

Memoryful Go with recursion

室屋 晃子 (東大)

Memoryful Gol [Hoshino, —, Hasuo '14]

effectful
terms

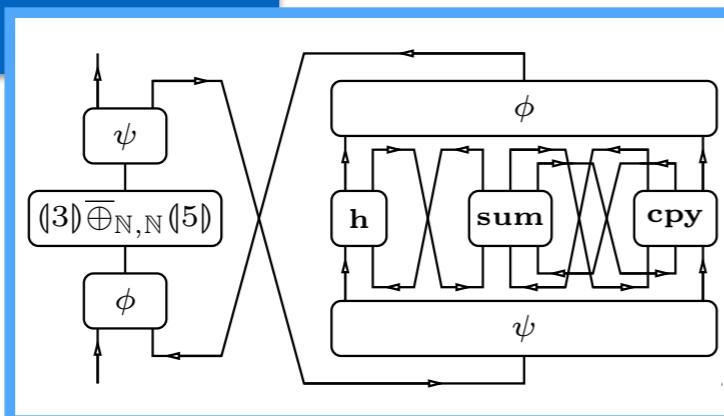
$(\lambda x : \text{nat}. x + x) (3 \sqcup 5) : \text{nat}$



translation

- based on Geometry of Interaction
- via coalgebraic component calculus

transducers



Geometry of Interaction (Gol)

- semantics of { linear logic proof [Girard '89],
functional programming}
- token machine presentation [Mackie '95]



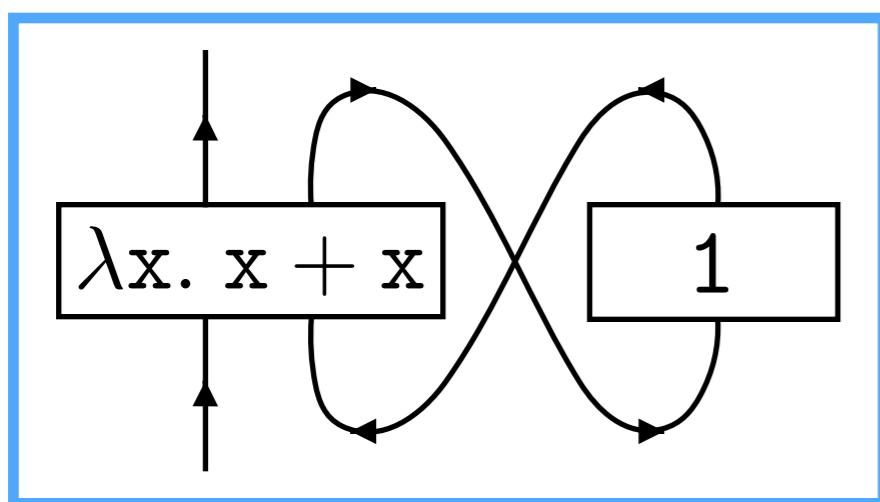
“Gol implementation”

compilation techniques and implementations

[Mackie '95] [Pinto '01] [Ghica '07]

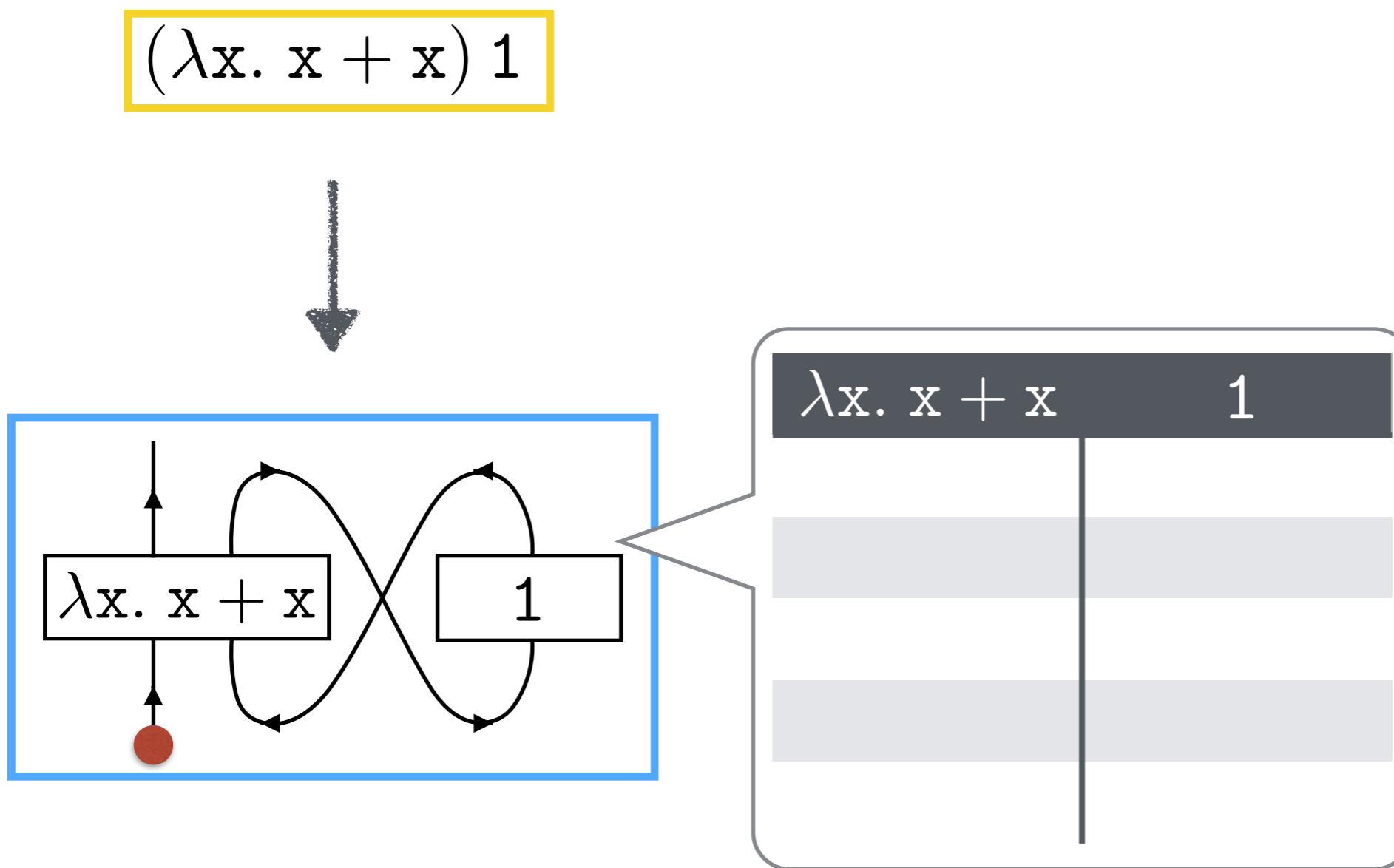
Geometry of Interaction (Gol)

- token machine semantics

$$(\lambda x. x + x) \ 1$$


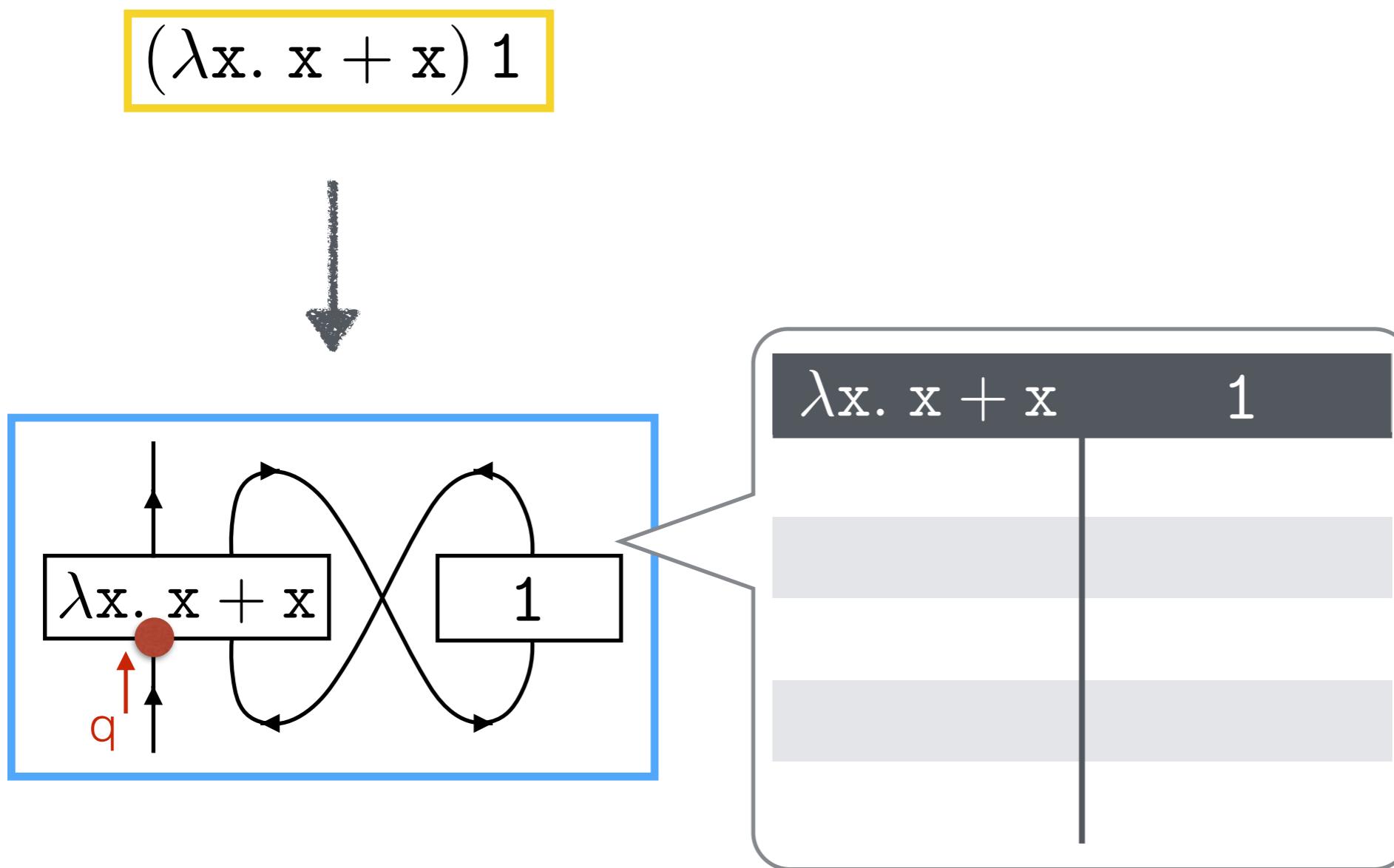
Geometry of Interaction (Goi)

- token machine semantics



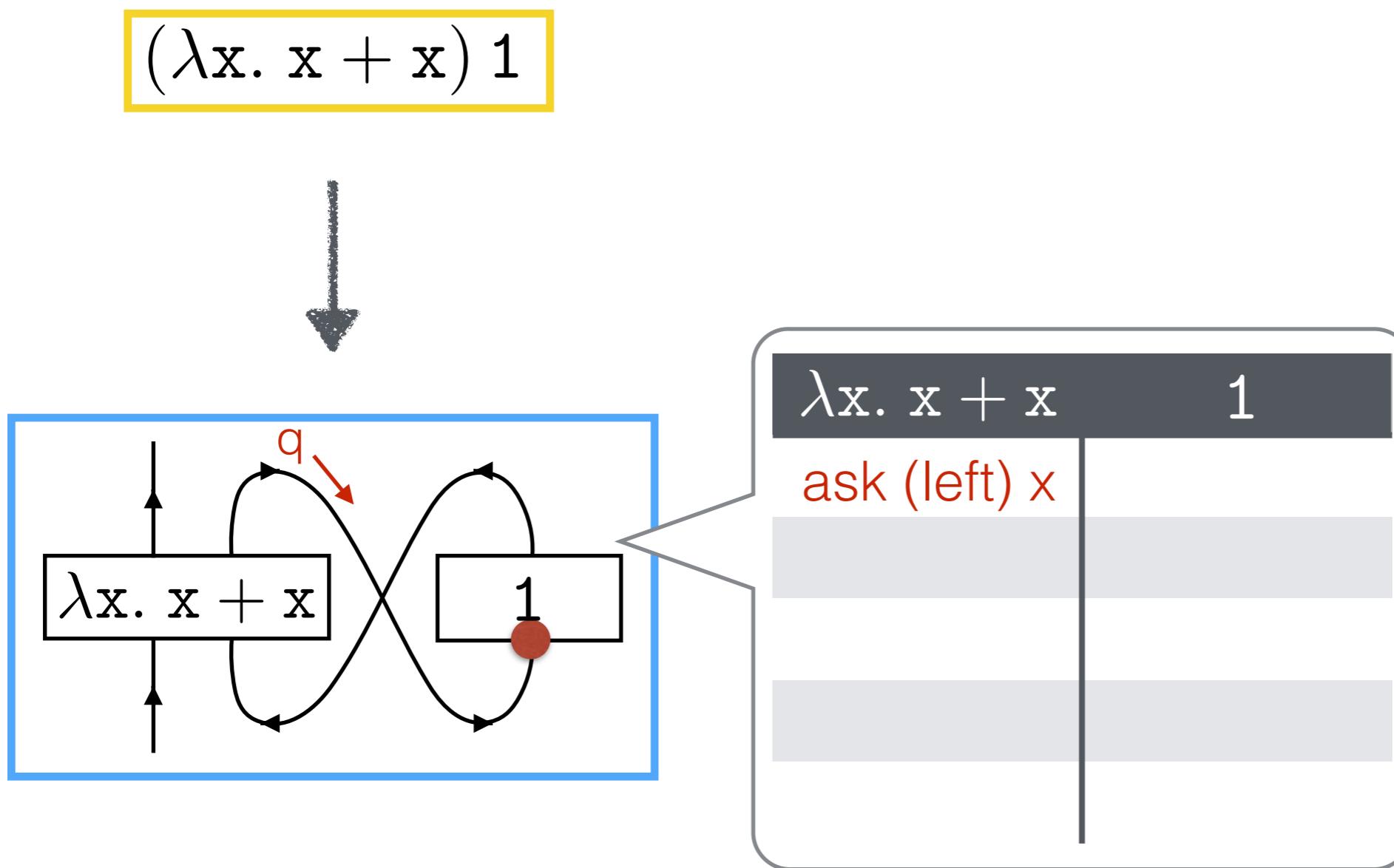
Geometry of Interaction (Goi)

- token machine semantics



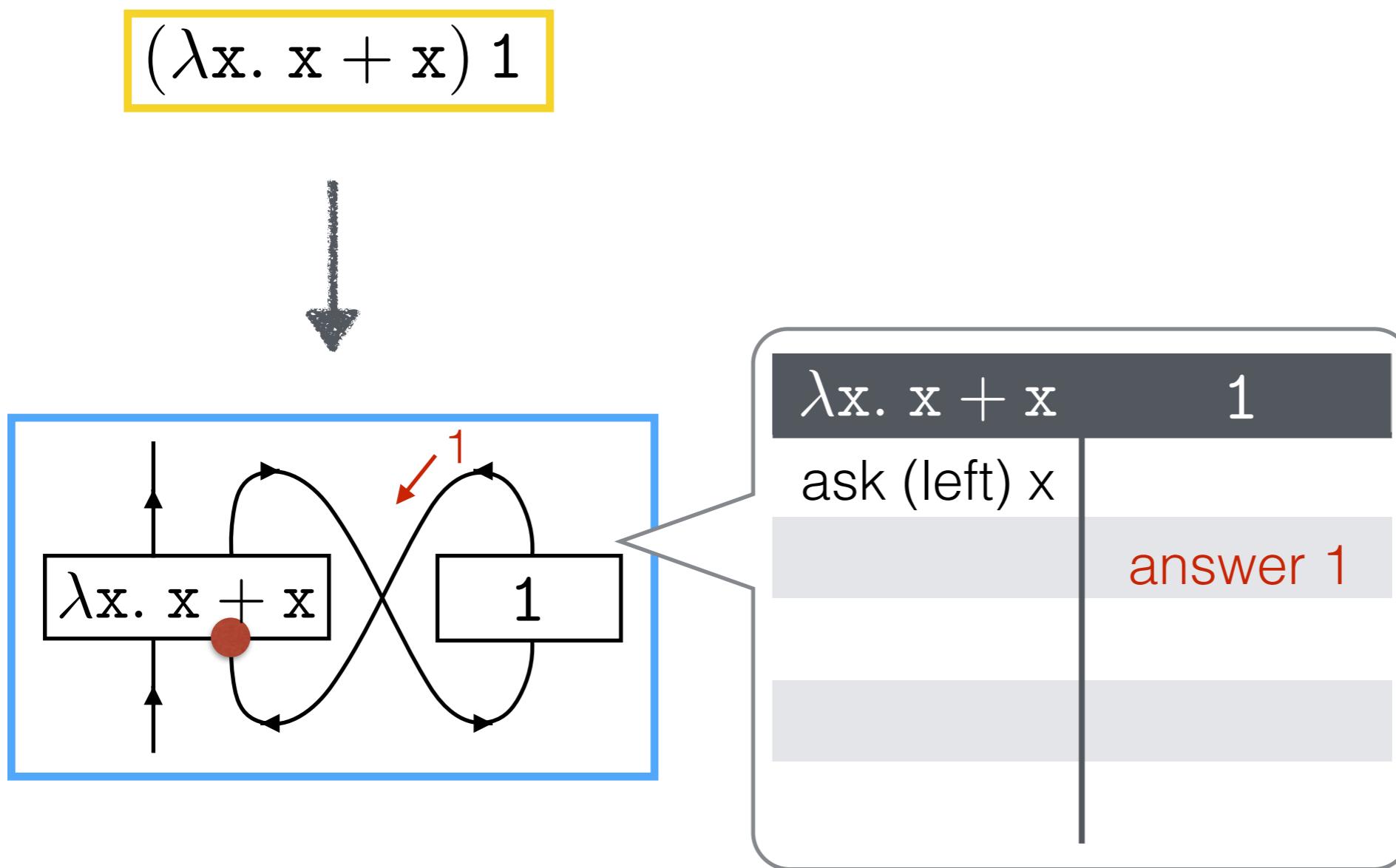
Geometry of Interaction (Gol)

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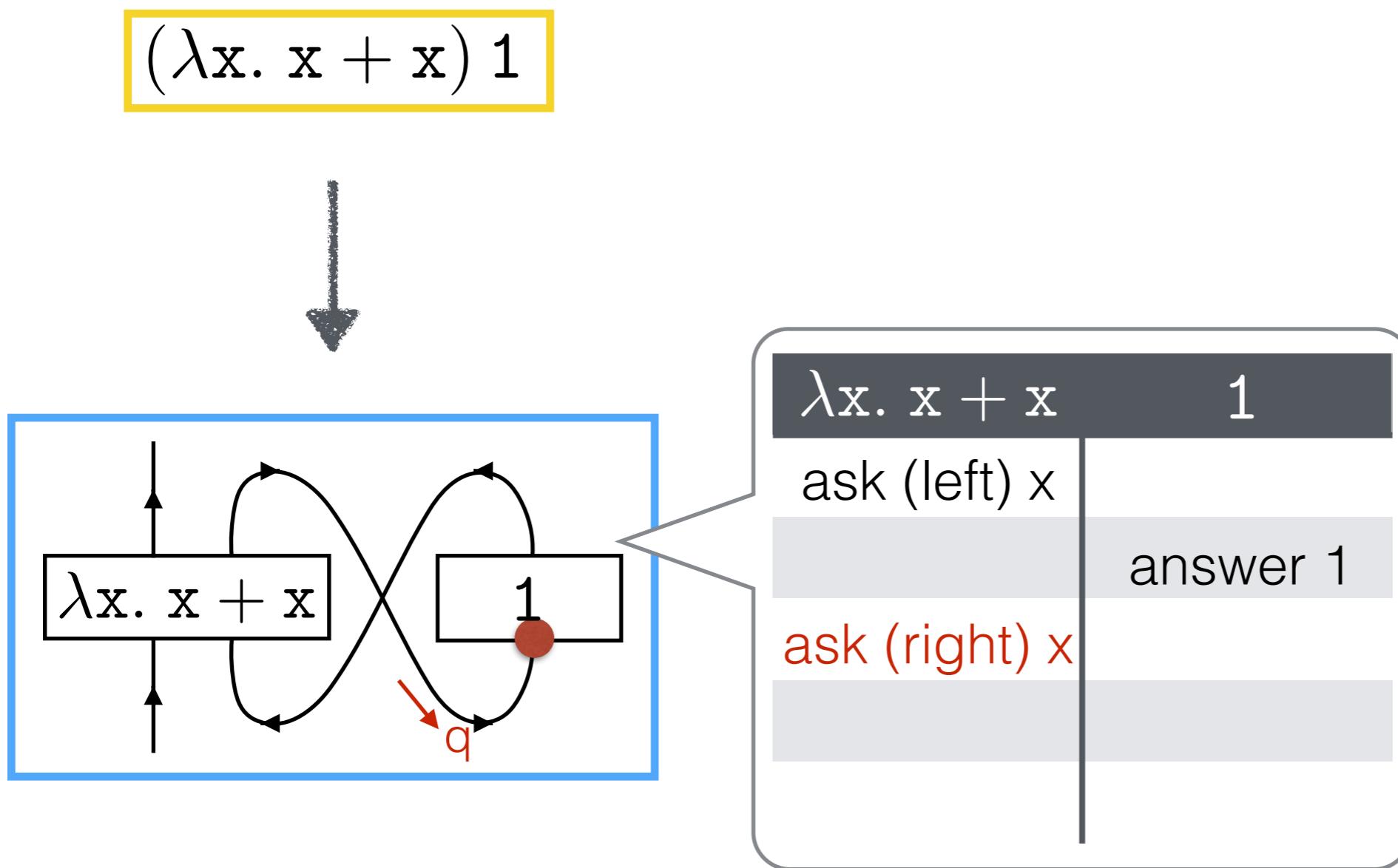
Geometry of Interaction (Gol)

- token machine semantics



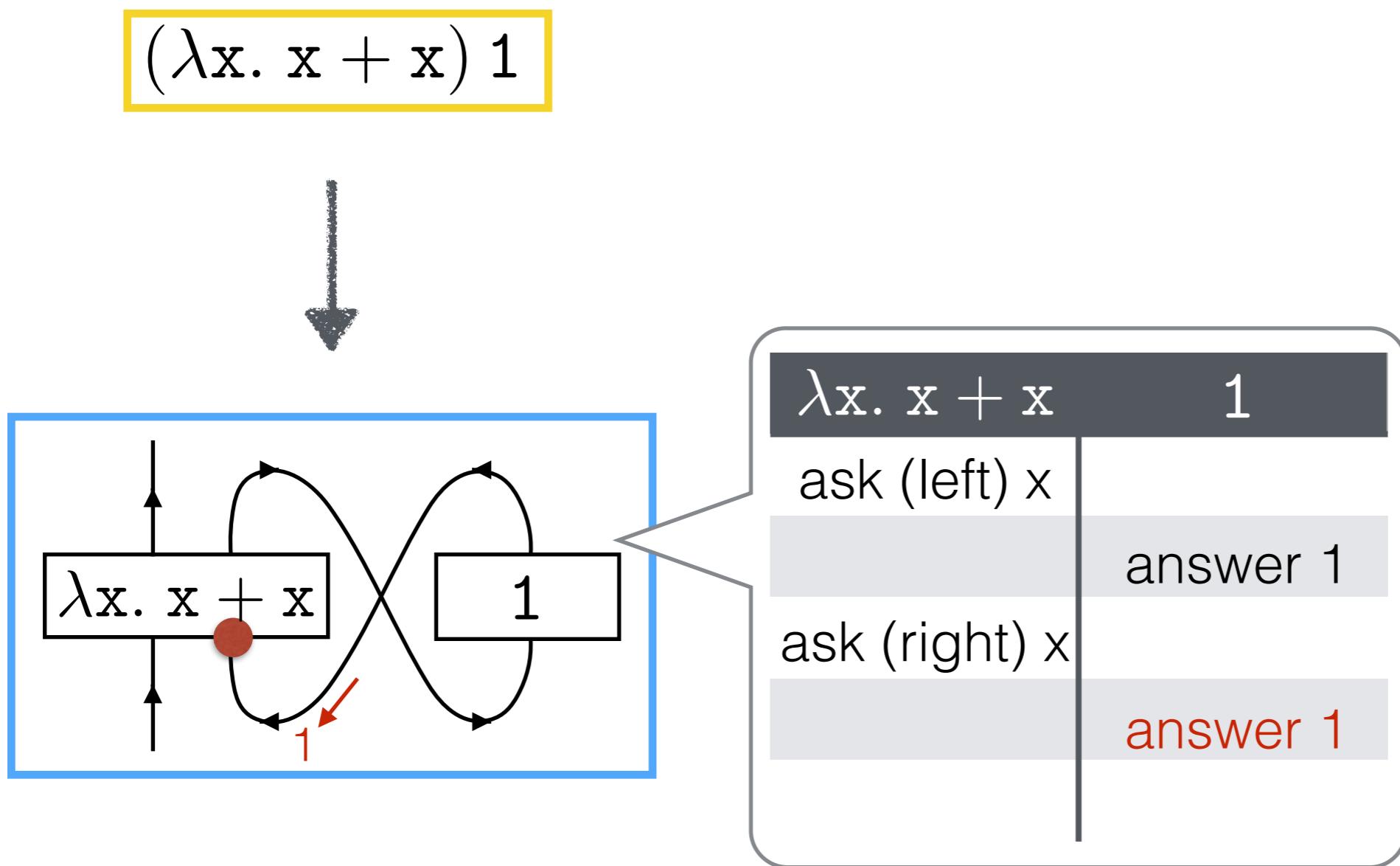
Geometry of Interaction (Gol)

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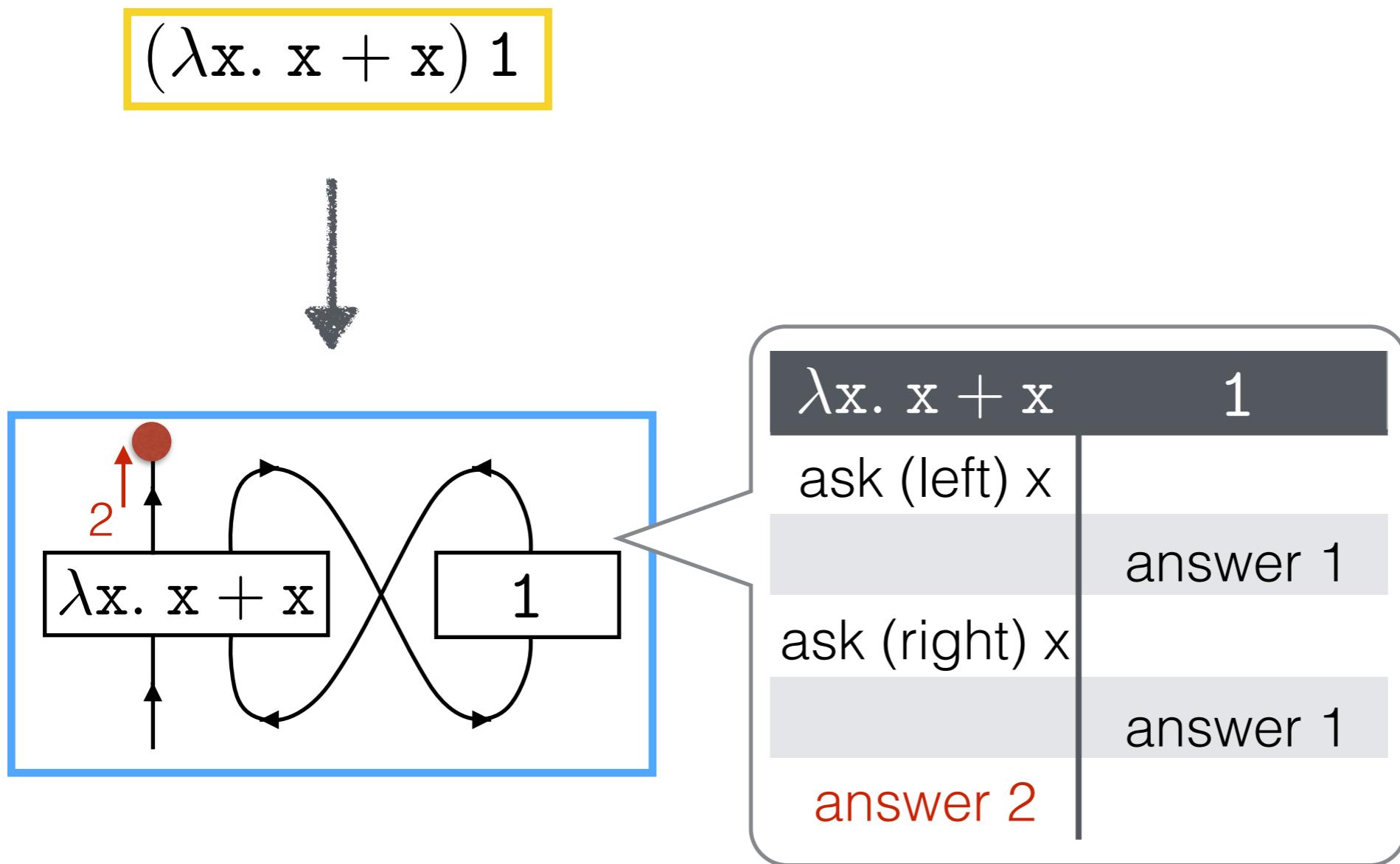
Geometry of Interaction (Gol)

- token machine semantics



Geometry of Interaction (Gol)

- token machine semantics



Memoryful Gol — Input

effectful
terms



transducers

CBV λ -terms with algebraic effects

algebraic operations [Plotkin, Power '01]

- nondeterministic choice $M \sqcup N$
- probabilistic choice $M \sqcup_p N$
- actions on global state
 - lookup_l(v : Val. M)
 - update_{l,v}(M)

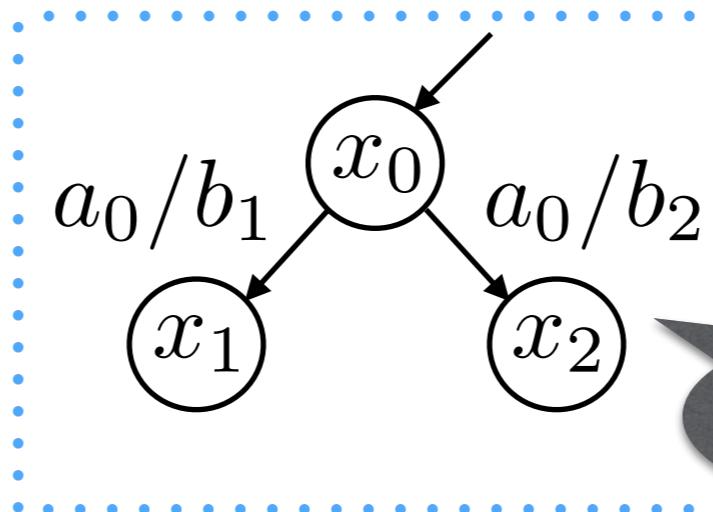
Memoryful Gol — Output

effectful
terms

transducers

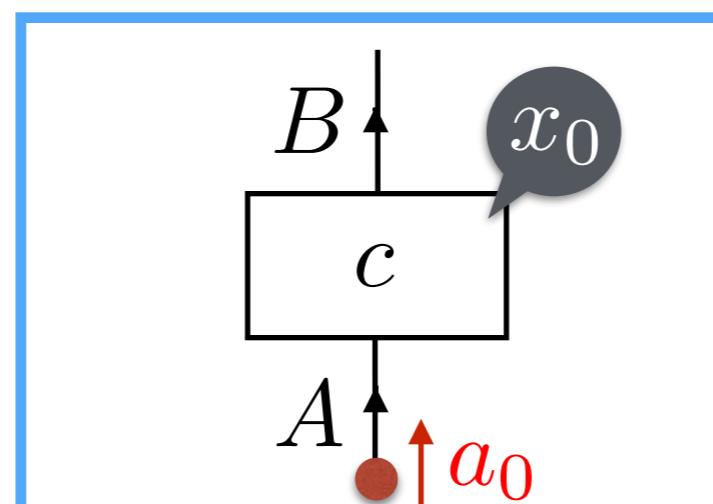
stream transducers (Mealy machines)

$$(X, c: X \times A \rightarrow T(X \times B), x_0 \in X): A \rightarrow B$$



automaton style

$$T = \mathcal{P}$$



string diagram style

Memoryful Gol — Output

effectful
terms

transducers

stream transducers (Mealy machines)

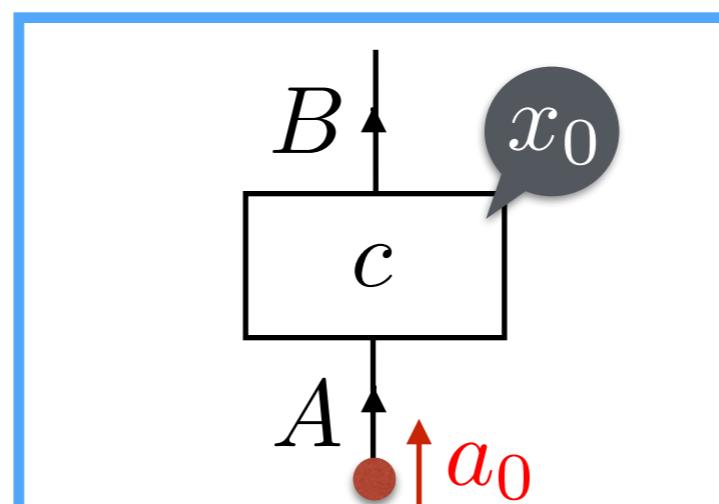
$$(X, c: X \times A \rightarrow T(X \times B), x_0 \in X): A \rightarrow B$$

Res(T)

objects: sets

arrows: transducers modulo behavioral equivalence

$$[(X, c: X \times A \rightarrow T(X \times B), x_0 \in X)]_{\sim}: A \rightarrow B$$



automaton style

string diagram style

Memoryful Gol — Output

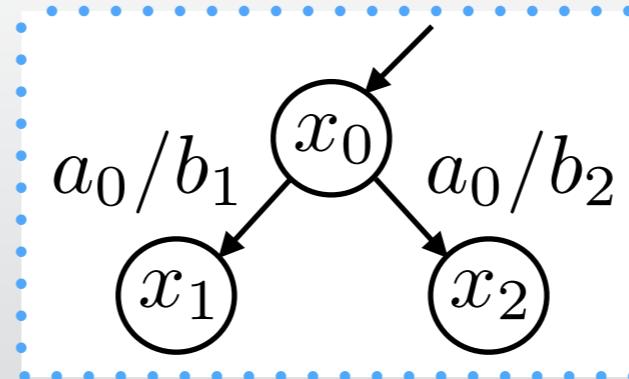
effectful
terms

stream transducers (Mealy machines)

$$(X, c: X \times A \rightarrow \mathcal{T}(X \times B), x_0 \in X): A \rightarrow B$$

transducers

$$T = \mathcal{P} \quad (x_0, a_0) \longmapsto \{(x_1, b_1), (x_2, b_2)\}$$

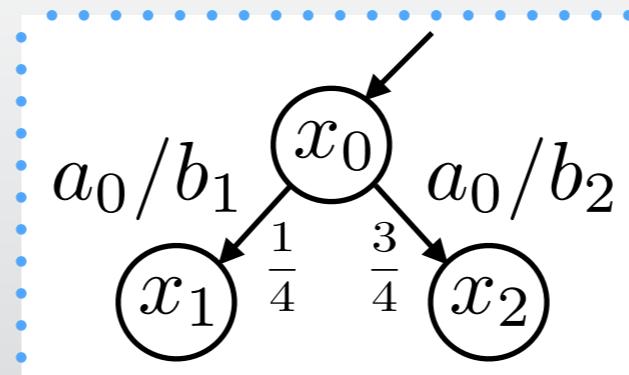


nondeterministic
computation

$$T = \mathcal{S} = (1 + (-) \times S)^S$$

computation with
global state

$$T = \mathcal{D} \quad (x_0, a_0) \longmapsto \left[\begin{array}{l} (x_1, b_1) \mapsto 1/4, \\ (x_2, b_2) \mapsto 3/4, \end{array} \right]$$



probabilistic
computation

Memoryful Gol — Translation

effectful
terms

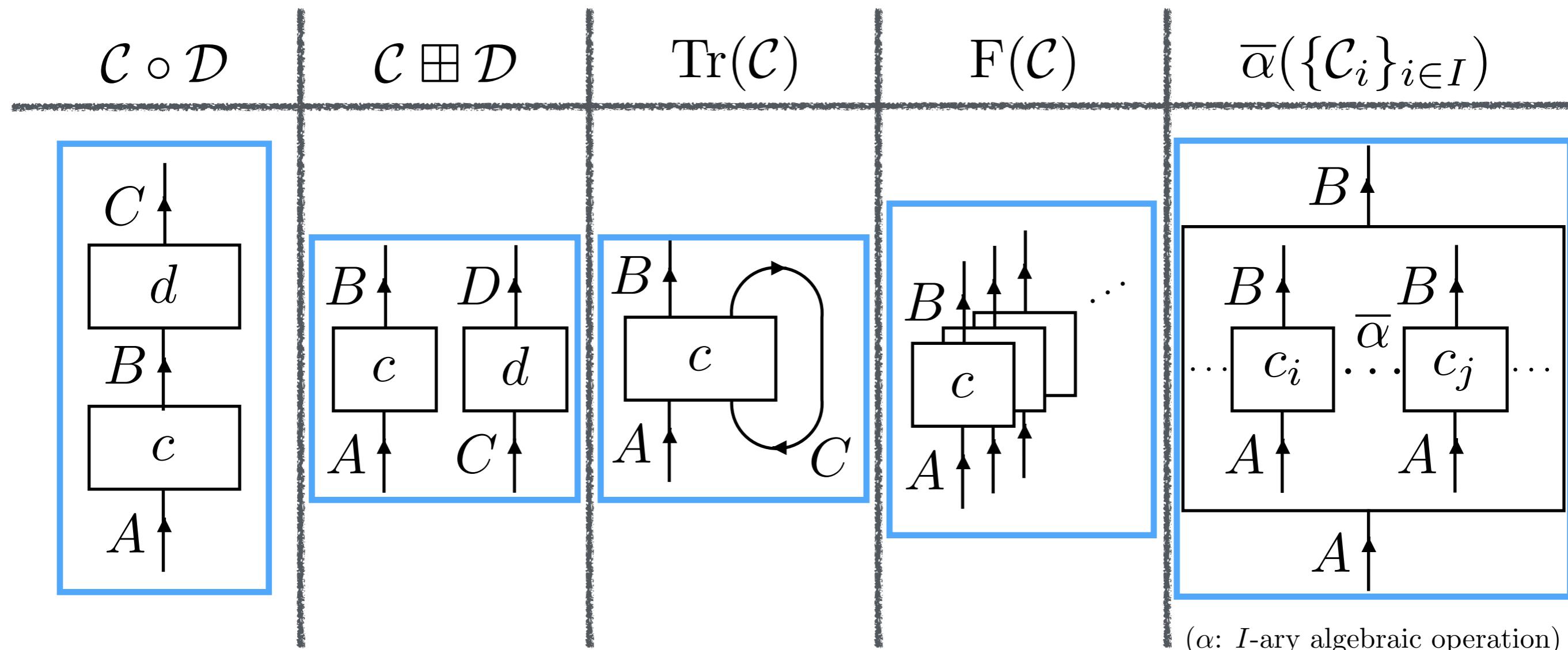


transducers

- idea: resumptions + categorical Gol
[Abramsky, Haghverdi, Scott '02]
- use **coalgebraic component calculus**
[Barbosa '03] [Hasuo, Jacobs '11]
 - composition operations for software components
 - (many-sorted) process calculus

Memoryful Gol — Translation

Def. (component calculus)



Memoryful Gol — Translation

Def. (component calculus)

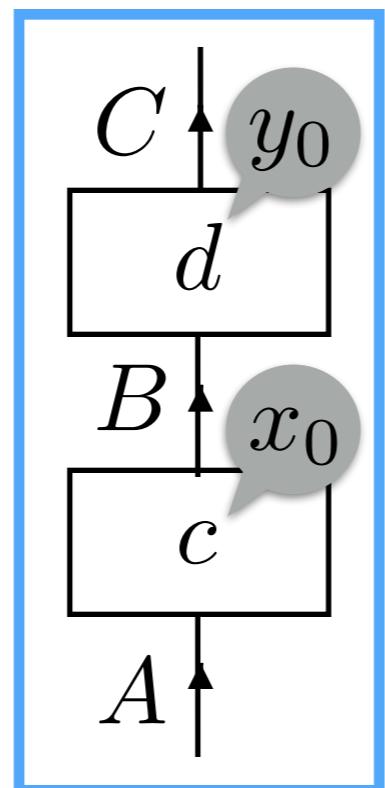
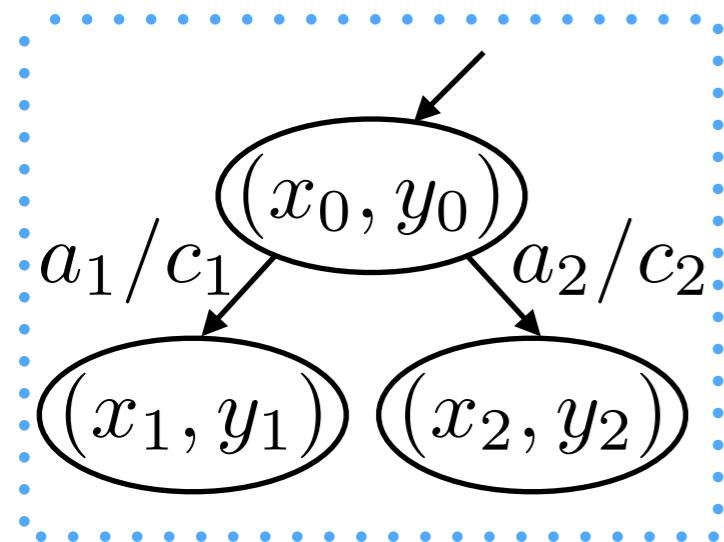
$\mathcal{C} \circ \mathcal{D}$

sequential
composition

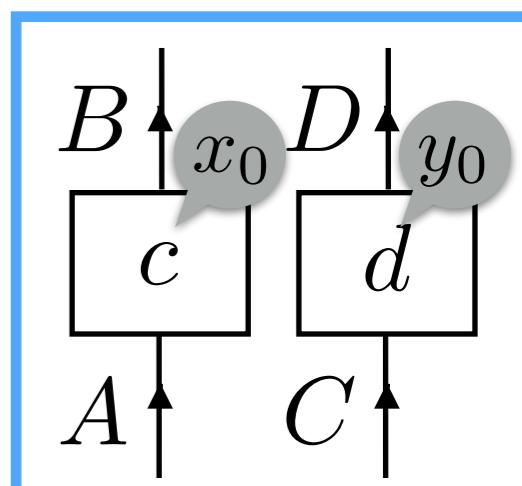
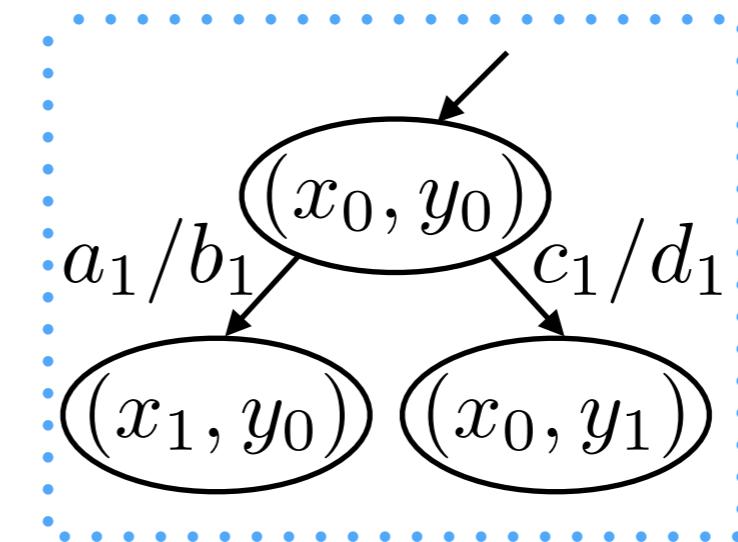
$\mathcal{C} \boxplus \mathcal{D}$

parallel
composition

$$\left(Y \times B \xrightarrow{d} T(Y \times C), \begin{array}{l} Y, \\ y_0 \in Y \end{array} \right) \circ \left(X \times A \xrightarrow{c} T(X \times B), \begin{array}{l} X, \\ x_0 \in X \end{array} \right) = \left(\begin{array}{l} X \times Y, \\ \dots \\ (x_0, y_0) \in X \times Y \end{array} \right)$$



$$\left(X \times A \xrightarrow{c} T(X \times B), \begin{array}{l} X, \\ x_0 \in X \end{array} \right) \boxplus \left(Y \times C \xrightarrow{d} T(Y \times D), \begin{array}{l} Y, \\ y_0 \in Y \end{array} \right) = \left(\begin{array}{l} X \times Y, \\ \dots \\ (x_0, y_0) \in X \times Y \end{array} \right)$$



Memoryful Gol — Translation

Def. (component calculus)

$\text{Tr}(\mathcal{C})$

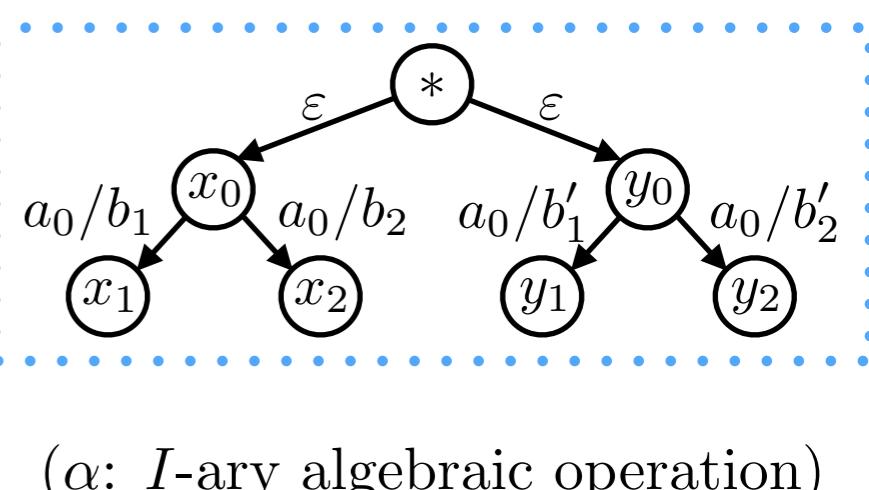
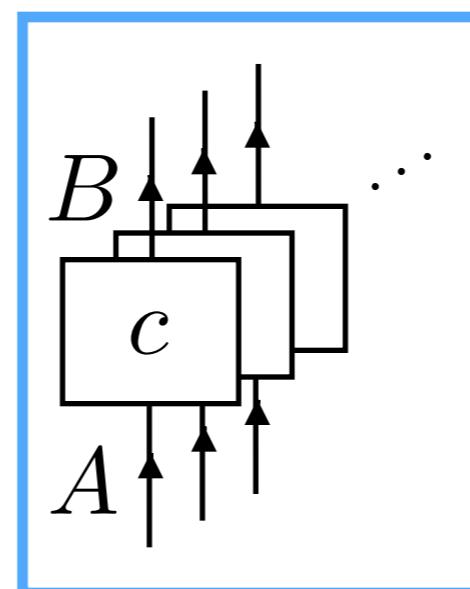
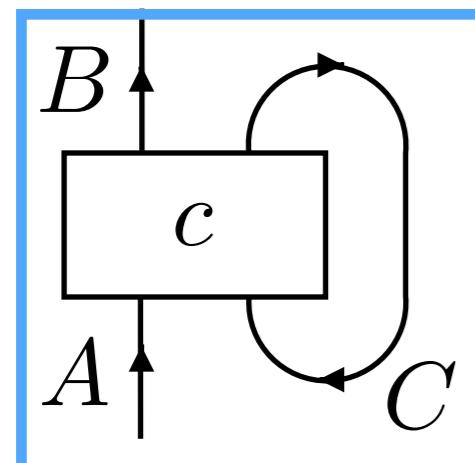
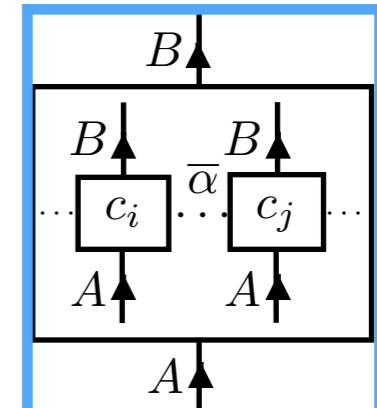
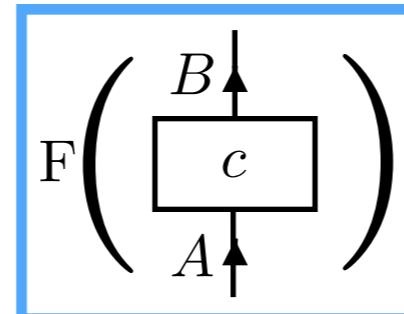
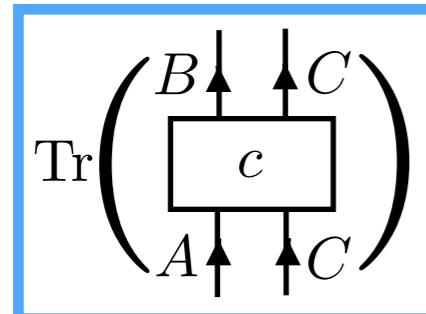
application

$F(\mathcal{C})$

!-modality

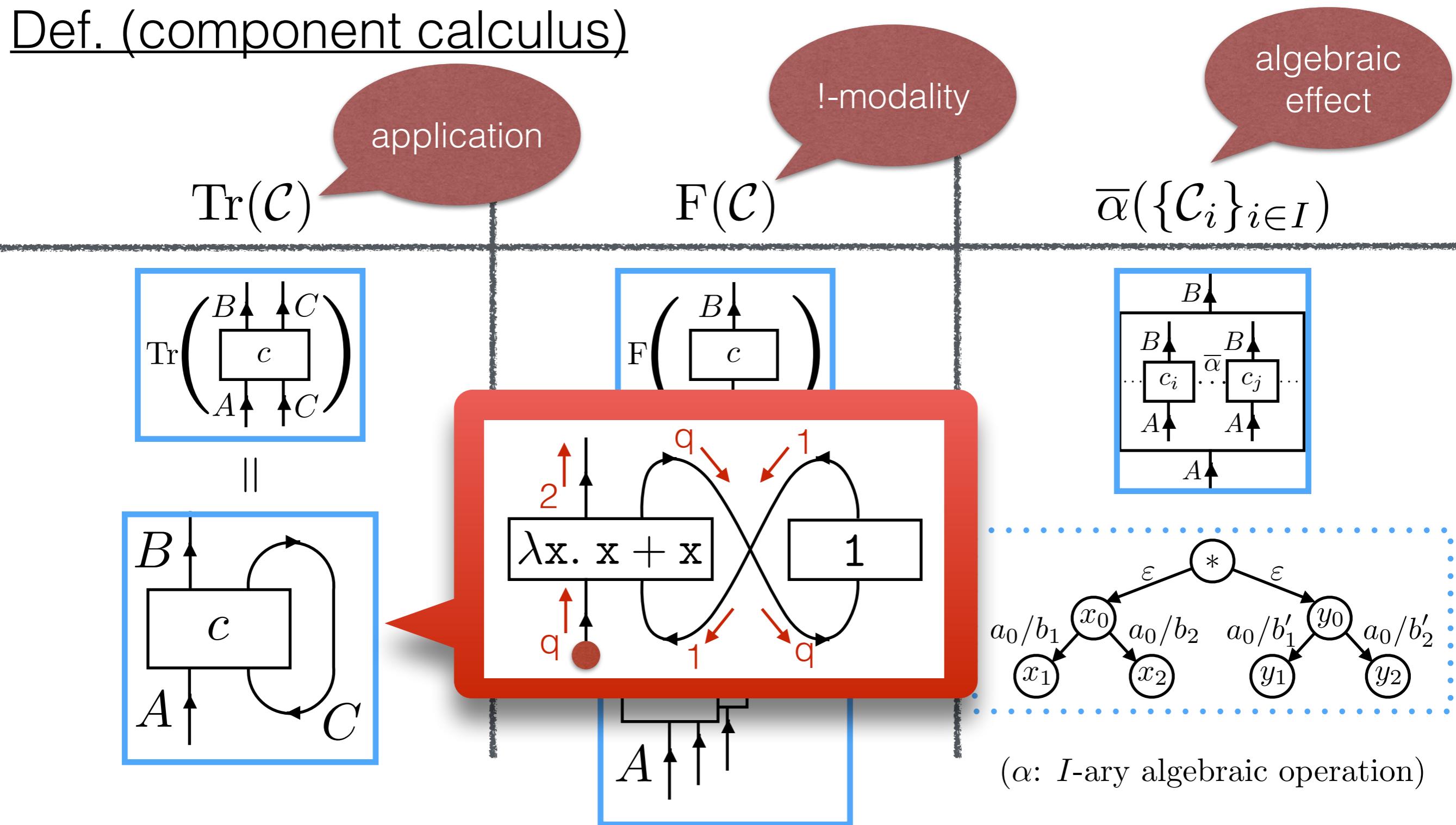
$\overline{\alpha}(\{\mathcal{C}_i\}_{i \in I})$

algebraic effect



Memoryful Gol — Translation

Def. (component calculus)



Memoryful Gol — Translation

Def. (component calculus)

$\text{Tr}(\mathcal{C})$

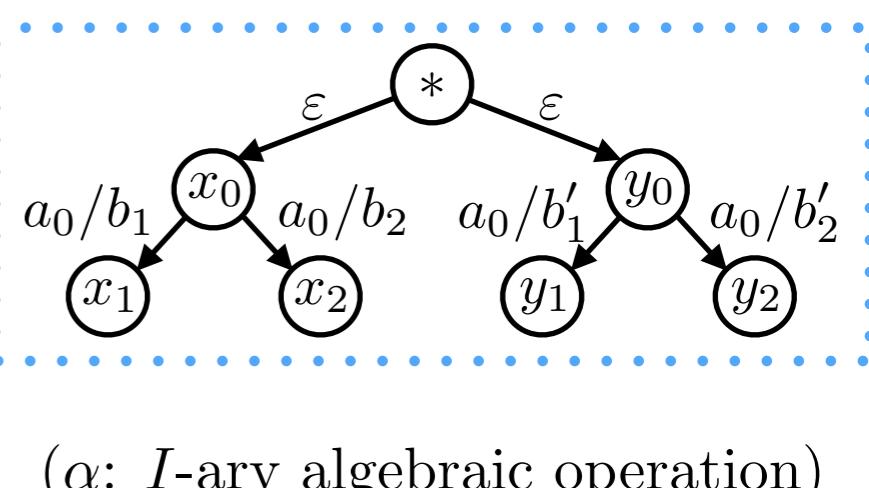
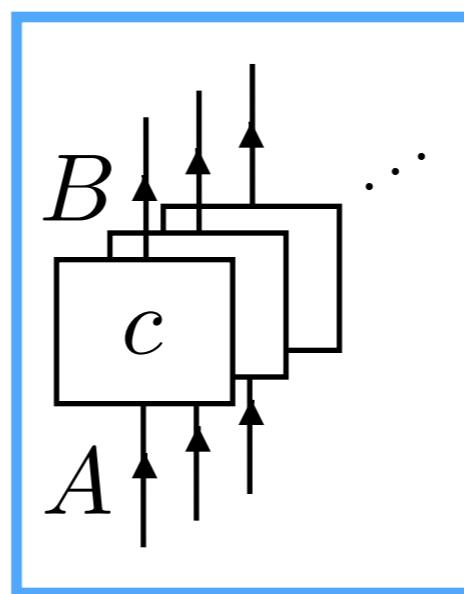
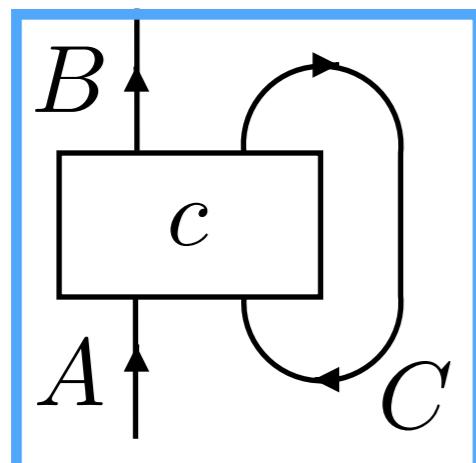
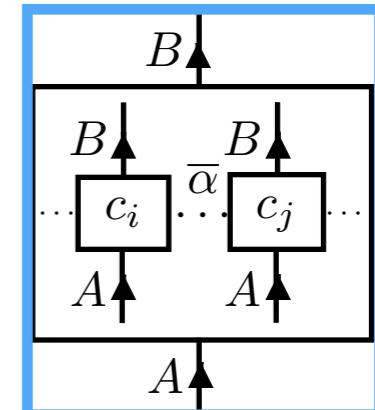
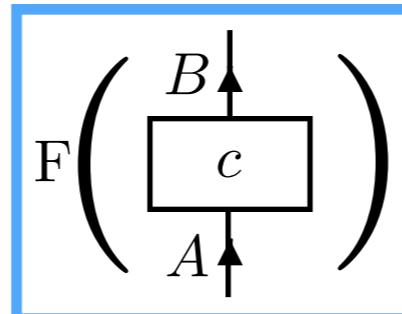
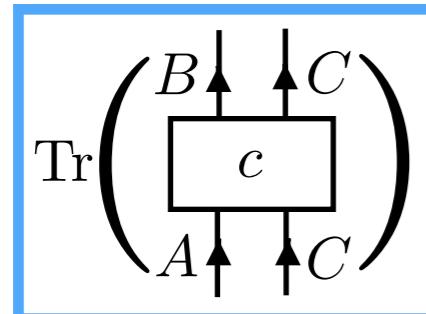
application

$F(\mathcal{C})$

!-modality

$\overline{\alpha}(\{\mathcal{C}_i\}_{i \in I})$

algebraic effect



Memoryful Gol — Translation

Def. (interpretation $(\Gamma \vdash t : \tau)$)

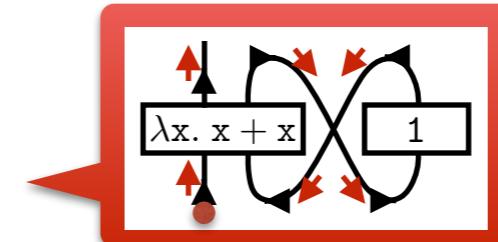
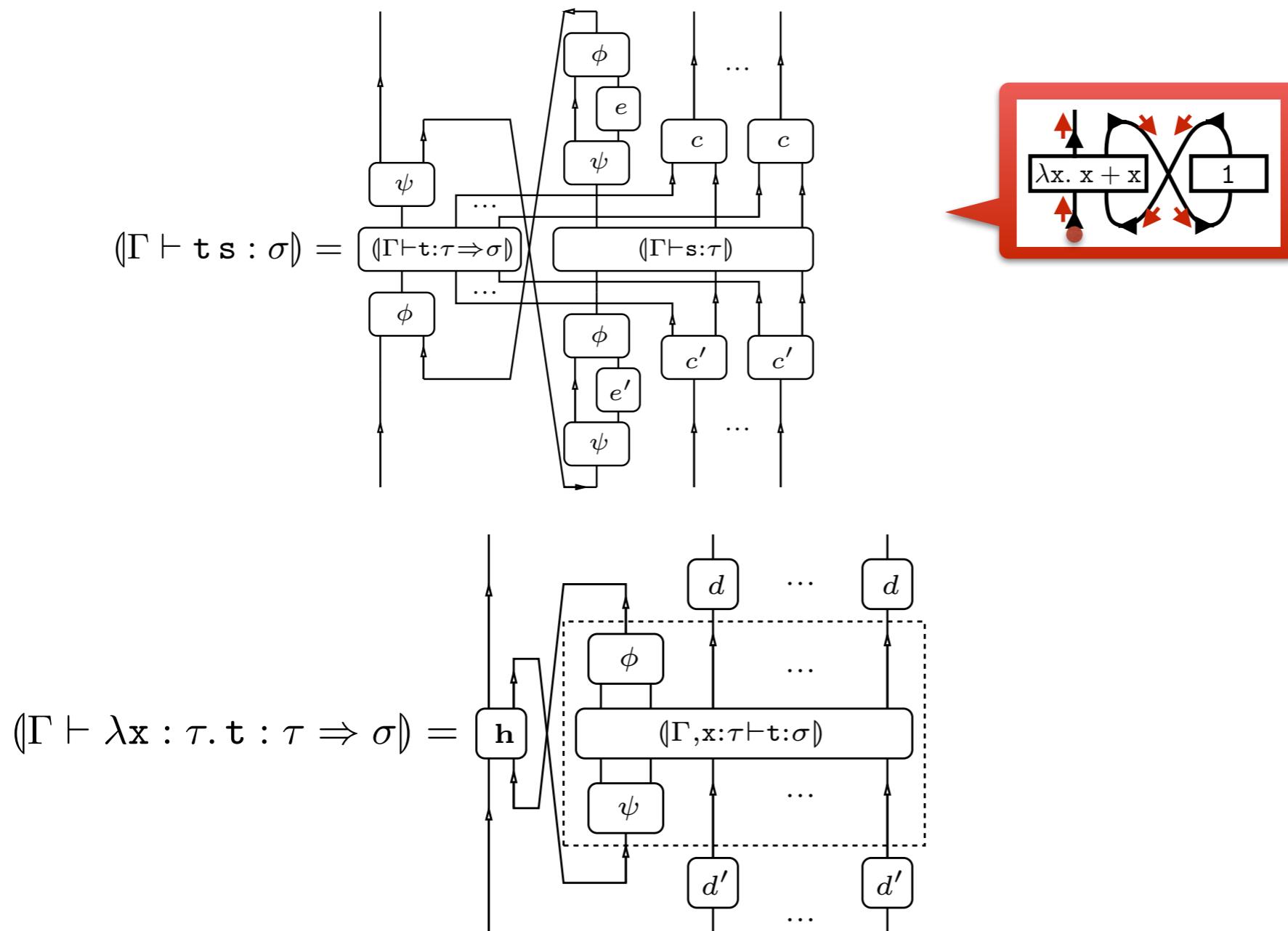
For a type judgement $(\Gamma \vdash t : \tau)$ ($\Gamma = x_1 : \tau_1, \dots, x_n : \tau_n$),

we inductively define

$$(\Gamma \vdash t : \tau) = \boxed{\begin{array}{c} n \\ \overbrace{\quad\quad\quad\quad}^{\text{---}} \\ N \uparrow N \uparrow \cdots \uparrow N \\ (\Gamma \vdash t : \tau) \\ \downarrow \quad \downarrow \quad \cdots \quad \downarrow \\ N \uparrow N \uparrow \cdots \uparrow N \end{array}} .$$

Memoryful Gol — Translation

Def. (interpretation $(\Gamma \vdash t : \tau)$)



Memoryful GoL — Translation

Def. (interpretation ($\Gamma \vdash t : \tau$))

$$(\Gamma \vdash n : \text{nat}) = \begin{array}{c} \text{Diagram showing two parallel wires labeled } h \text{ and } k_n \text{ merging at a junction, with arrows pointing upwards. To the right, there are three boxes labeled } w, w', \text{ and } w' \text{ with arrows pointing upwards, followed by ellipses.} \\ \dots \\ w \\ \dots \\ w' \\ \dots \\ w' \end{array}$$

$$(\Gamma, x : \text{nat}, y : \text{nat} \vdash x + y : \text{nat}) = \begin{array}{c} \text{Diagram showing a computation graph for addition. The root node is labeled 'sum'. It has two children, which are labeled 'h' and 'w'. Node 'h' has one child, 'w', which has one child, 'w''. Node 'sum' also has two children, which are labeled 'w' and 'w''. Each 'w' node has one child, 'w'', which has one child, 'w'''. The connections are as follows: 'h' connects to 'w', 'w' connects to 'w''', and 'sum' connects to both 'w' and 'w''. Arrows indicate the flow of data from bottom to top.} \\ \text{Diagram showing a computation graph for addition. The root node is labeled 'sum'. It has two children, which are labeled 'h' and 'w'. Node 'h' has one child, 'w', which has one child, 'w''. Node 'sum' also has two children, which are labeled 'w' and 'w''. Each 'w' node has one child, 'w'', which has one child, 'w'''. The connections are as follows: 'h' connects to 'w', 'w' connects to 'w''', and 'sum' connects to both 'w' and 'w''. Arrows indicate the flow of data from bottom to top.} \end{array}$$

$$(\Gamma \vdash t + s : \text{nat}) = (\Gamma \vdash (\lambda xy : \text{nat}. x + y) t s : \text{nat})$$

$$(\mathbf{x}_1 : \tau_1, \dots, \mathbf{x}_n : \tau_n \vdash \mathbf{x}_i : \tau_i) = \begin{array}{c} \text{Diagram showing a sequence of boxes labeled } w, w', \dots, w' \text{ and } w \text{ arranged in two rows. The first row contains } w, w', w', \dots, w' \text{ and the second row contains } w, w, w, \dots, w. \text{ Arrows indicate connections between adjacent boxes in each row and between the two rows. A box labeled } h \text{ is connected to the first box in the first row. Brackets above the boxes group them into pairs: } (w, w'), (w', w'), \dots, (w', w') \text{ in the first row and } (w, w), (w, w), \dots, (w, w) \text{ in the second row. Ellipses indicate continuation. The label } i-1 \text{ is placed above the second row.} \\ \hline \end{array}$$

Memoryful Gol — Translation

Thm. (soundness)

For closed terms M and N of type τ ,

- $\vdash M = N : \tau$ implies $([(M)]_\sim, [(N)]_\sim) \in \Phi[\tau]$
- $\vdash M = N : \text{nat}$ implies $(M) \simeq (N)$.

behavioral equivalence

- Moggi's equations for computational lambda-calculus
- equations for algebraic operations

$$M \sqcup M = M$$

$$E[\text{opr}(M_1, \dots, M_n)] = \text{opr}(E[M_1], \dots, E[M_n])$$

$$(\lambda x. M) (N_1 \sqcup N_2) = (\lambda x. M) N_1 \sqcup (\lambda x. M) N_2$$

Memoryful Gol [Hoshino, —, Hasuo '14]

effectful
terms

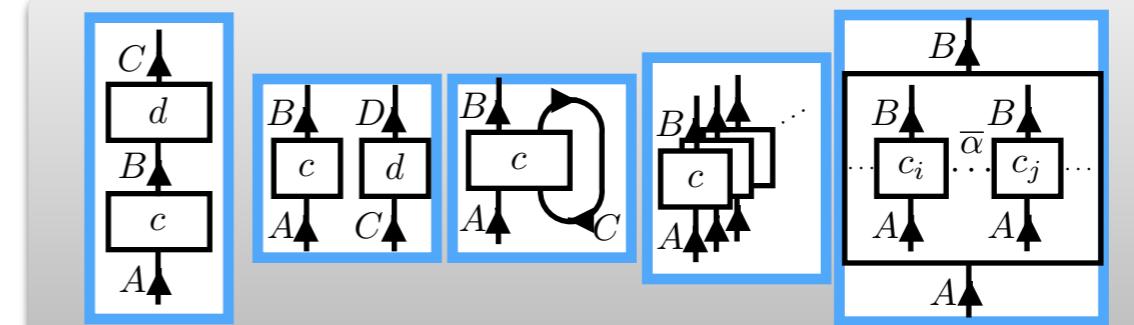
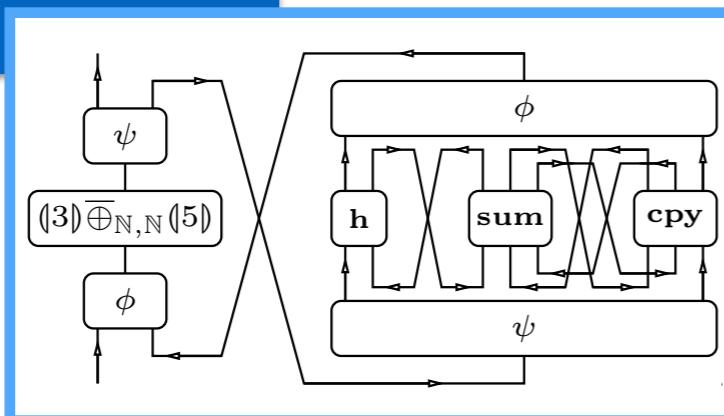
$$(\lambda x : \text{nat}. x + x) (3 \sqcup 5) : \text{nat}$$

$$\langle\Gamma \vdash t : \tau\rangle = \boxed{\begin{array}{c} n \\ \overbrace{\mathbb{N} \uparrow \mathbb{N} \uparrow \cdots \uparrow \mathbb{N}} \\ \langle\Gamma \vdash t : \tau\rangle \\ \mathbb{N} \uparrow \mathbb{N} \uparrow \cdots \uparrow \mathbb{N} \end{array}}$$

translation

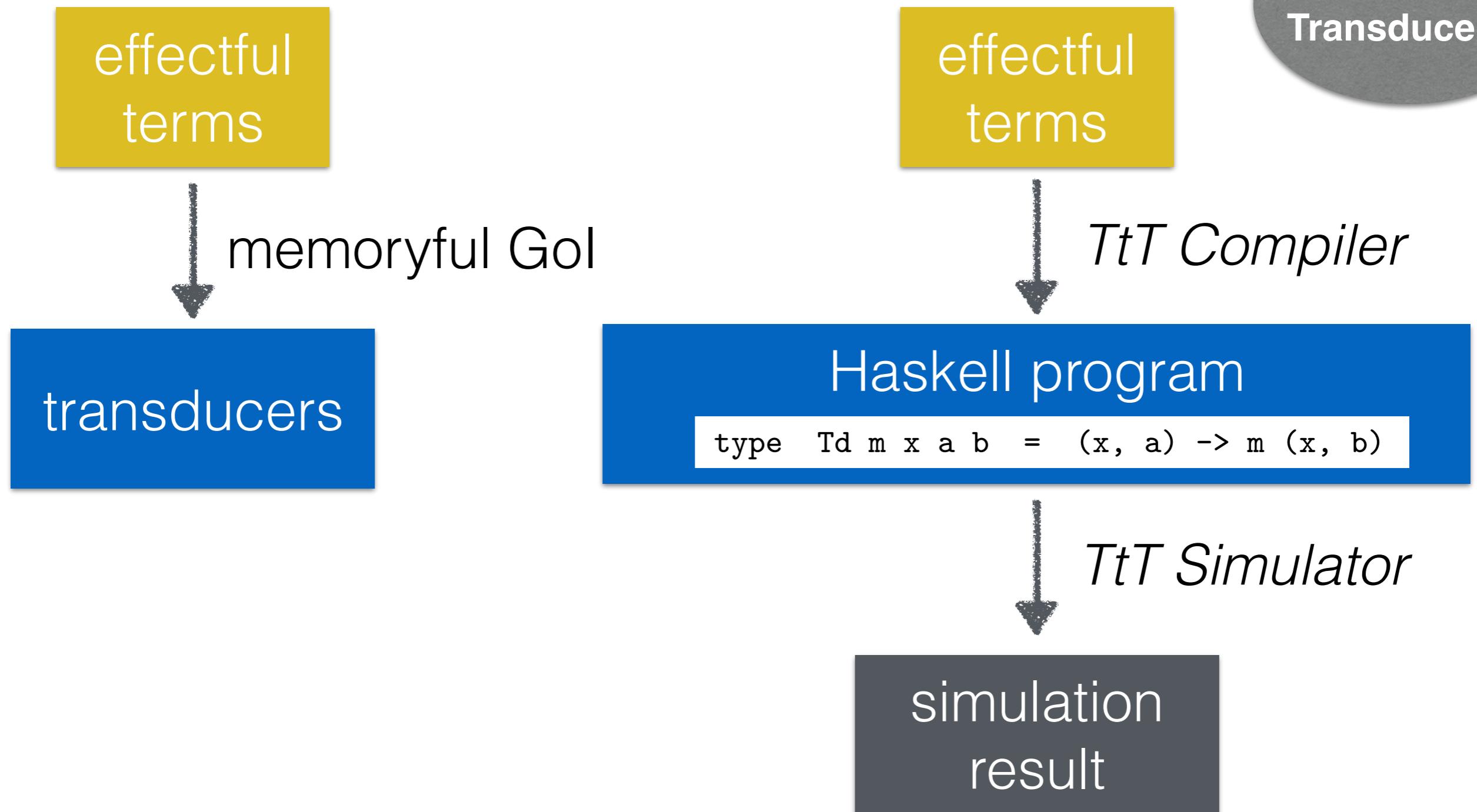
- based on Geometry of Interaction
- via coalgebraic component calculus

transducers



Prototype Implementation: *TtT*

“Terms to Transducers”



Prototype Implementation: *TtT*

$3 \sqcup 5$

$(\lambda x. x) 1$

$(\lambda f. f 0 + f 1) (\lambda x. 3 \sqcup 5)$



```

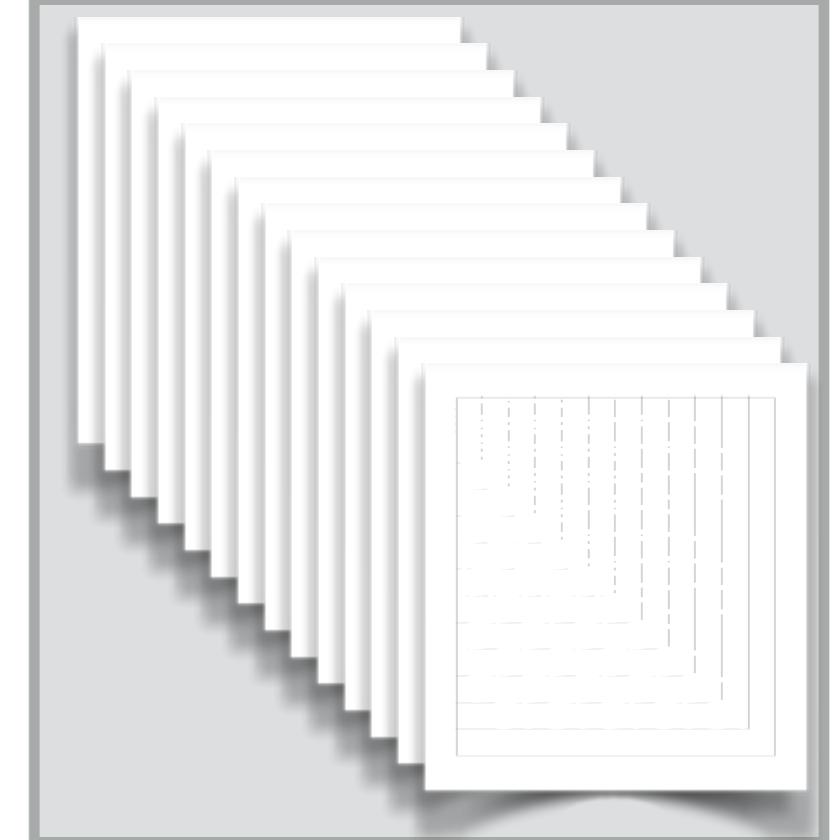
. ---- dd<42,137> --->
| Query:[ [3]_|[5] @ Nothing ]
+--. ---- dd<42,137> --->
| | | Query:[ [3]_|[5] @ * ]
| | | h; k_3; h
| | | [ [3]_|[5] @ * ]:Answer
| | | ---- dd<42,3> --->
| | | [ [3]_|[5] @ Just (Left (*)) ]:Answer
| | | ---- dd<42,3> --->
| Result: 3 / State: Just (Left (*))
`-. ---- dd<42,137> --->
| | | Query:[ 3_|[5] @ * ]
| | | h; k_5; h
| | | [ 3_|[5] @ * ]:Answer
| | | ---- dd<42,5> --->
| | | [ [3]_|[5] @ Just (Right (*)) ]:Answer
| | | ---- dd<42,5> --->
Result: 5 / State: Just (Right (*))

```

```

---- dd<42,137> --->
Query:[ [(\x.x) 1] @ {_, *} , * ]
phi
---- gdd<42,137> --->
Query:[ [\x.x] 1 @ {_, *} ]
h
[ [\x.x] 1 @ {_, *} ]:Answer
---- dgdd<42,137> --->
psi; psi; phi
---- gdd<42,137> --->
Query:[ (\x.x) [1] @ * ]
h
[ (\x.x) [1] @ * ]:Answer
---- dgdd<42,137> --->
psi; e; phi; phi
---- dd<0,gdd<42,137>> --->
Query:[ [\x.x] 1 @ {_, *} ]
h; v
o
psi
---- dd<42,137> --->
Query:[ (\x.[x]) 1 @ * ]
h
[ (\x.[x]) 1 @ * ]:Query x
---- <42,137> --->
phi
} 0
u; h
[ [\x.x] 1 @ {_, *} ]:Answer
---- dd<0,<42,137>> --->
psi; psi; e'; phi
---- dd<42,137> --->
Query:[ (\x.x) [1] @ * ]
h; k_1; h
[ (\x.x) [1] @ * ]:Answer
---- dd<42,1> --->
psi; e; phi; phi
---- dd<0,<42,1>> --->
Query:[ [\x.x] 1 @ {_, *} ]
h; v
o {
psi
---- <42,1> --->
Answer x:[ (\x.[x]) 1 @ * ]
h
[ (\x.[x]) 1 @ * ]:Answer
---- dd<42,1> --->
phi
} 0
u; h
[ [\x.x] 1 @ {_, *} ]:Answer
---- dd<0,gdd<42,1>> --->
psi; psi; e'; phi
---- dgdd<42,1> --->
Query:[ (\x.x) [1] @ * ]
h
[ (\x.x) [1] @ * ]:Answer
---- gdd<42,1> --->
psi; phi; phi
---- dgdd<42,1> --->
Query:[ [\x.x] 1 @ {_, *} ]
h
[ [\x.x] 1 @ {_, *} ]:Answer
---- gdd<42,1> --->
psi
[ [(\x.x) 1] @ {_, *} , * ]:Answer
---- dd<42,1> --->
Result: 1 / State: {_, *} , *

```



(4,526 lines)

Memoryful Gol [Hoshino, —, Hasuo '14]

effectful
terms

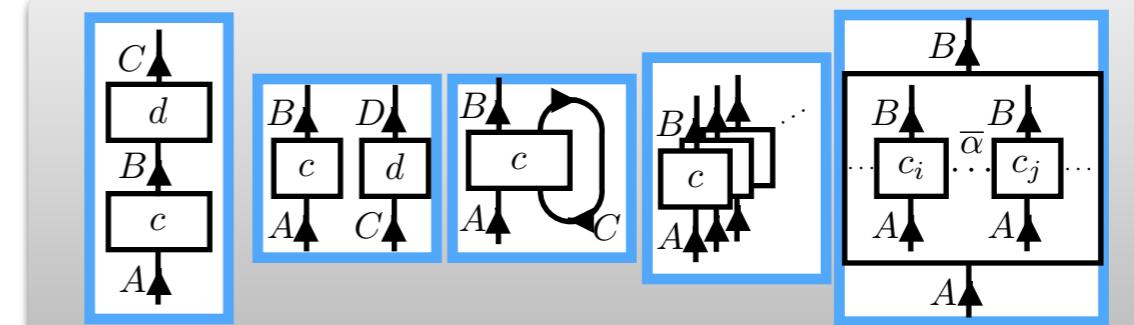
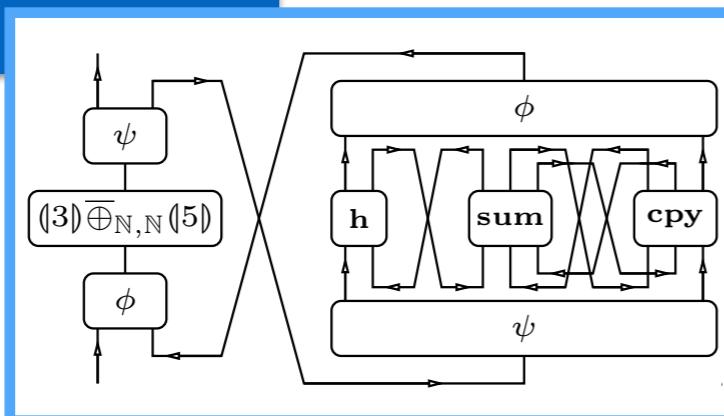
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$$\langle\Gamma \vdash t : \tau\rangle = \boxed{\begin{array}{c} n \\ \overbrace{\mathbb{N} \uparrow \mathbb{N} \uparrow \cdots \uparrow \mathbb{N}} \\ \boxed{\Gamma \vdash t : \tau} \\ \mathbb{N} \uparrow \mathbb{N} \uparrow \cdots \uparrow \mathbb{N} \end{array}}$$

translation

- based on Geometry of Interaction
- via coalgebraic component calculus

transducers



Memoryful Gol with recursion

effectful
terms

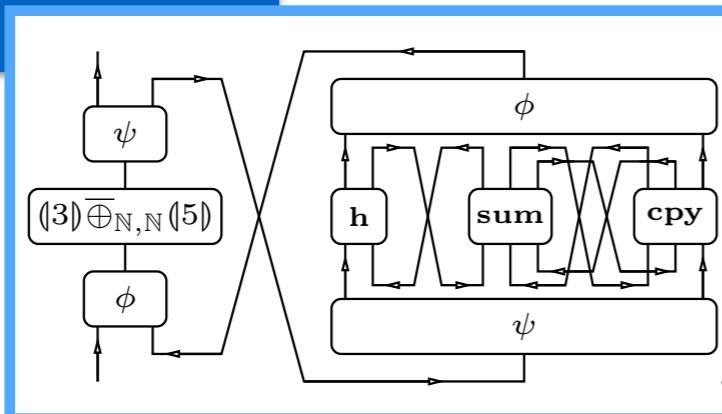
recursion

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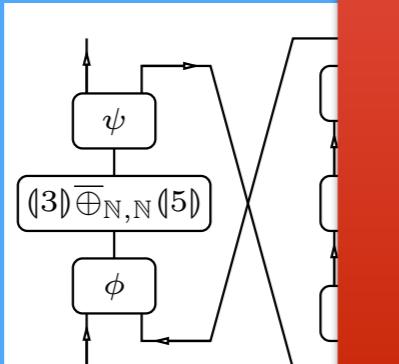
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recursion

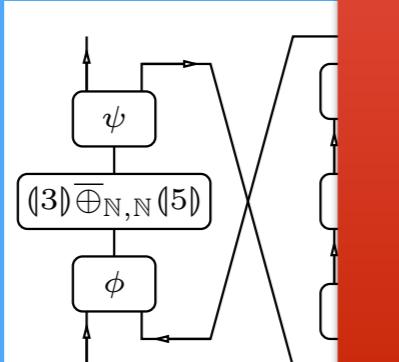
$(\lambda x : \text{nat}. x + x) (3 \sqcup 5) : \text{nat}$

translation

- based on Geometry of Interaction
- via coalgebraic component calculus

$\text{fix}(F) = F(F(F(\dots)))$

transducers



Memoryful Gol with recursion

effectful
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recursion

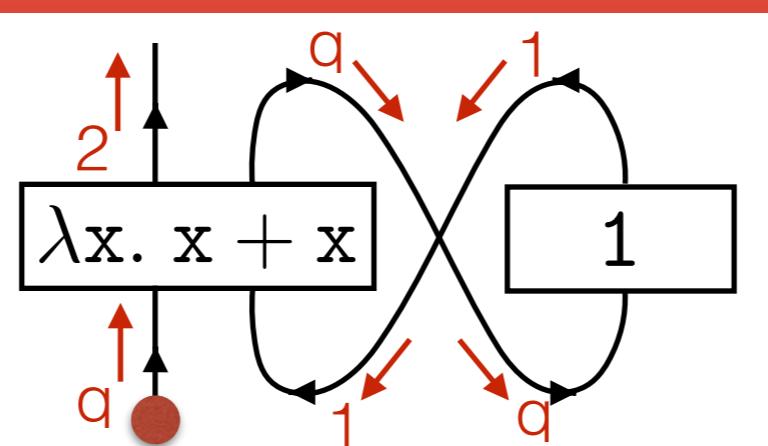
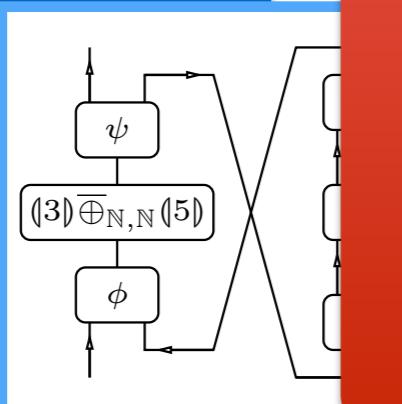
$(\lambda x : \text{nat}. x + x) (3 \sqcup 5) : \text{nat}$

translation

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$\text{fix}(F) = F(F(F(\dots)))$

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terms

recursion

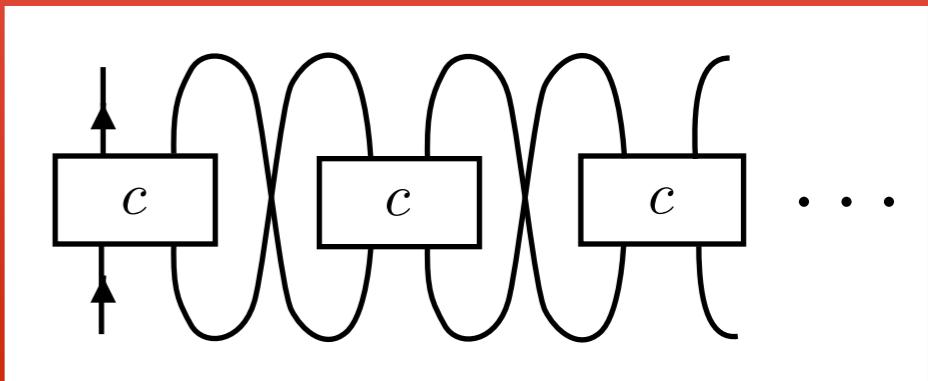
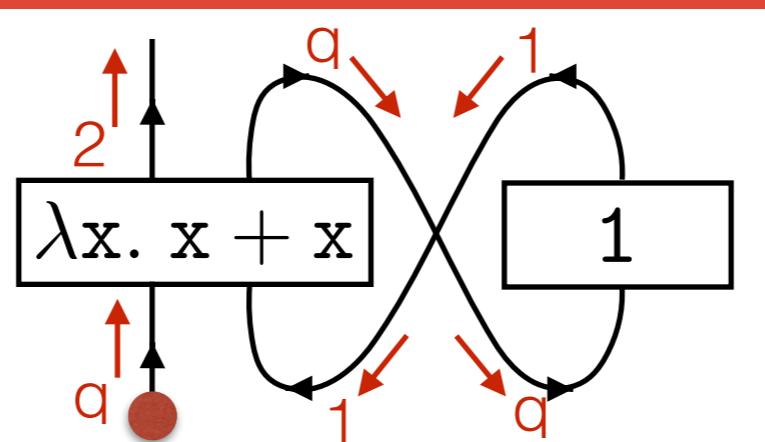
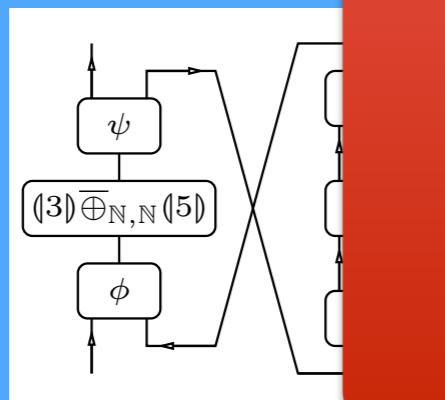
$(\lambda x : \text{nat}. x + x) (3 \sqcup 5) : \text{nat}$

translation

- based on Geometry of Interaction
- via coalgebraic component calculus

$\text{fix}(F) = F(F(F(\dots)))$

transducers



Memoryful Gol with recursion

effectful
terms

recursion

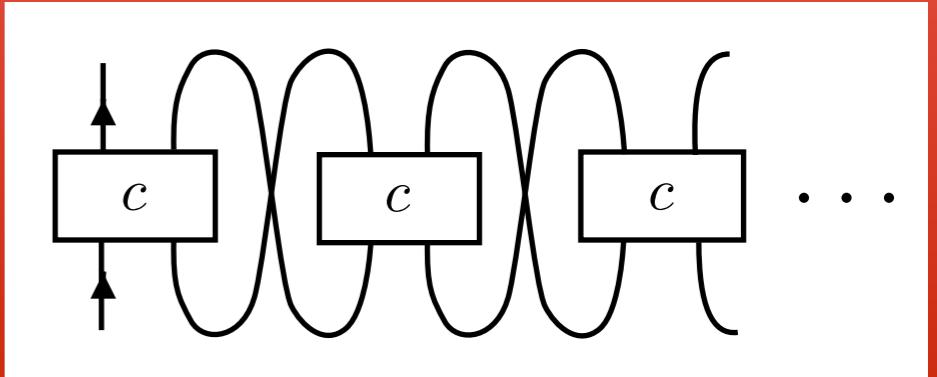
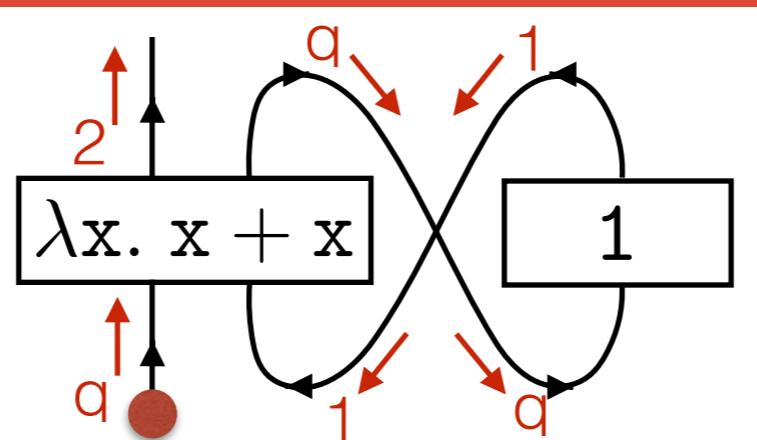
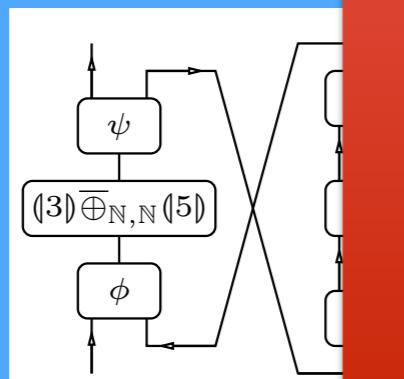
$(\lambda x : \text{nat}. x + x) (3 \sqcup 5) : \text{nat}$

translation

- based on Geometry of Interaction
- via **extended** coalgebraic component calculus

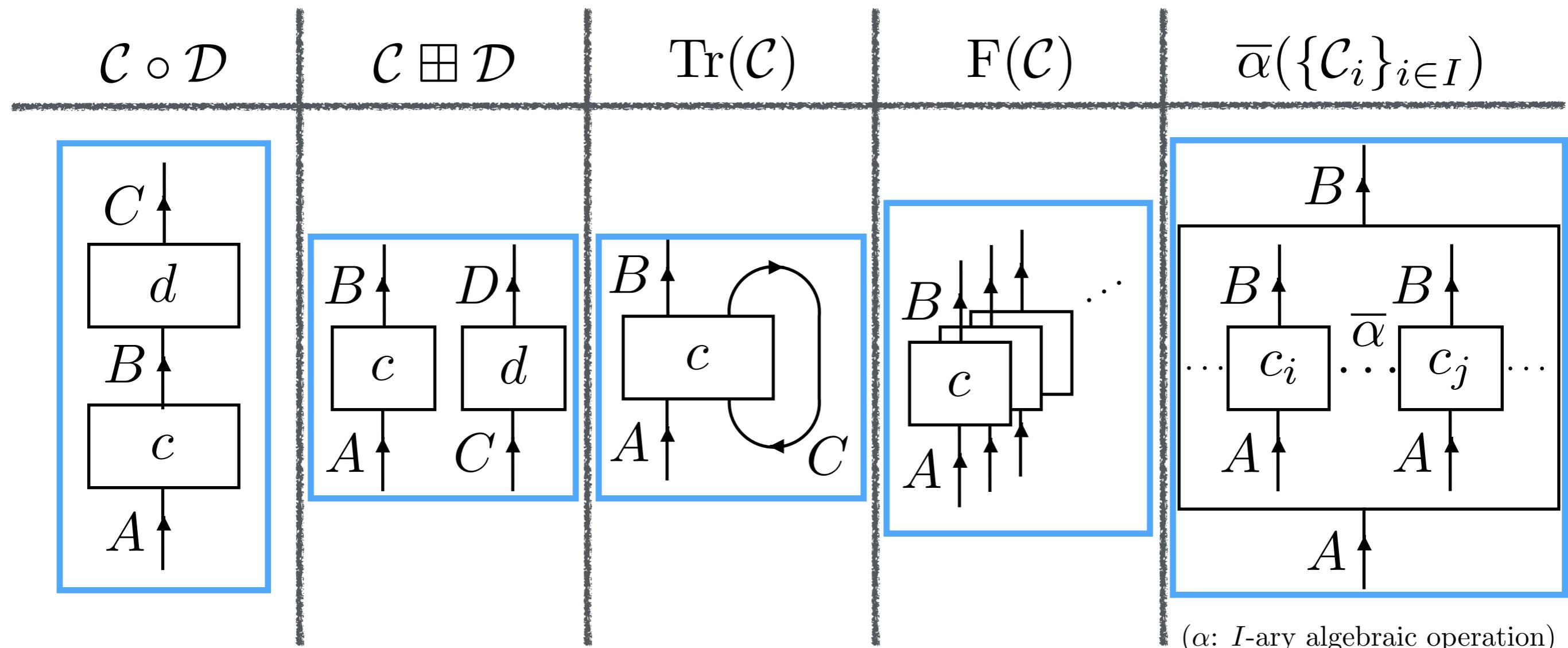
$$\text{fix}(F) = F(F(F(\dots)))$$

transducers



Memoryful Gol with recursion

Def. (component calculus)

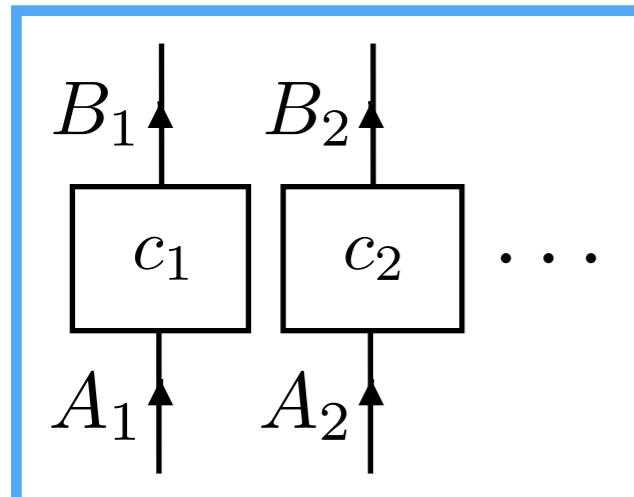


Memoryful Gol with recursion

Def. (**extended** component calculus)

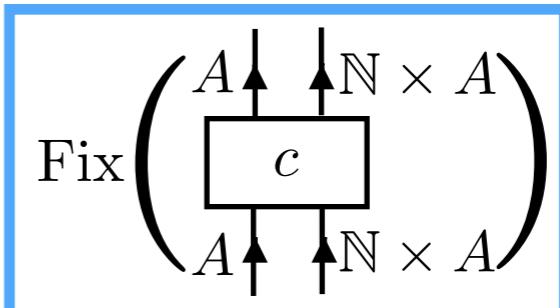
$$\bigoplus_{i \in I} C_i$$

countable
parallel
composition

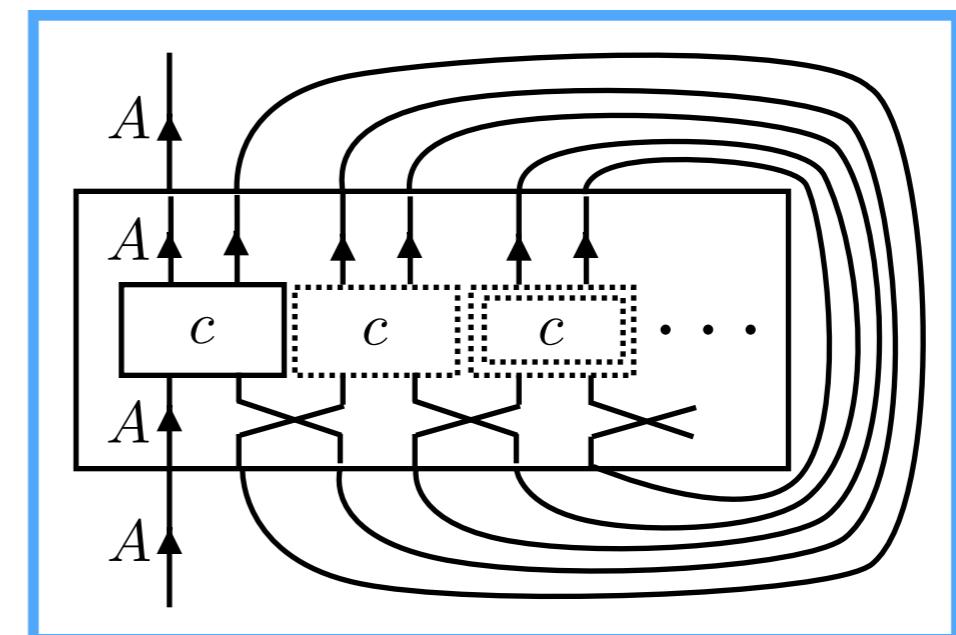


$$\text{Fix}(C)$$

fixpoint



||



Memoryful Gol with recursion

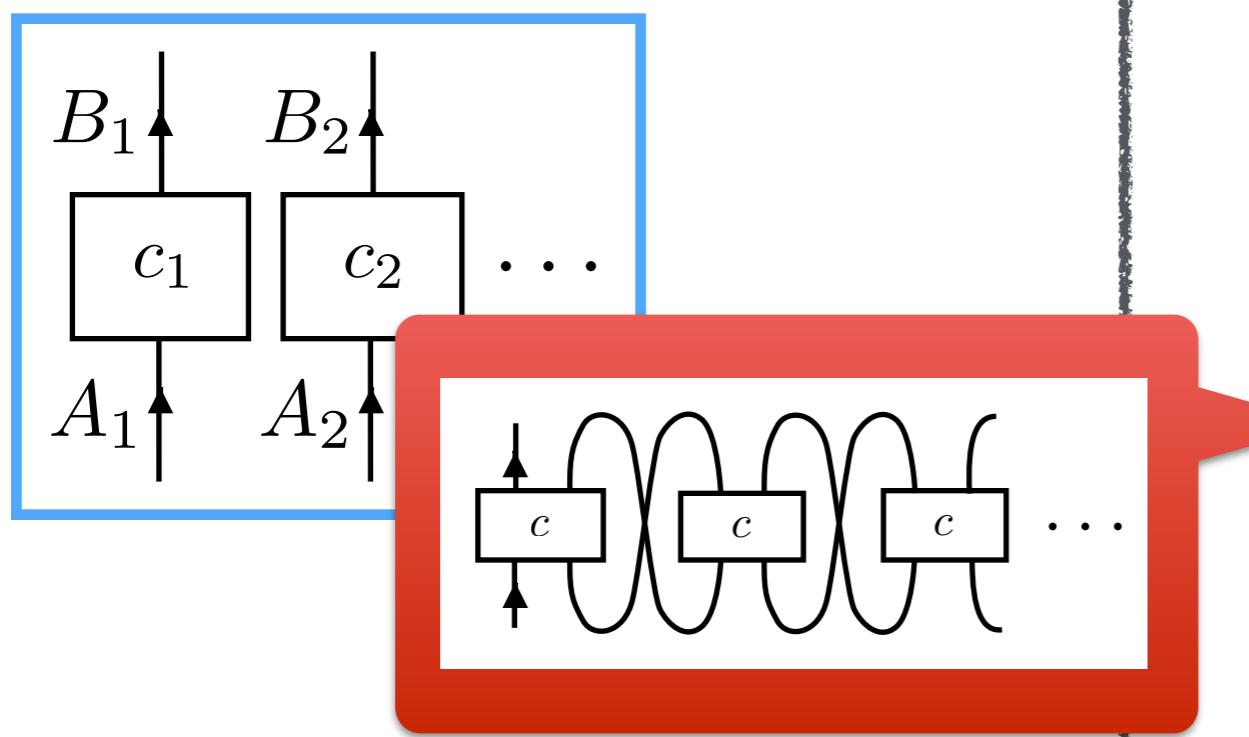
Def. (**extended** component calculus)

$$\bigoplus_{i \in I} C_i$$

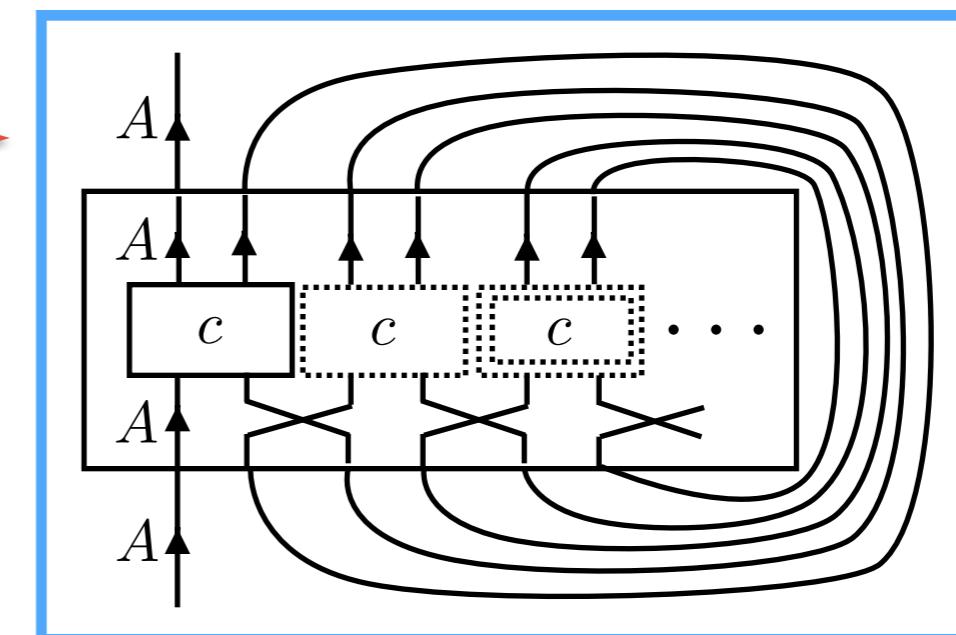
countable
parallel
composition

$$\text{Fix}(C)$$

fixpoint

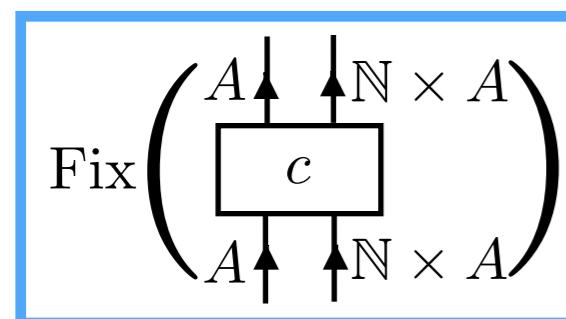


$$\text{Fix} \left(\begin{array}{c} A \uparrow \\ \boxed{c} \\ A \uparrow \end{array} \right) \quad \text{||} \quad \text{Fix} \left(\begin{array}{c} \mathbb{N} \times A \uparrow \\ \boxed{c} \\ \mathbb{N} \times A \uparrow \end{array} \right)$$

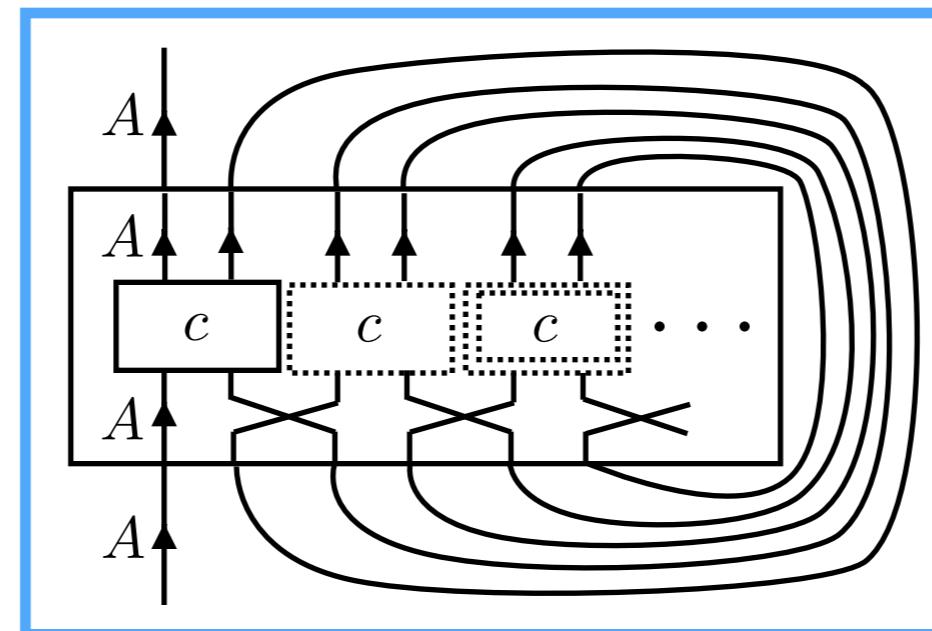


Memoryful Gol with recursion

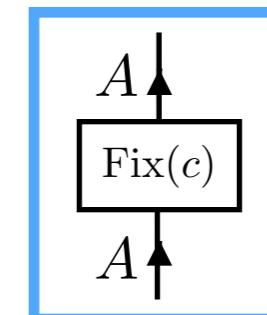
Lem.



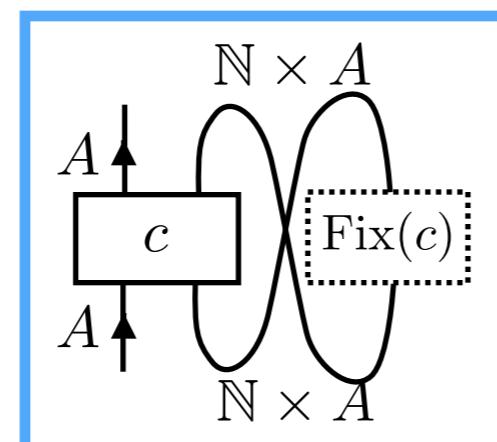
=



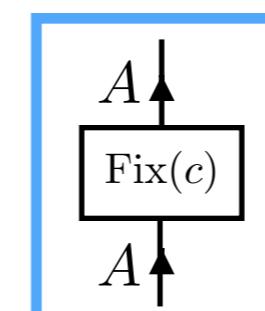
=:



satisfies



\simeq



Memoryful Gol with recursion

Def. (interpretation $(\Gamma \vdash t : \tau)$)

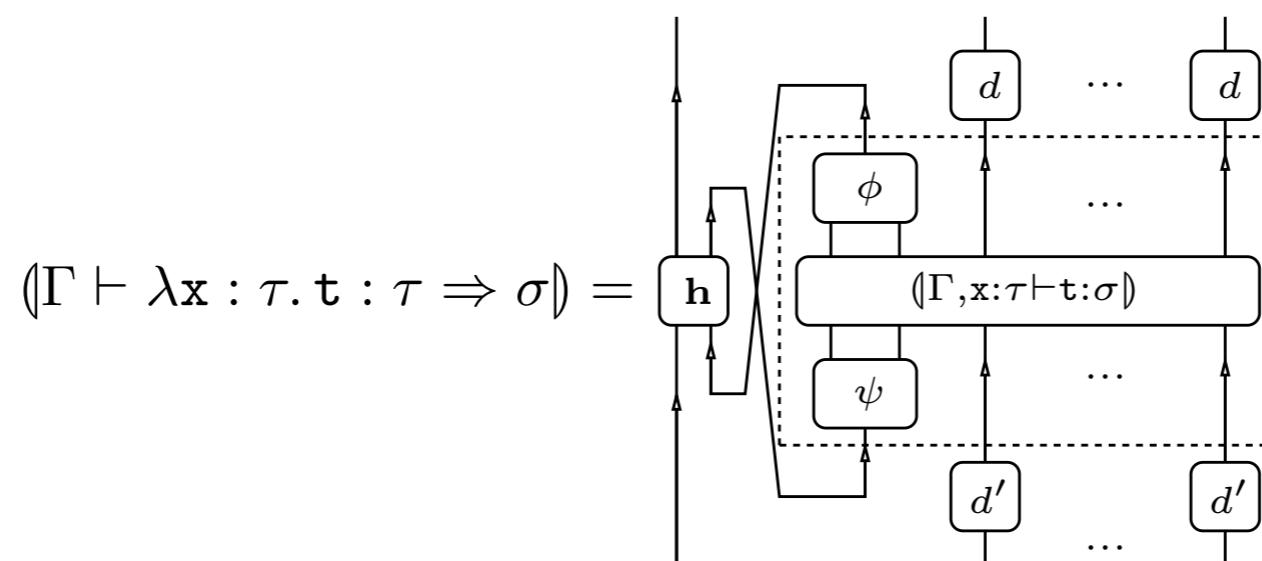
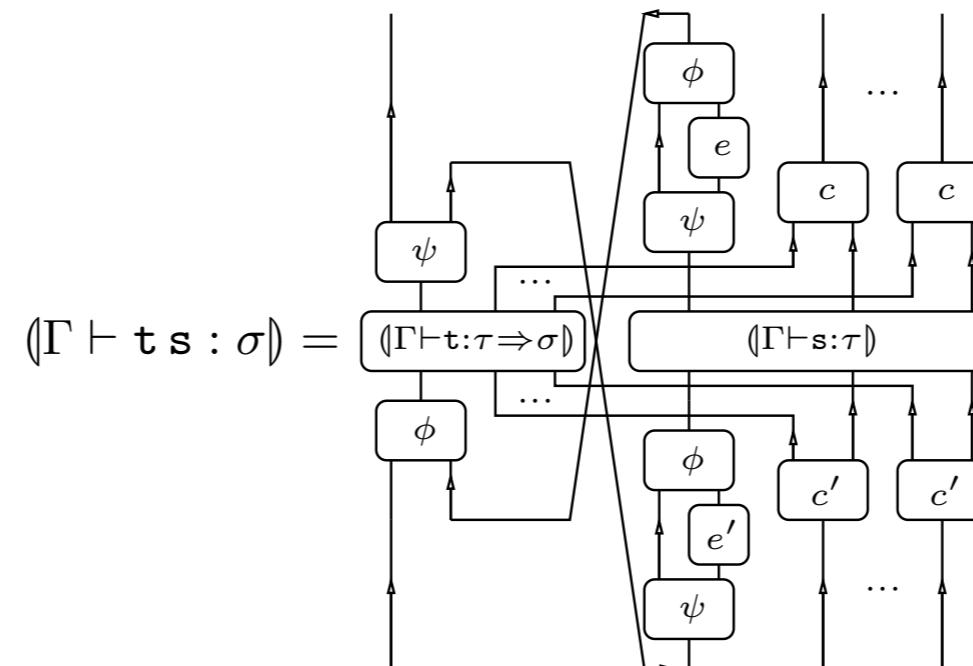
For a type judgement $(\Gamma \vdash t : \tau)$ ($\Gamma = x_1 : \tau_1, \dots, x_n : \tau_n$),

we inductively define

$$(\Gamma \vdash t : \tau) = \boxed{\begin{array}{c} n \\ \overbrace{\quad\quad\quad\quad}^{\text{---}} \\ N \uparrow N \uparrow \cdots \uparrow N \\ (\Gamma \vdash t : \tau) \\ \downarrow \quad \downarrow \quad \cdots \quad \downarrow \\ N \uparrow N \uparrow \cdots \uparrow N \end{array}} .$$

Memoryful Gol with recursion

Def. (interpretation $(\Gamma \vdash t : \tau)$)



Memoryful Gol with recursion

Def. (interpretation ($\Gamma \vdash t : \tau$))

$$(\Gamma \vdash n : \text{nat}) = \begin{array}{c} \text{Diagram showing a box labeled } h \text{ connected to a box labeled } k_n \text{ by two crossing wires. The output wire from } k_n \text{ splits into two wires, one leading to a box labeled } w \text{ and another leading to a box labeled } w'. \text{ Ellipses indicate continuation.} \\ \dots \\ \dots \end{array}$$

$$(\Gamma, x : \text{nat}, y : \text{nat} \vdash x + y : \text{nat}) = \begin{array}{c} \text{Diagram showing a computation graph for addition. The root node is labeled 'sum'. It has two children, which are the nodes 'h' and 'w'. Node 'h' has one child, 'w'. Node 'w' has three children, which are the nodes 'w', 'w'', and 'w'' (the last two are part of an ellipsis). Each node is represented by a rounded rectangle with an upward-pointing arrow. The connections between nodes are shown as lines with arrows indicating flow from left to right. The labels 'h', 'sum', 'w', 'w'', and 'w'' are placed near their respective nodes. Ellipses '...' are used to indicate additional nodes in the sequence. The entire diagram is contained within a large rounded rectangle.} \end{array}$$

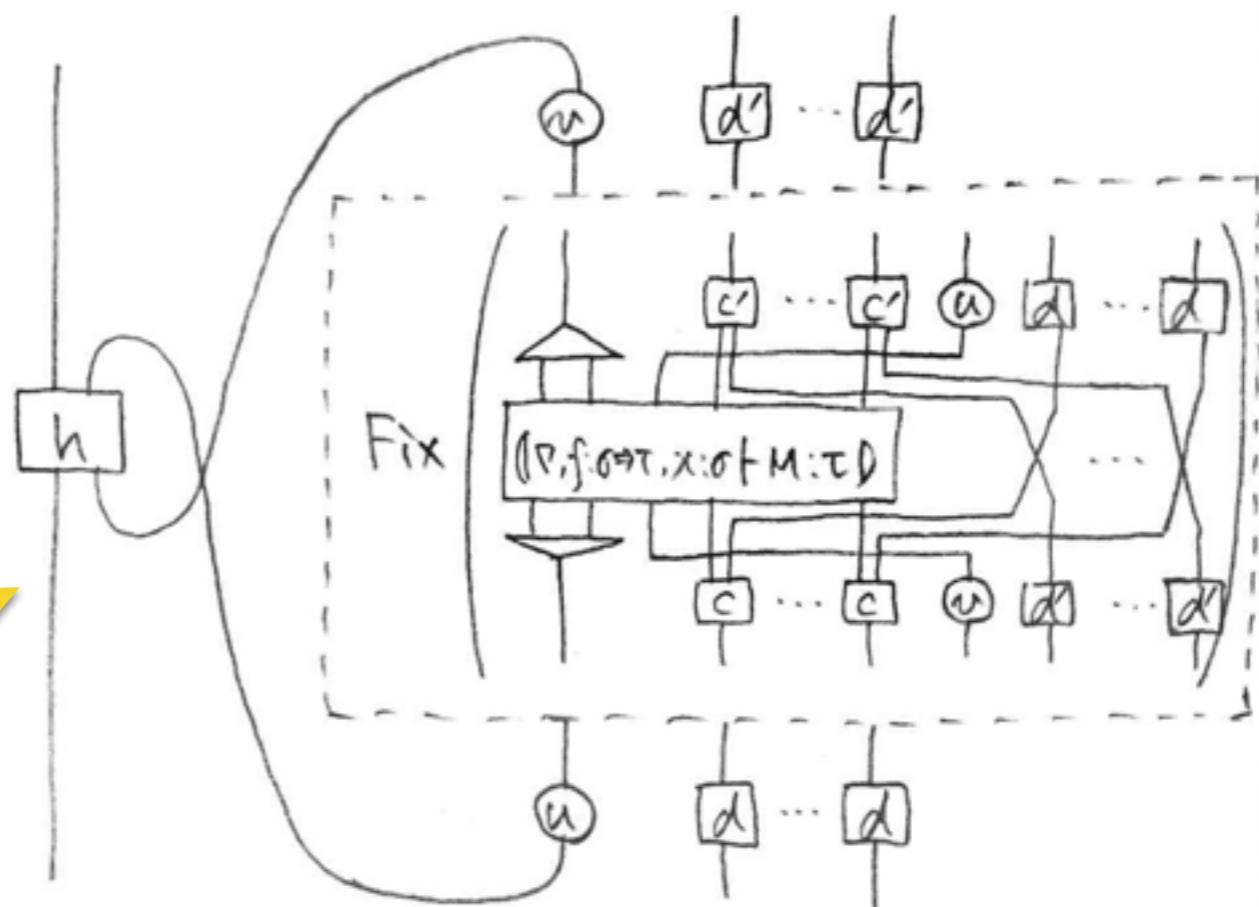
$$(\Gamma \vdash t + s : \text{nat}) = (\Gamma \vdash (\lambda xy : \text{nat}. x + y) t s : \text{nat})$$

$$(\mathbf{x}_1 : \tau_1, \dots, \mathbf{x}_n : \tau_n \vdash \mathbf{x}_i : \tau_i) = \begin{array}{c} \text{Diagram showing a sequence of boxes labeled } w \text{ and } w' \text{ connected by arrows. The top row has boxes } w \text{ and the bottom row has boxes } w'. \text{ Arrows point from left to right between adjacent boxes in each row. Vertical arrows also connect the rows. Brackets on the right side group the boxes into pairs, with labels } i-1 \text{ above the top bracket and } \dots \text{ below it.} \\ \text{Diagram showing a sequence of boxes labeled } w \text{ and } w' \text{ connected by arrows. The top row has boxes } w \text{ and the bottom row has boxes } w'. \text{ Arrows point from left to right between adjacent boxes in each row. Vertical arrows also connect the rows. Brackets on the right side group the boxes into pairs, with labels } i-1 \text{ above the top bracket and } \dots \text{ below it.} \end{array}$$

Memoryful Gol with recursion

Def. (interpretation $(\Gamma \vdash t : \tau)$)

$$(\Gamma \vdash \text{rec}(f:\sigma \Rightarrow \tau, x:\sigma.M) : \sigma \Rightarrow \tau) =$$



$$\frac{\begin{array}{c} [\![\Gamma]\!] \times [\![\sigma \Rightarrow \tau]\!] \times [\![\sigma]\!] \longrightarrow \Phi[\![\tau]\!] \\ \hline [\![\Gamma]\!] \times [\![\sigma \Rightarrow \tau]\!] \longrightarrow [\![\sigma \Rightarrow \tau]\!] \end{array}}{\begin{array}{c} [\![\Gamma]\!] \Rightarrow [\![\sigma \Rightarrow \tau]\!] \longrightarrow [\![\Gamma]\!] \Rightarrow [\![\sigma \Rightarrow \tau]\!] \\ \hline [\![\Gamma]\!] \longrightarrow [\![\sigma \Rightarrow \tau]\!] \end{array}}$$

Memoryful Gol with recursion

Thm. (soundness)

For closed terms M and N of type τ ,

- $\vdash M = N : \tau$ implies $([(M)]_{\sim}, [(N)]_{\sim}) \in \Phi[\tau]$
- $\vdash M = N : \text{nat}$ implies $(M) \simeq (N)$.

behavioral equivalence

- Moggi's equations for computational lambda-calculus
- equations for algebraic operations

$$M \sqcup M = M$$

$$E[\text{opr}(M_1, \dots, M_n)] = \text{opr}(E[M_1], \dots, E[M_n])$$

$$(\lambda x. M) (N_1 \sqcup N_2) = (\lambda x. M) N_1 \sqcup (\lambda x. M) N_2$$

$$\text{rec}(f : \sigma \Rightarrow \tau, x : \sigma. M) = \lambda x. M[\text{rec}(f : \sigma \Rightarrow \tau, x : \sigma. M)/f]$$

Memoryful Gol with recursion

Thm. (domain-theoretic characterization of Fix)

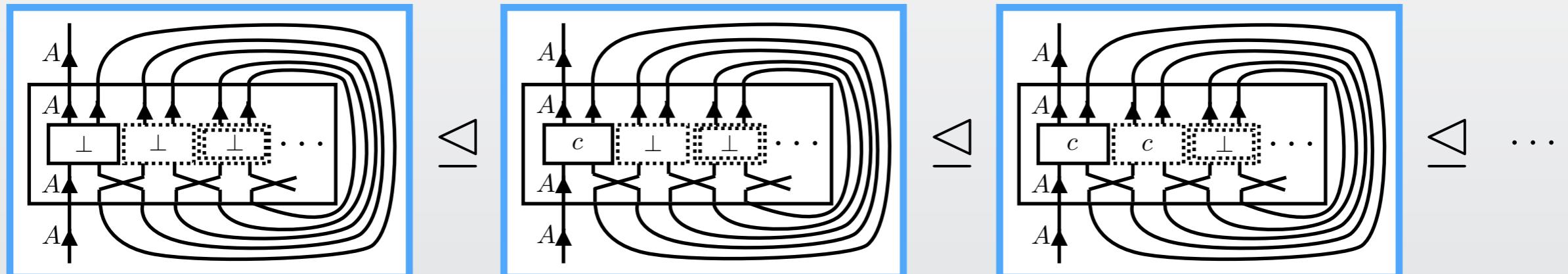
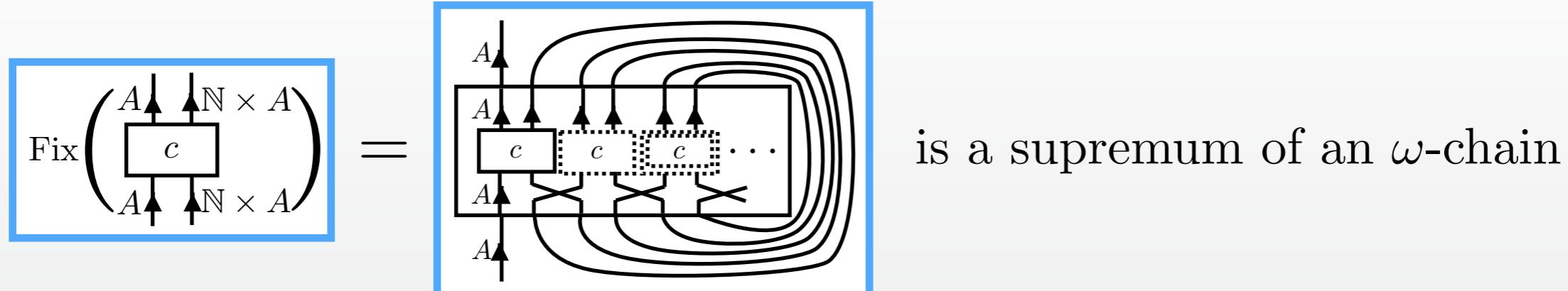
Under the assumption that

- \mathbf{Set}_T is a **Cppo**-enriched category with **Cppo**-enriched (countable) couplings
- compositions \circ_T of \mathbf{Set}_T is strict in the restricted form: $f \circ_T \perp = \perp$ and $\perp \circ_T (\eta_Y \circ g) = \perp$ hold for any $f: X \rightarrow TY$ and $g: X \rightarrow Y$ in \mathbf{Set}
- premonoidal structures $X \otimes -, - \otimes X$ of \mathbf{Set}_T is locally continuous and strict for any X in \mathbf{Set}

it holds that:

Memoryful Gol with recursion

Thm. (domain-theoretic characterization of Fix)



where $(X, c: X \times A \rightarrow T(X \times B), x_0 \in X) \leq (Y, c: Y \times A \rightarrow T(Y \times B), y_0 \in Y)$

$$\stackrel{\text{def}}{\iff} X = Y \wedge x = y \wedge c \sqsubseteq d \text{ in } \mathbf{Set}_T(X \times A, X \times B)$$

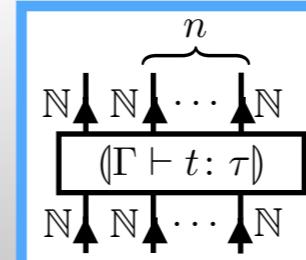
Memoryful Gol with recursion

effectful terms

$$(\lambda x : \text{nat}. x + x) (3 \sqcup 5) : \text{nat}$$

recursion

$\langle\Gamma \vdash t : \tau\rangle =$



translation

- based on Geometry of Interaction
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transducers

