then, consider the standard simplex

$$\Delta_{n-1} \;=\; \left\{ x \in \mathbb{R}^n \;\middle|\; x_i \geq 0,\; |x| = x_1 + \cdots + x_n = 1 \right\}$$

and the map $\Delta_{n-1} \to \Delta_{n-1}$, $x \mapsto Ax \big/ |Ax|$. Conclude that Δ_{n-1} is homeomorphic to the closed ball D^{n-1} and therefore the map has a fixed point.

Milnor: *Topology from the Differentiable Viewpoint* §1·§2 用語集

boundary	境界.
commutative	可換な.
complement	補集合. 補空間. orthogonal — 直交補空間
constant	定数（の）.
continuous	連続な.
coordinate	座標.
critical	臨界の. — point 臨界点 — value 臨界値
dense	稠密な.
derivative	微分. partial — 偏微分
diffeomorphism	微分同相.
dimension	次元.
euclidean space	ユークリッド空間.
fixed point	不動点.
homogeneous	斉次の.
homeomorphism	同相. 位相同型.
hyperplace	超平面.
image	像.
interior	内包.
manifold	多様体.
map, mapping	写像. 関数. identity — 恒等写像 inclusion — 包含写像 linear — 線形写像
measure	測度. Lebesgue — ルベーグ測度
neighborhood	近傍.
normal vector	法ベクトル.
open set	開集合.
polynomial	多項式. complex — 複素多項式
regular	正則な. — point 正則点 — value 正則値
singular	特異な.
smooth	なめらかな.
tangent space	接空間.
tangent vector	接ベクトル.
topology	位相幾何. トポロジー. differential — 微分位相幾何