Compiling Effectful Terms to Transducers

Prototype Implementation of
Memoryful Geometry of Interaction

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LOLA (Vienna), July 13, 2014

Compiling Effectful Terms to Transdu

TtT
("Terms to Transducers")

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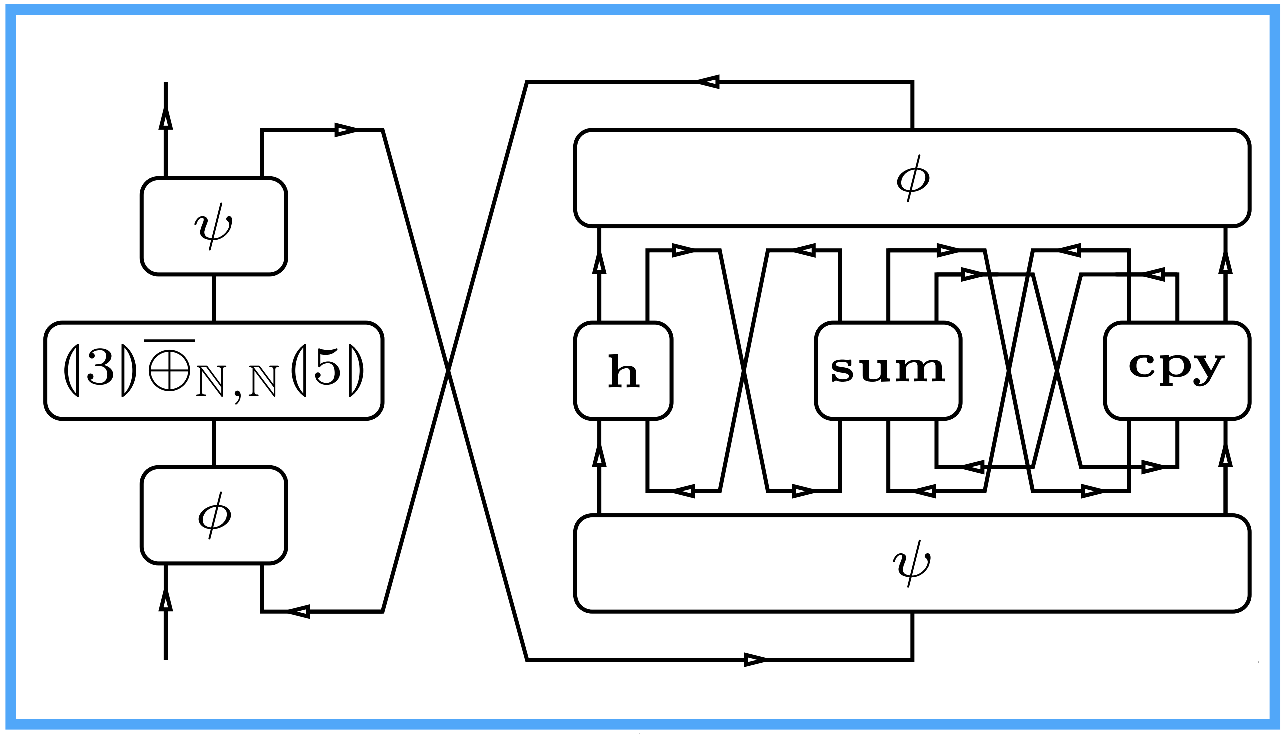
Our Tool *TtT*

"Terms to Transducers"

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$

TtT Compiler



TtT Simulator

terms

transducers

Overview

terms

← λ-terms with algebraic effects

TtT Compiler

← **memoryful GoI**
[Hoshino, —, Hasuo CSL-LICS '14]

transducers

← stream transducers

TtT Simulator

simulation result

Geometry of Interaction (GoI)

- semantics of linear logic proof [Girard ’89],

  functional programming

- token machine presentation [Mackie ’95]

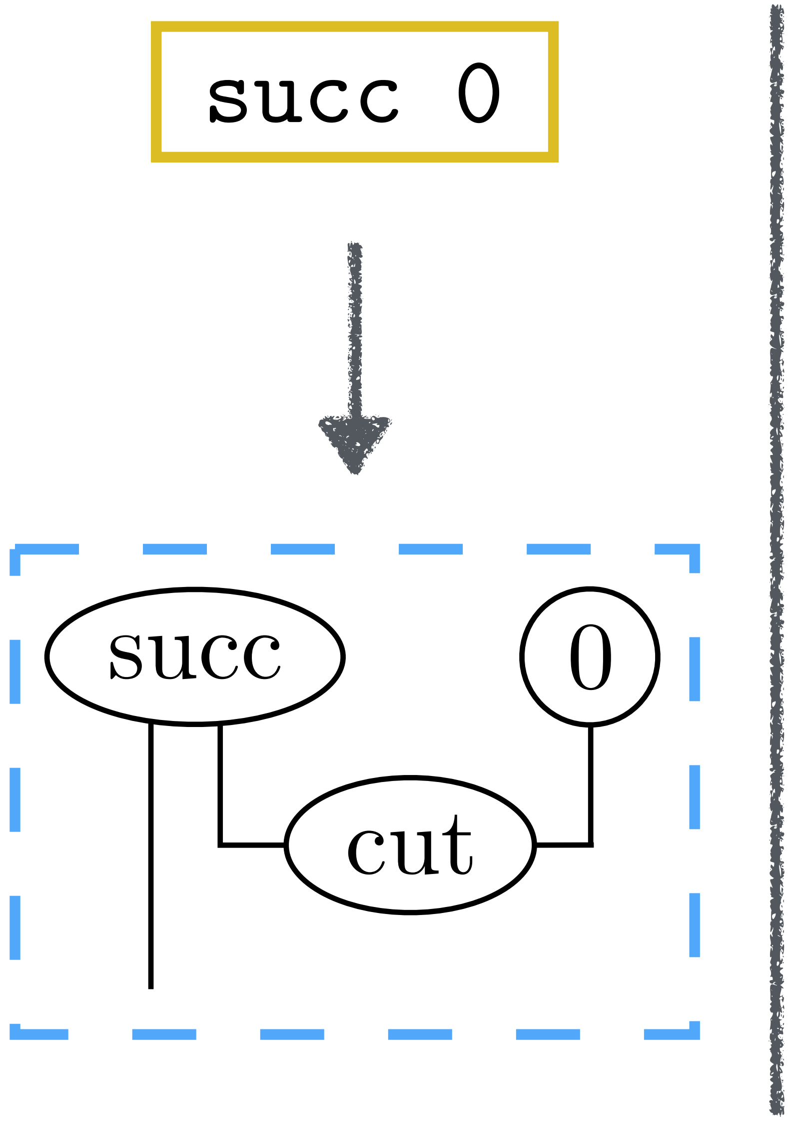
“GoI implementation”

compilation techniques and implementations

[Mackie ’95] [Pinto ’01] [Ghica ’07]

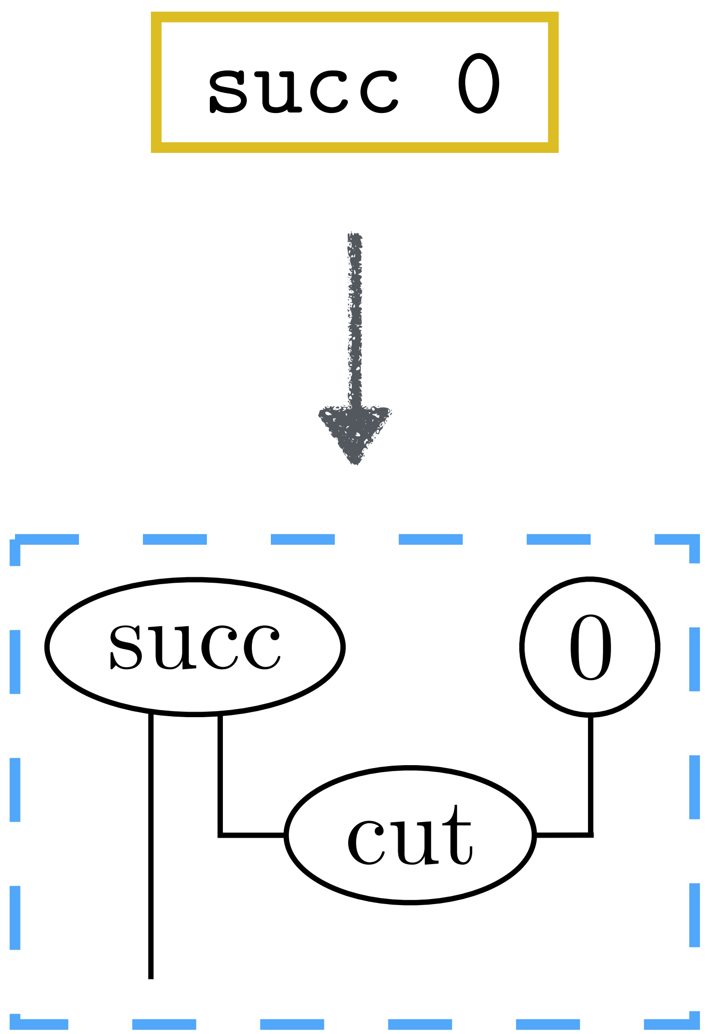
Geometry of Interaction (GoI)

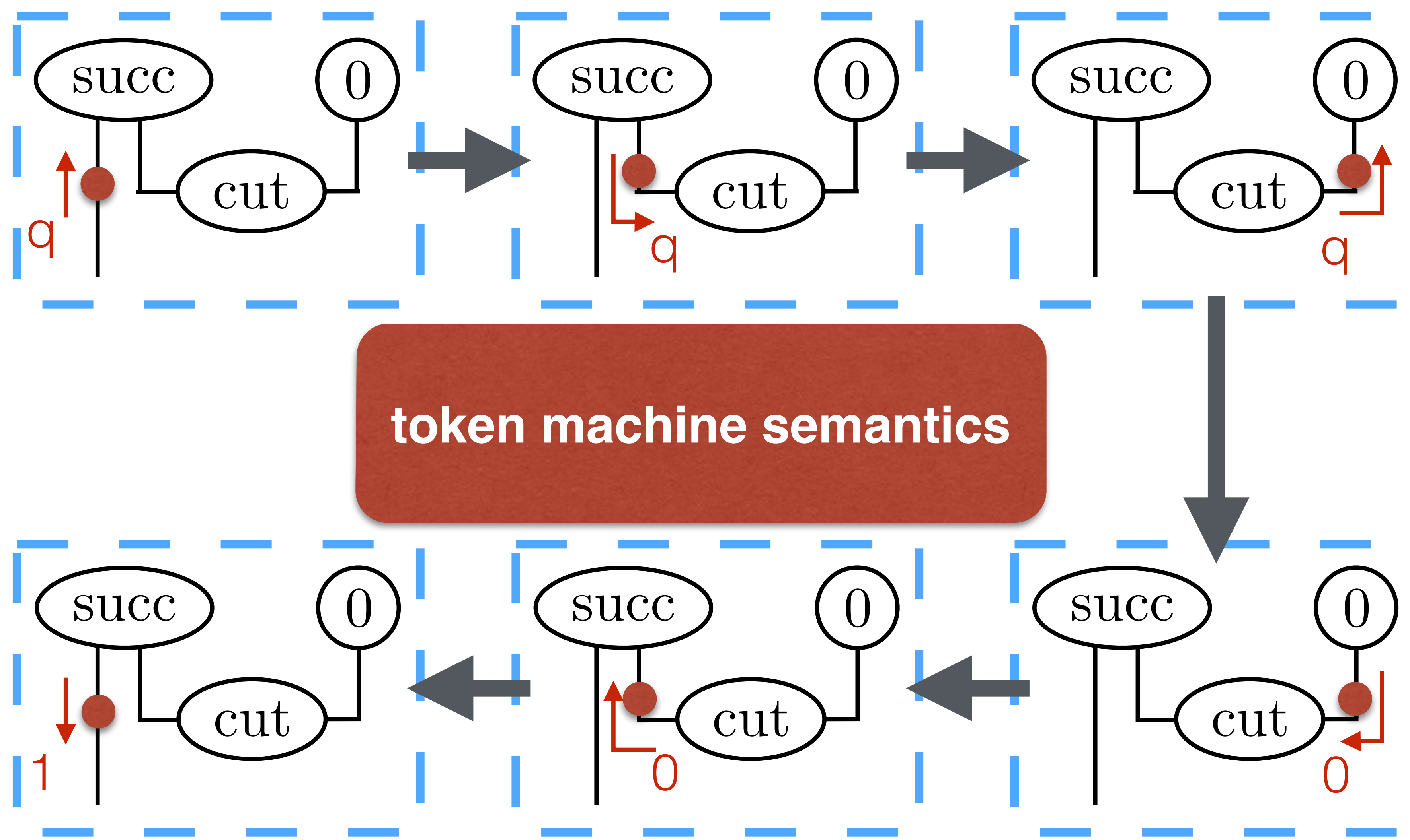
- token machine presentation [Mackie '95]



Geometry of Interaction (GoI)

- token machine presentation [Mackie '95]





Geometry of Interaction (GoI)

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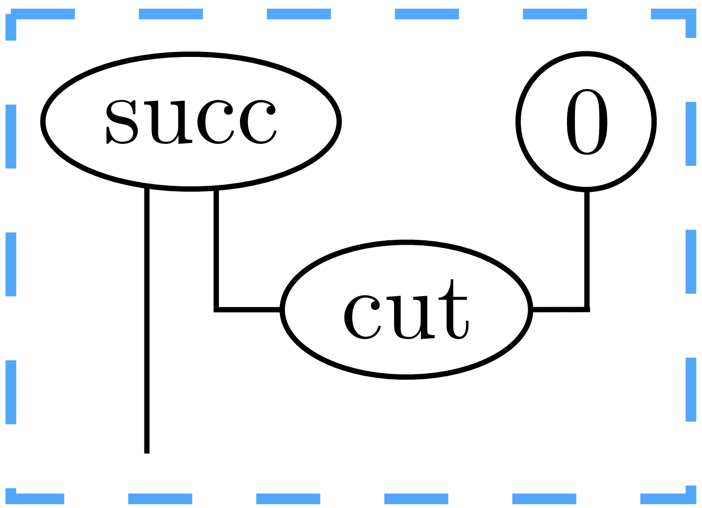
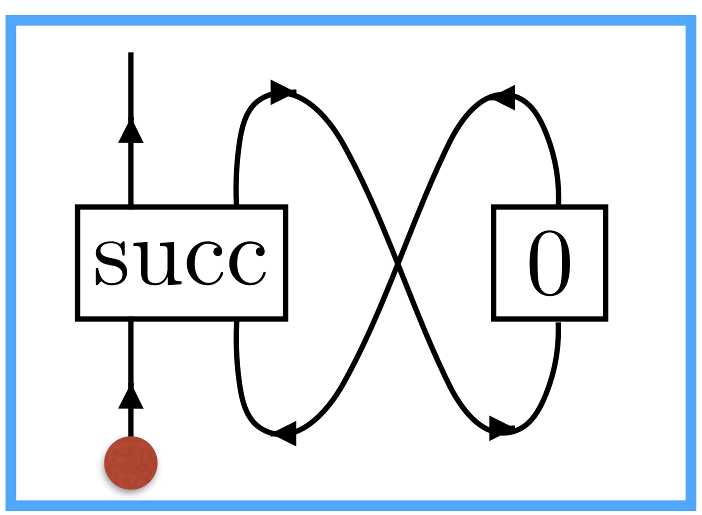


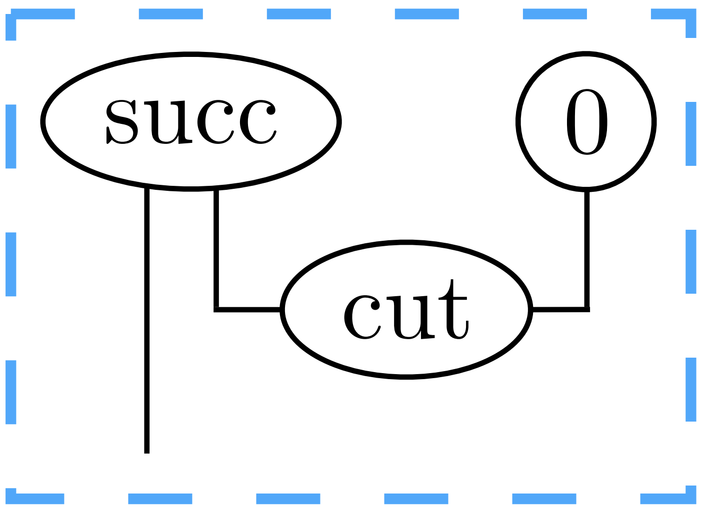
proof net style



string diagram style
in traced monoidal category

Geometry of Interaction (GoI)

- token machine presentation [Mackie '95]



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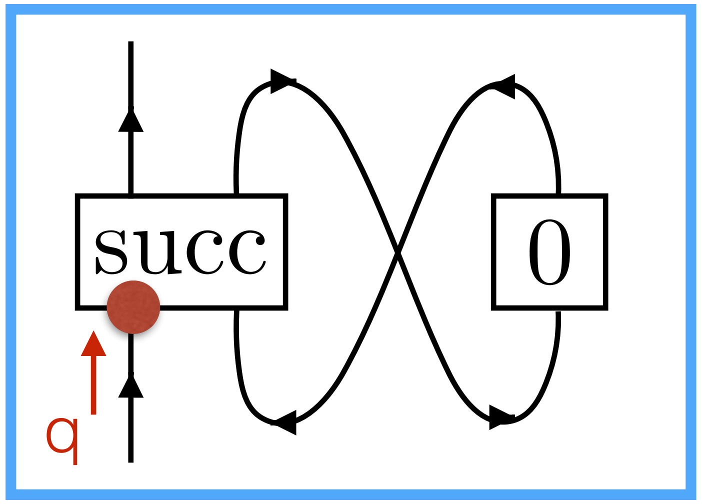

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Geometry of Interaction (GoI)

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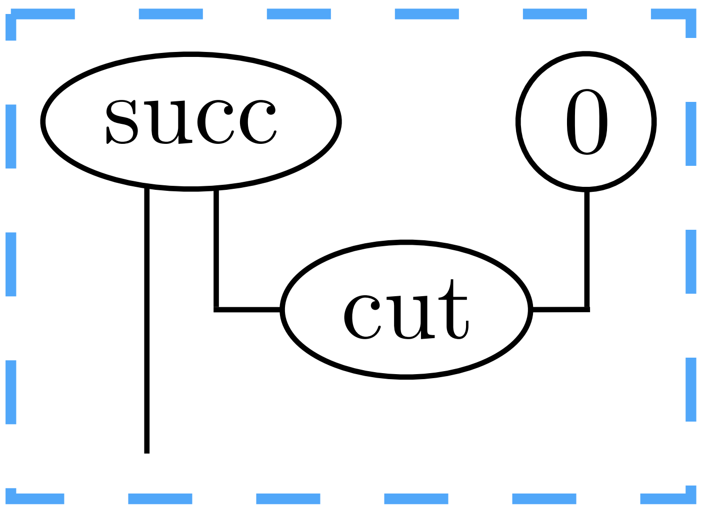
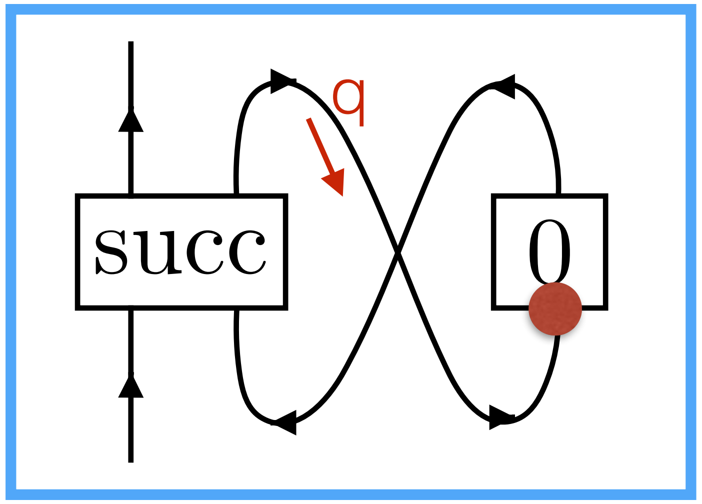


proof net style



string diagram style
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Geometry of Interaction (GoI)

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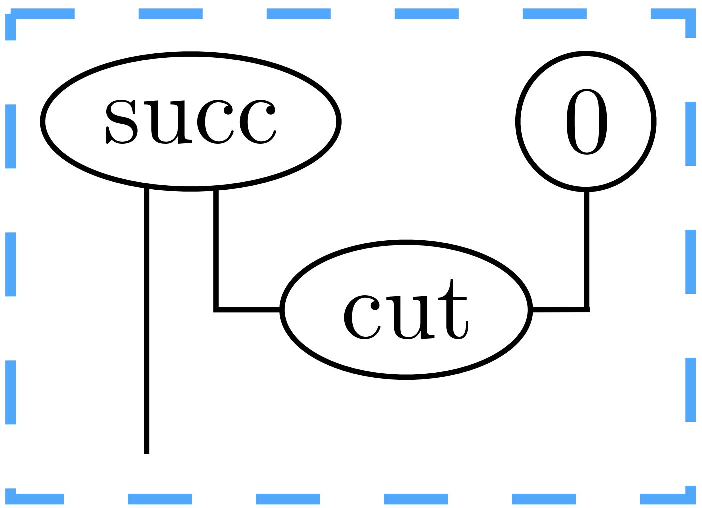
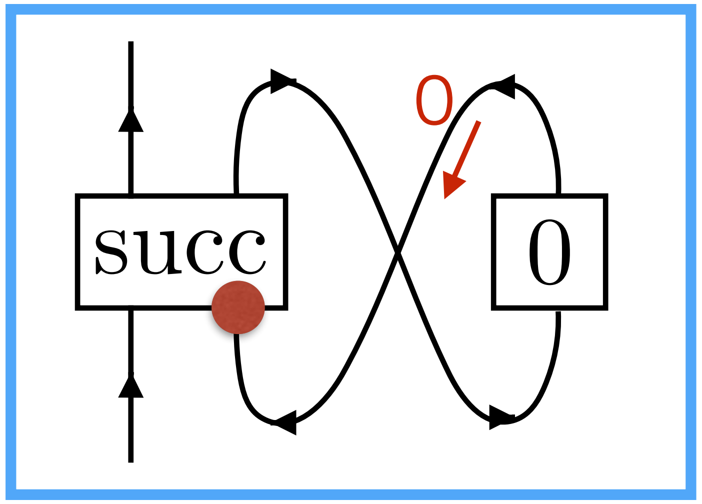


proof net style



string diagram style
in traced monoidal category

Geometry of Interaction (GoI)

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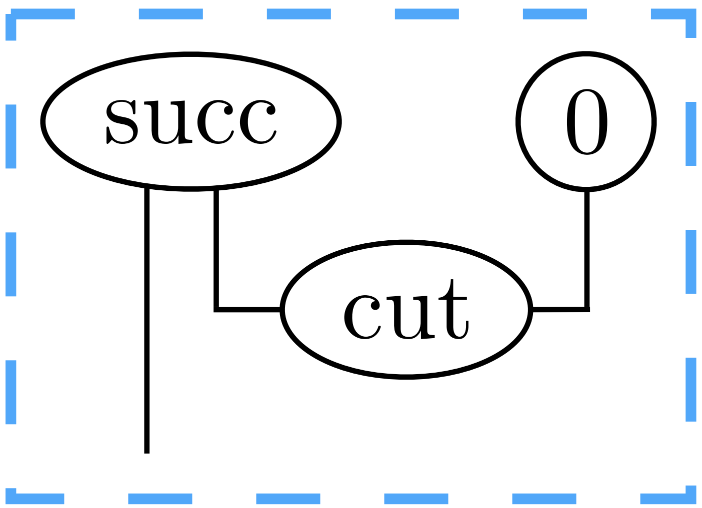

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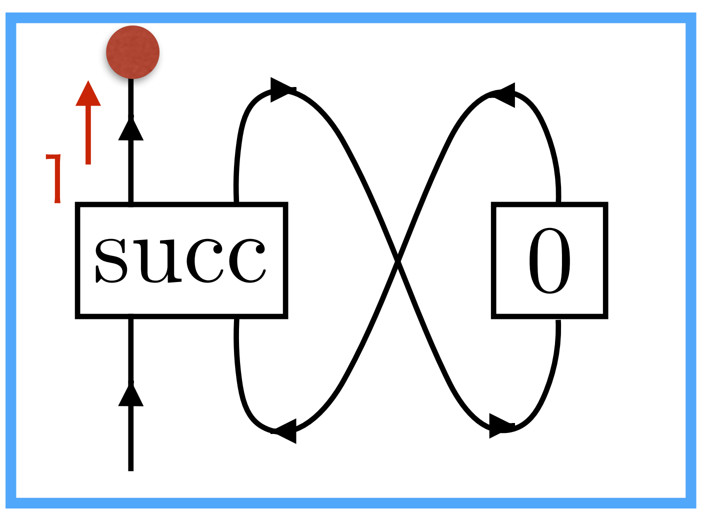
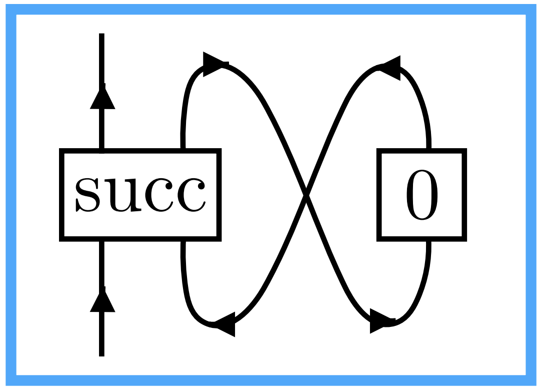


string diagram style
in traced monoidal category

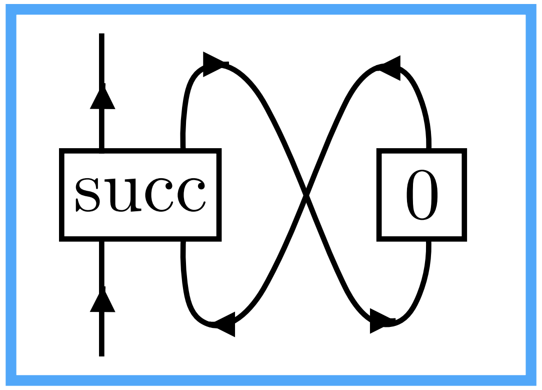
GoI is “memoryless”



- advantage: simplicity
- challenges
  - additive connectives $\&, \oplus$
  - computational effects

GoI is "memoryless"

- advantage: simplicity



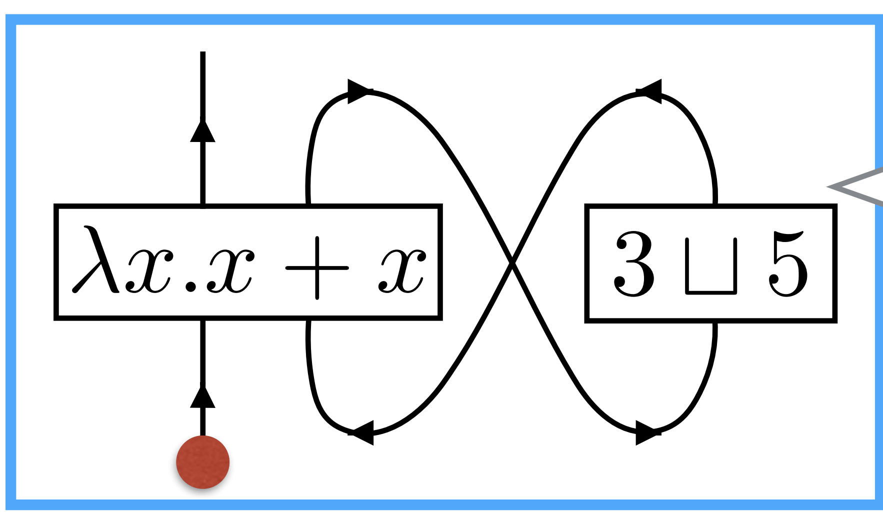
- challenges
  - additive connectives $\&, \oplus$
  - computational effects

additive slices
[Laurent '01]

GoI is "memoryless"

- challenge: computational effects

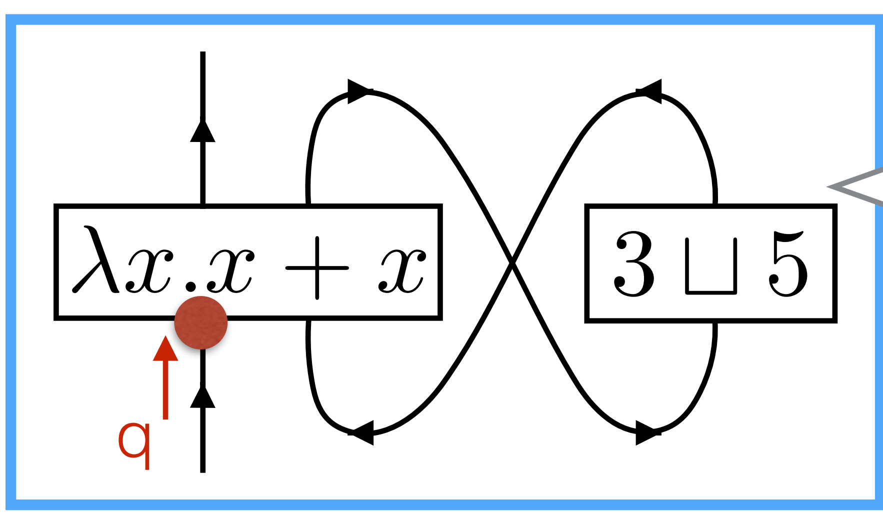
$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$



GoI is "memoryless"

- challenge: computational effects

$$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$$

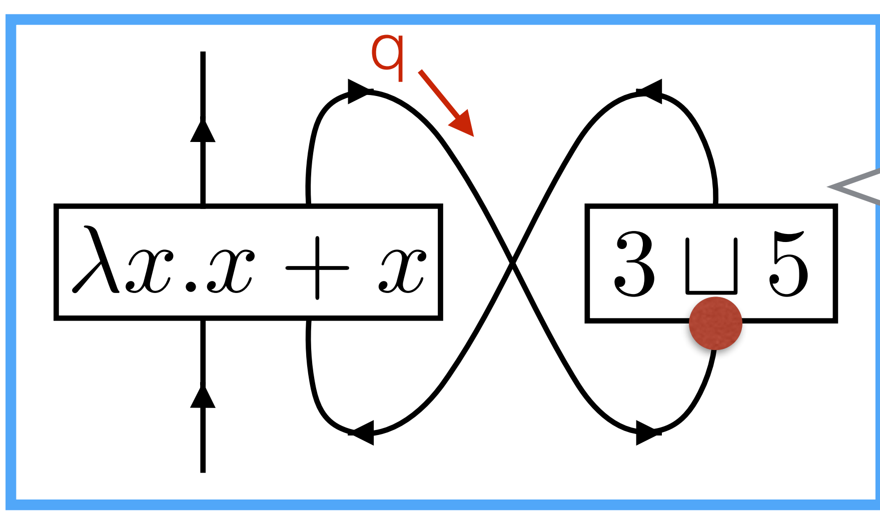


$\lambda x.x + x$	$3 \sqcup 5$

GoI is “memoryless”

- challenge: computational effects

$$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$$

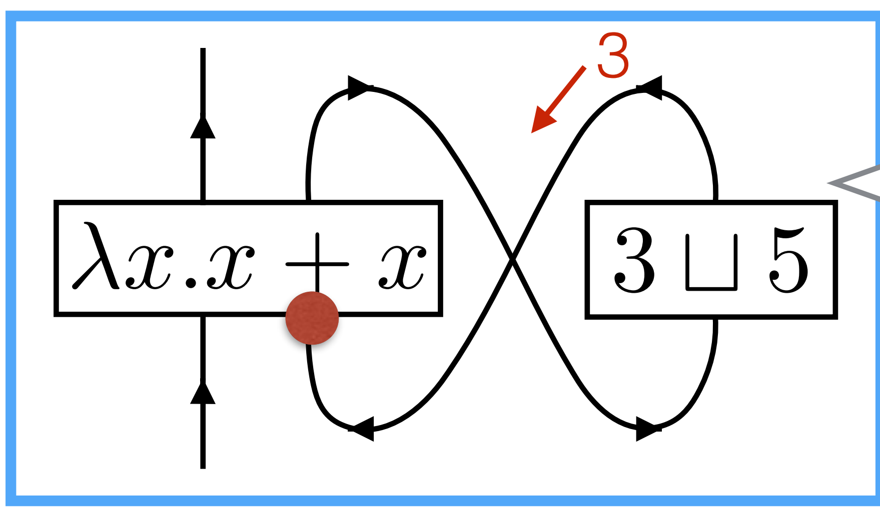


$\lambda x.x + x$	$3 \sqcup 5$
ask (left) x	

GoI is “memoryless”

- challenge: computational effects

$\boxed{(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}}$

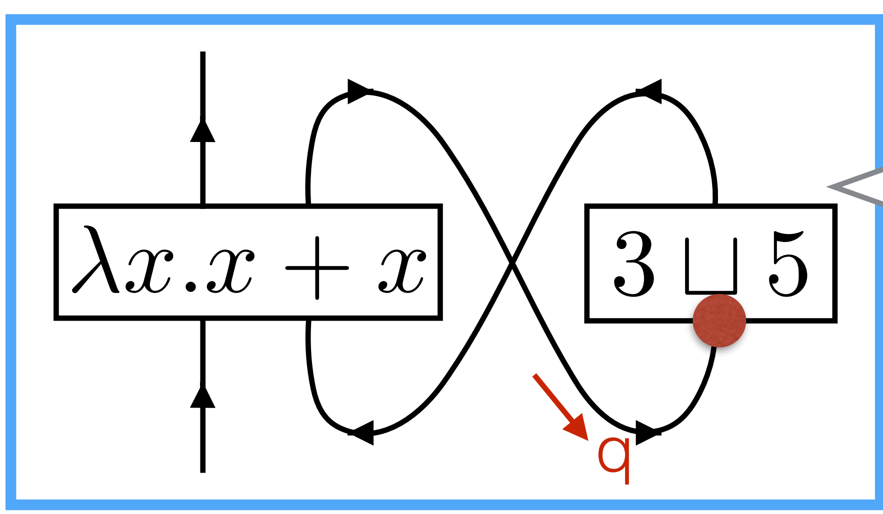


$\lambda x.x + x$	$3 \sqcup 5$
ask (left) x	
	answer 3

GoI is “memoryless”

- challenge: computational effects

$$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \qquad : \mathtt{nat}$$

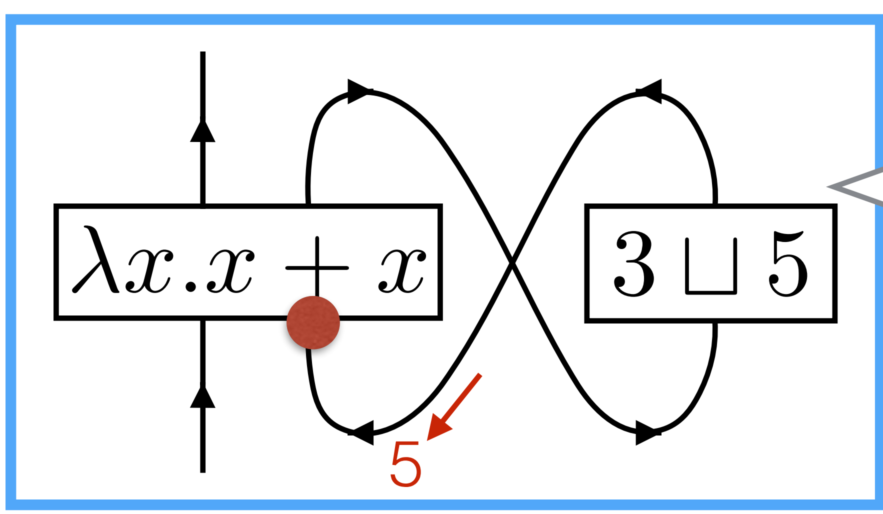


$\lambda x.x + x$	$3 \sqcup 5$
ask (left) x	
	answer 3
ask (right) x	

GoI is "memoryless"

- challenge: computational effects

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(\mathtt{3} \sqcup \mathtt{5}) \quad : \mathtt{nat}$

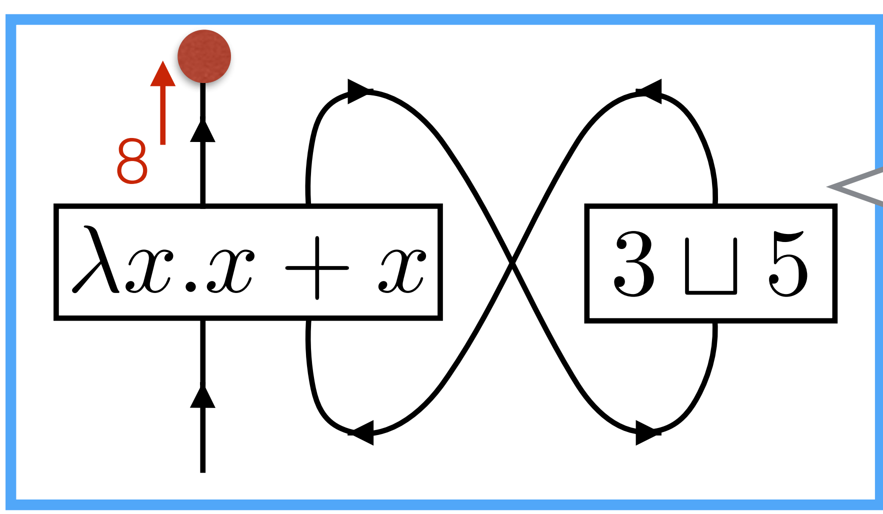


$\lambda x.x + x$	$3 \sqcup 5$
ask (left) x	
	answer 3
ask (right) x	
	answer 5

GoI is “memoryless”

- challenge: computational effects

$$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$$



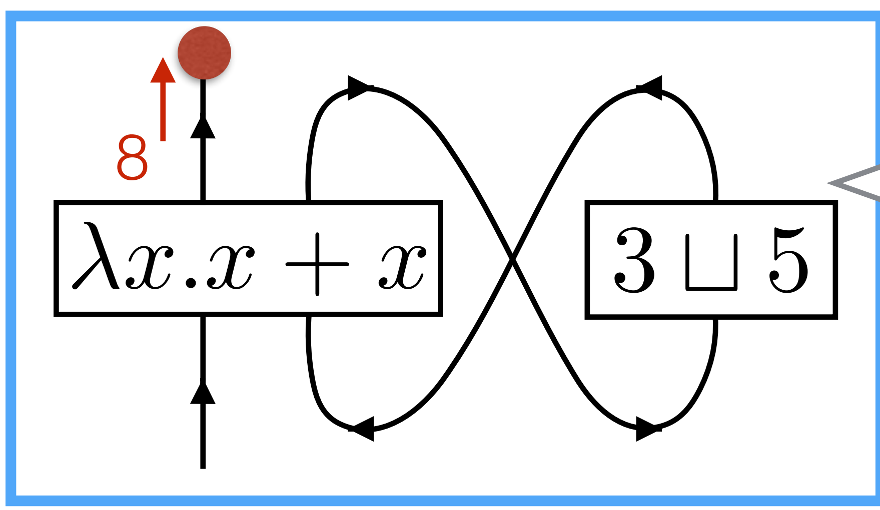
$\lambda x.x + x$	$3 \sqcup 5$
ask (left) x	
	answer 3
ask (right) x	
	answer 5
answer 8	

GoI is “memoryless”

memoryful GoI
[Hoshino, —, Hasuo CSL-LICS ’14]

- challenge: computational effects

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$



$\lambda x.x + x$	$3 \sqcup 5$
ask (left) x	
	answer 3
ask (right) x	
	answer 5
answer 8	

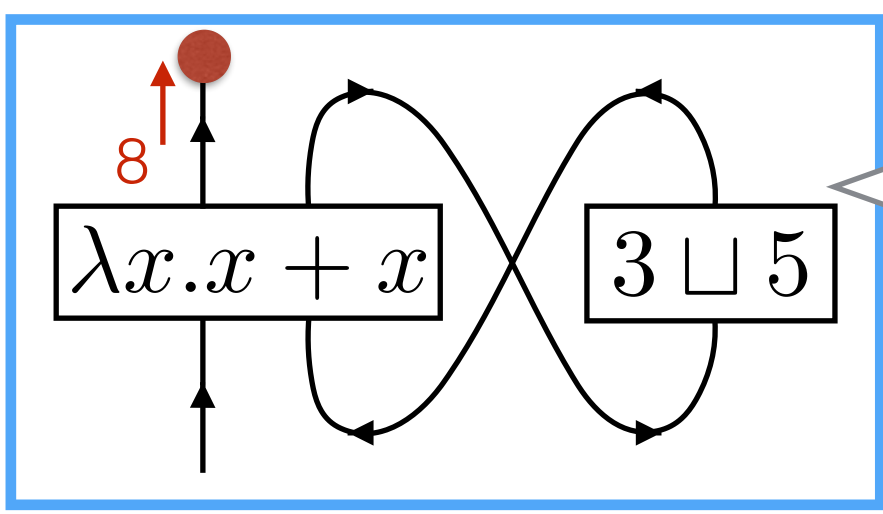
GoI is "memoryless"

memoryful GoI
[Hoshino, —, Hasuo CSL-LICS '14]

Thu., July 17
11:45 -

- challenge: computational effects

$$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$$



$\lambda x.x+x$	$3 \sqcup 5$
ask (left) x	
	answer 3
ask (right) x	
	answer 5
answer 8	

GoI is “memoryless”

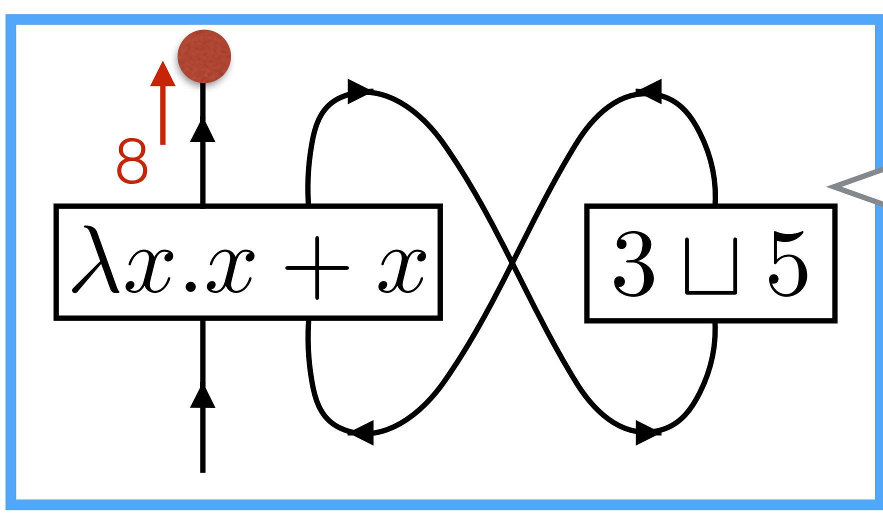
- challenge: computational effects

memoryful GoI
[Hoshino, —, Hasuo CSL-LICS ’14]

Thu., July 17
11:45 -

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup$

idea: equip each node with “memory”



$\lambda x.x + x$	$3 \sqcup 5$
ask (left) x	
	answer 3
ask (right) x	
	answer 5
answer 8	

Memoryful GoI — Input

terms

transducers

λ-terms with <u>algebraic effects</u>

algebraic operations [Plotkin, Power '03]

- nondeterministic choice
- probabilistic choice
- action on global state

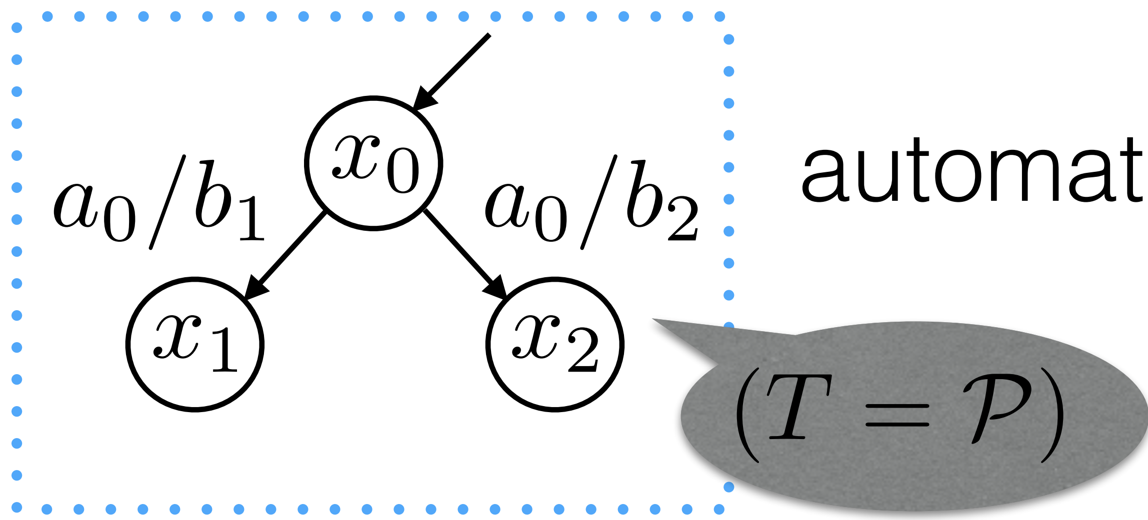
Memoryful GoI — Output

terms

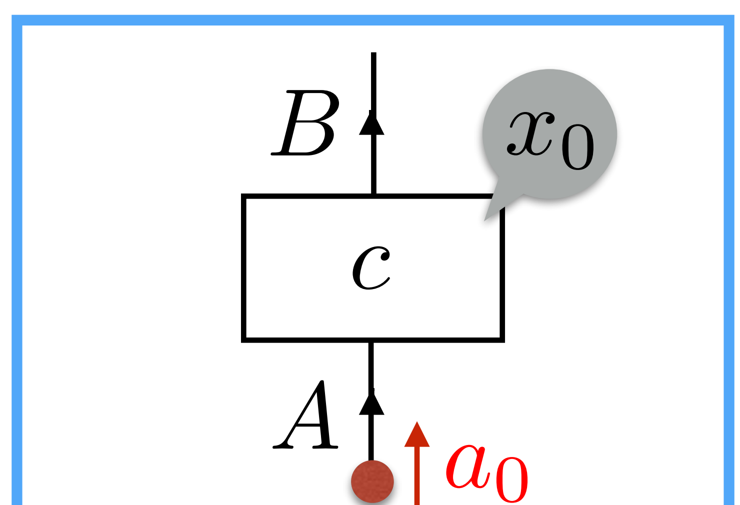
transducers

stream transducers (Mealy machines)

$$\mathcal{C} = (X, X \times A \xrightarrow{c} T(X \times B), x_0 \in X)$$



automaton style



string diagram style

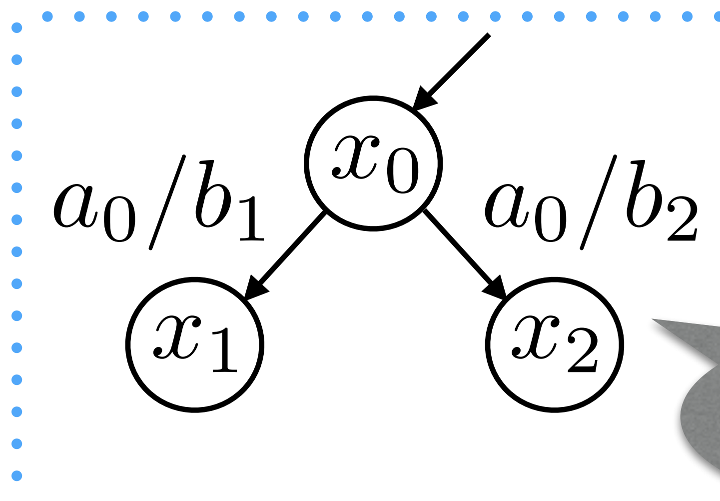
Memoryful GoI — Output

terms

transducers

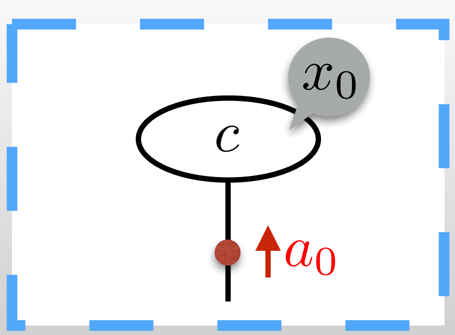
stream transducers (Mealy machines)

$$\mathcal{C} = (X, X \times A \xrightarrow{c} T(X \times B), x_0 \in X)$$

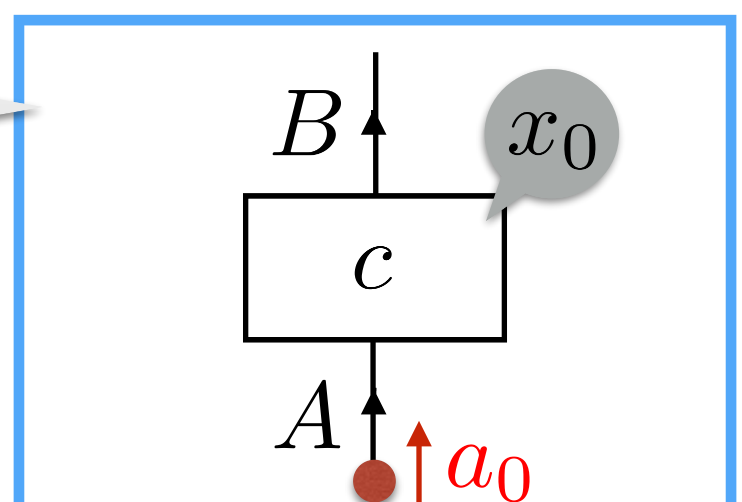


$(T = \mathcal{P})$

automaton style



proof net style



string diagram style

Memoryful GoI — Output

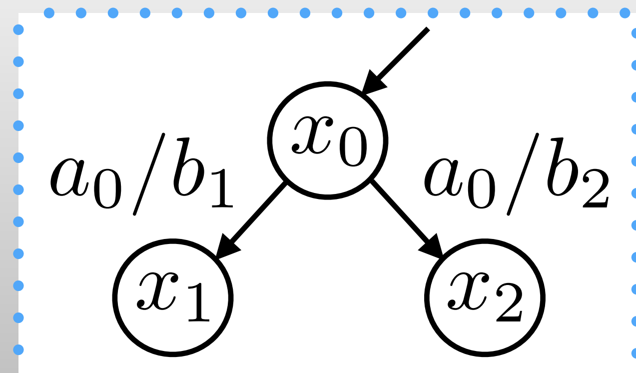
terms

transducers

stream transducers (Mealy machines)

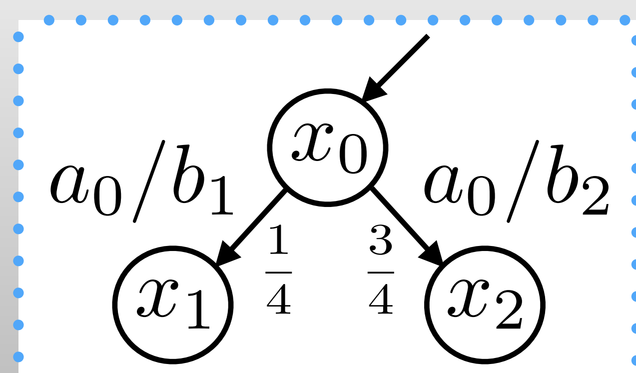
$$\mathcal{C} = (X,\ X \times A \xrightarrow{c} T(X \times B),\ x_0 \in X)$$

$T = \mathcal{P}$ $\quad (x_0, a_0) \longmapsto \{(x_1, b_1), (x_2, b_2)\}$



nondeterministic computation

$T = \mathcal{D}$ $\quad (x_0, a_0) \longmapsto \begin{bmatrix} (x_1, b_1) \mapsto 1/4, \\ (x_2, b_2) \mapsto 3/4 \end{bmatrix}$



probabilistic computation

Memoryful GoI — Translation

terms

↓

transducers

- idea: resumptions + categorical GoI

  [Abramsky, Haghverdi, Scott '02]

- use **coalgebraic component calculus**

  [Barbosa '03] [Hasuo, Jacobs '11]

  - **composition operations for software components**
  - **(many-sorted) process calculus**

Memoryful GoI — Translation

1. introduce component calculus over transducers

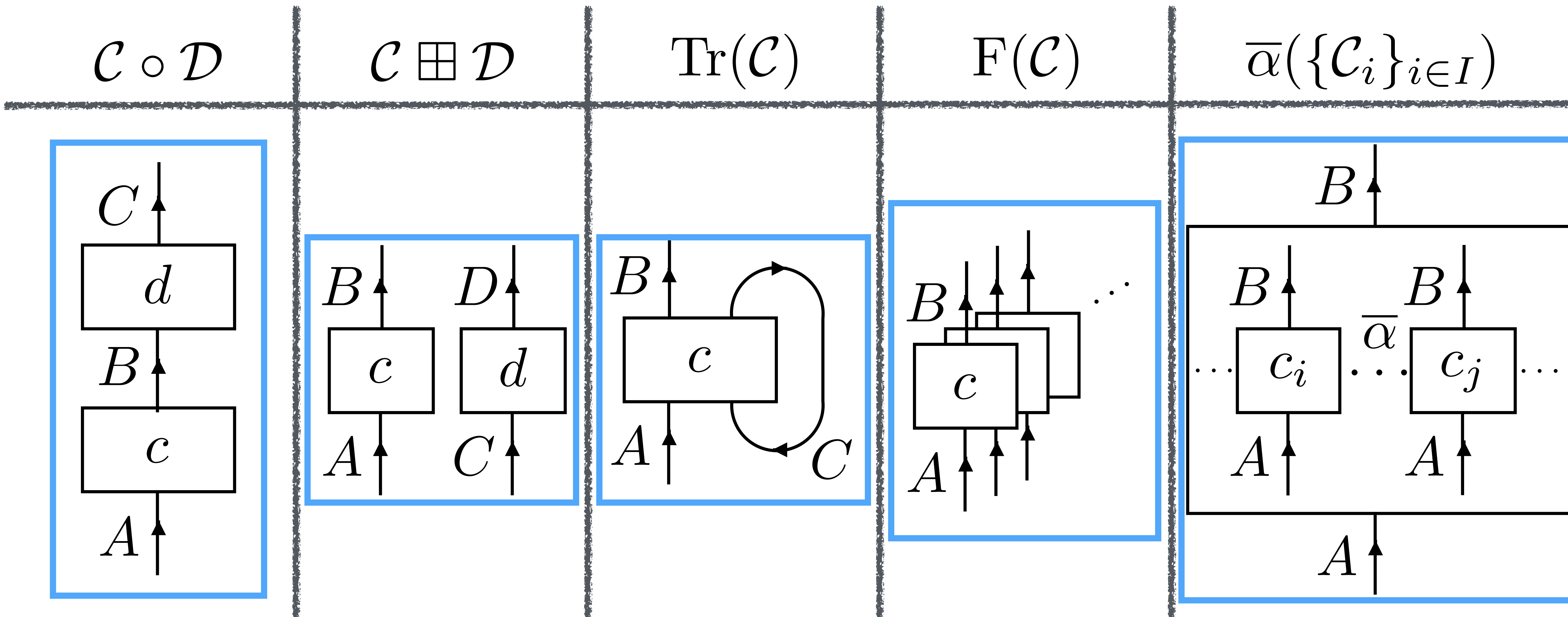
2. define interpretation inductively $(\!|\Gamma \vdash t\colon \tau|\!)$

$$(\!|\Gamma \vdash t\ \ s\colon \tau|\!)$$
$$= (\!|\Gamma \vdash t\colon \sigma \Rightarrow \tau|\!) \bullet (\!|\Gamma \vdash s\colon \sigma|\!)$$

3. prove soundness of interpretation $(\!|\Gamma \vdash t\colon \tau|\!)$

Memoryful GoI — Translation

Def. (component calculus)



(α: I-ary algebraic operation)

Memoryful GoI — Translation

Def. (component calculus)

sequential composition

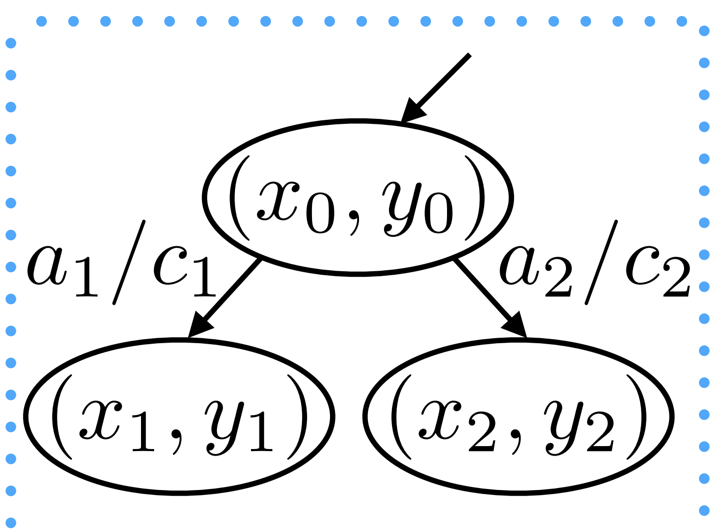
$\mathcal{C} \circ \mathcal{D}$

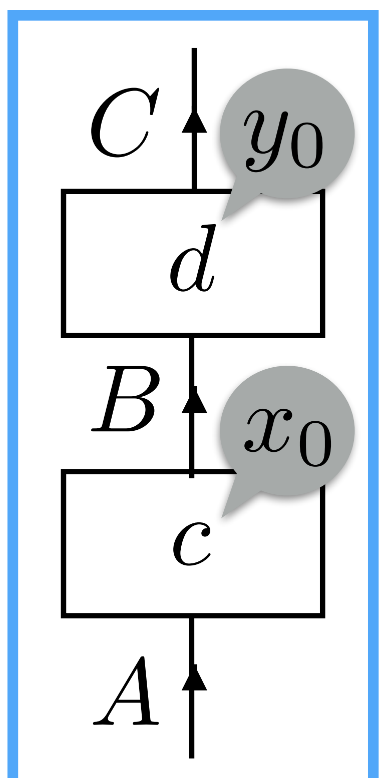
parallel composition

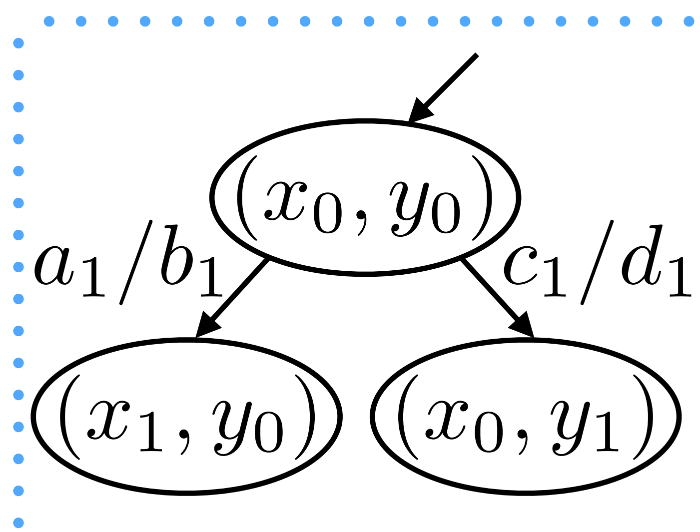
$\mathcal{C} \boxplus \mathcal{D}$

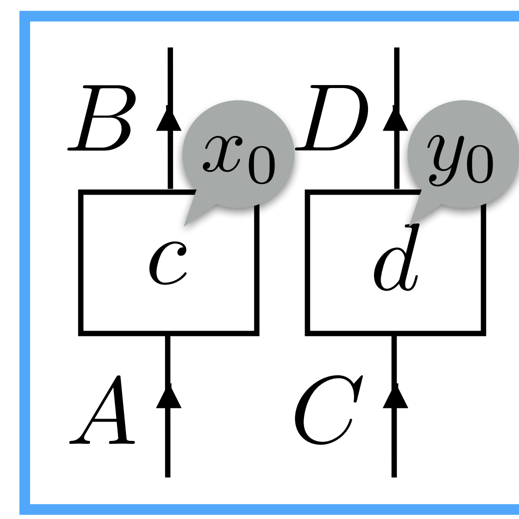
$$\begin{pmatrix} Y, \\ Y \times B \xrightarrow{d} T(Y \times C), \\ y_0 \in Y \end{pmatrix} \circ \begin{pmatrix} X, \\ X \times A \xrightarrow{c} T(X \times B), \\ x_0 \in X \end{pmatrix} = \begin{pmatrix} X \times Y, \\ \cdots \\ (x_0, y_0) \in X \times Y \end{pmatrix}$$

$$\begin{pmatrix} X, \\ X \times A \xrightarrow{c} T(X \times B), \\ x_0 \in X \end{pmatrix} \boxplus \begin{pmatrix} Y, \\ Y \times C \xrightarrow{d} T(Y \times D), \\ y_0 \in Y \end{pmatrix} = \begin{pmatrix} X \times Y, \\ \cdots \\ (x_0, y_0) \in X \times Y \end{pmatrix}$$









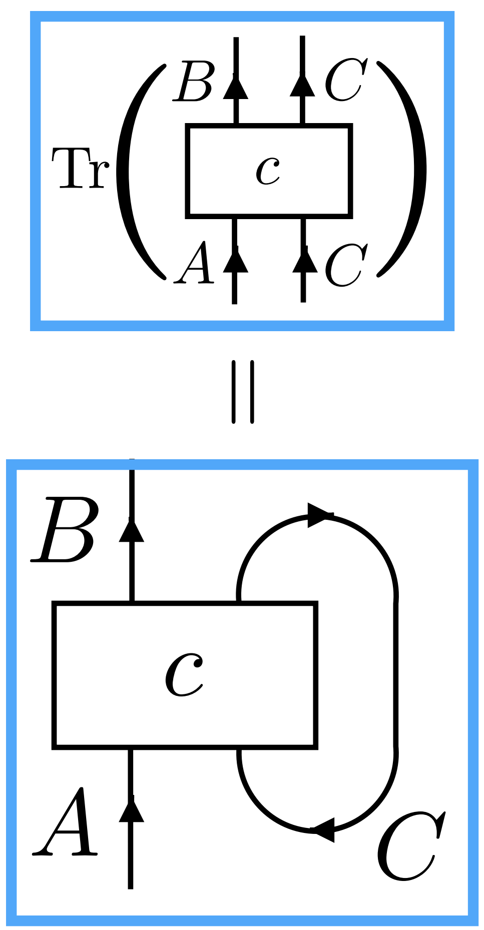
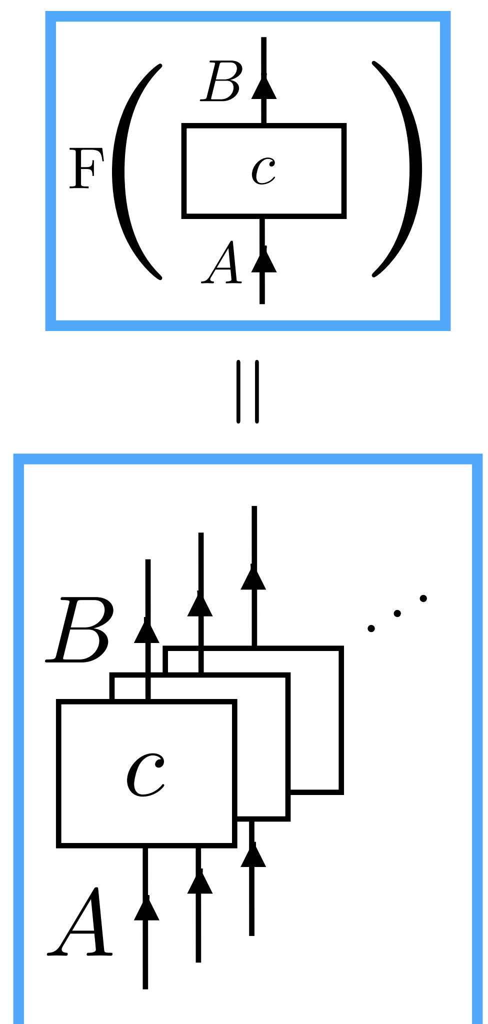
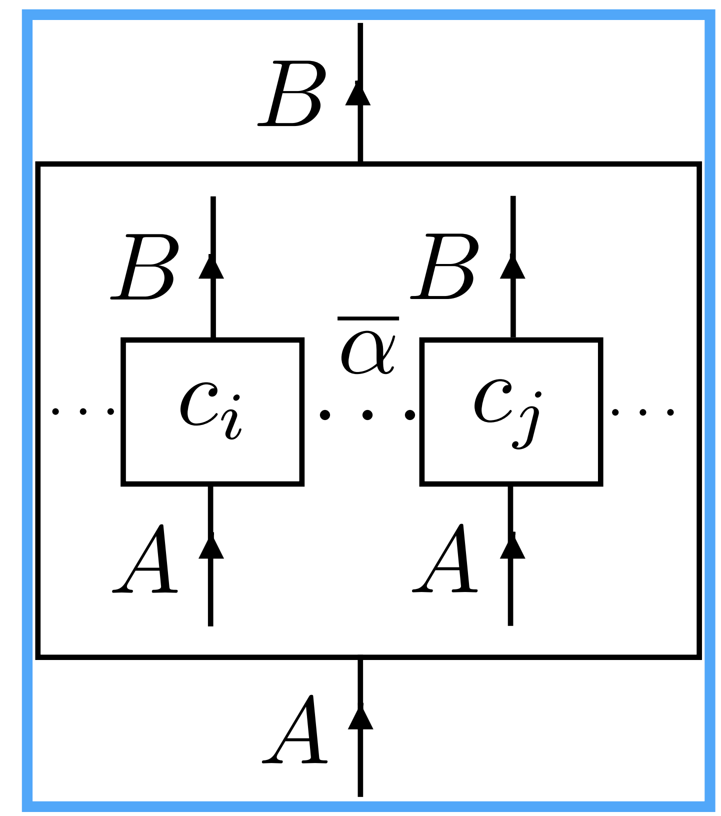
Memoryful GoI — Translation

Def. (component calculus)

application

!-modality

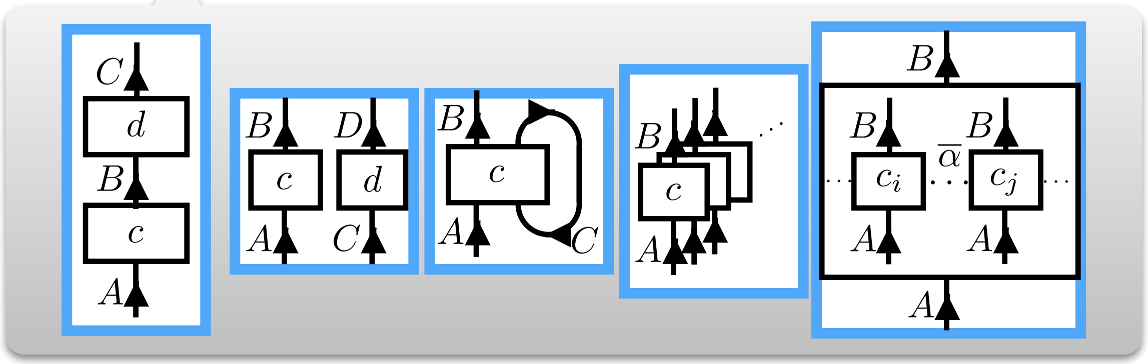
algebraic effect

$\mathrm{Tr}(\mathcal{C})$	$\mathrm{F}(\mathcal{C})$	$\overline{\alpha}(\{\mathcal{C}_i\}_{i\in I})$
		

(α: I-ary algebraic operation)

Memoryful GoI — Translation

1. introduce component calculus over transducers



2. define interpretation inductively $(\!|\Gamma \vdash t \colon \tau|\!)$

$$(\!|\Gamma \vdash t\ \ s \colon \tau|\!) = (\!|\Gamma \vdash t \colon \sigma \Rightarrow \tau|\!) \bullet (\!|\Gamma \vdash s \colon \sigma|\!)$$

3. prove soundness of interpretation $(\!|\Gamma \vdash t \colon \tau|\!)$

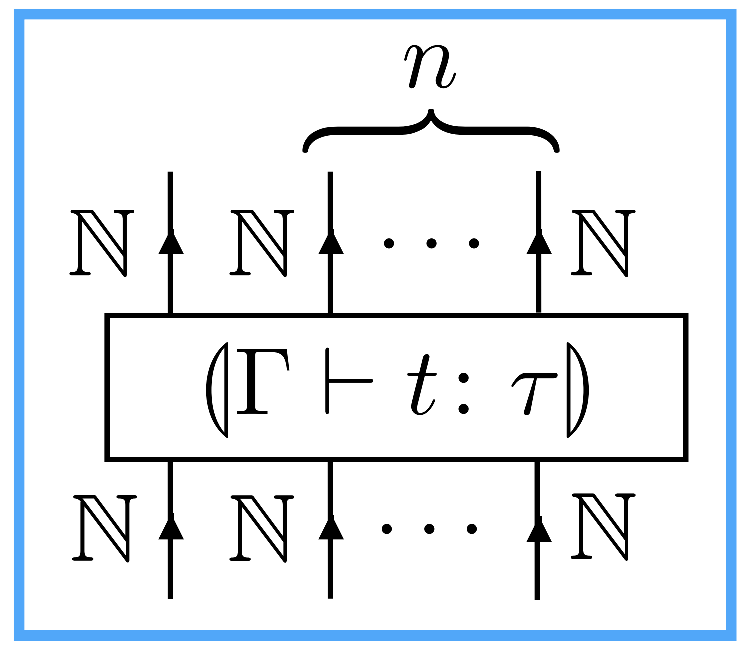
Memoryful GoI — Translation

Def. (interpretation $(\!|\Gamma \vdash t \colon \tau|\!)$)

For a type judgement $\Gamma \vdash t \colon \tau \;\; (\Gamma = x_1 \colon \tau_1, \ldots, x_n \colon \tau_n)$,

we inductively define

$$(\!|\Gamma \vdash t \colon \tau|\!) =$$



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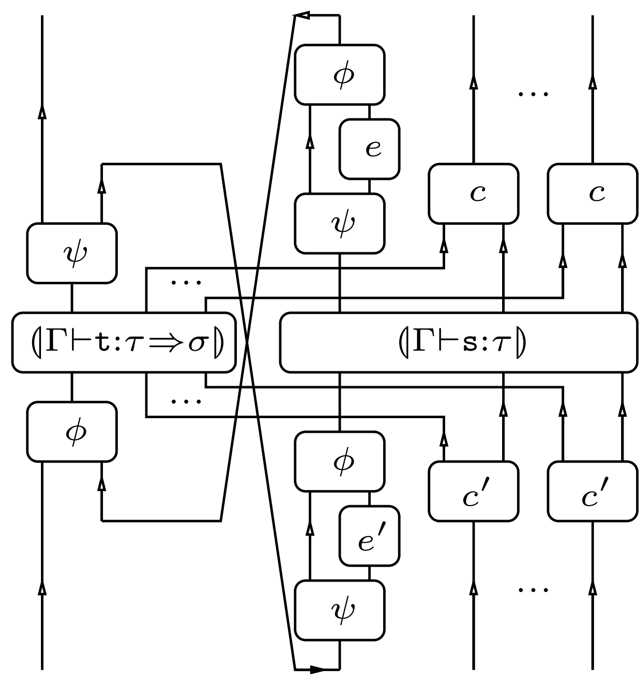
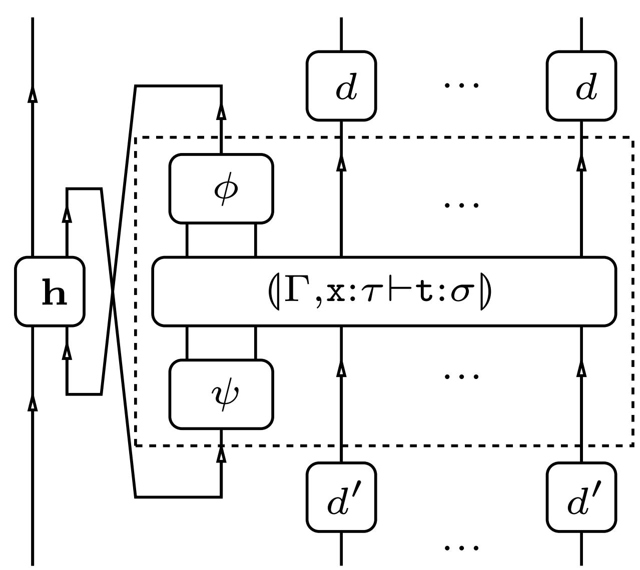
Memoryful GoI — Translation

Def. (interpretation $(\!|\Gamma \vdash t : \tau|\!)$)

$(\!|\Gamma \vdash \mathtt{t}\ \mathtt{s} : \sigma|\!) =$



$(\!|\Gamma \vdash \lambda\mathtt{x} : \tau.\, \mathtt{t} : \tau \Rightarrow \sigma|\!) =$



$$\frac{A \Rightarrow B}{!A \multimap B}$$

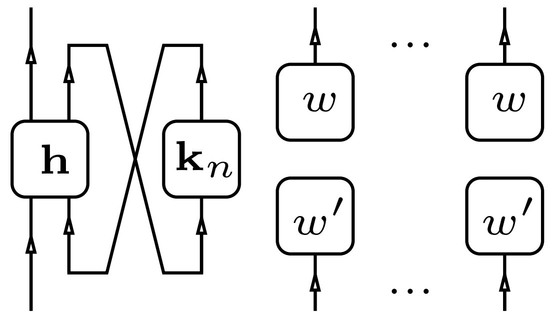
intuitionistic logic

linear logic

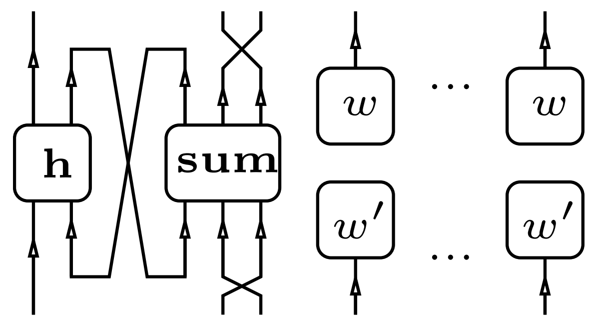
Memoryful GoI — Translation

<u>Def. (interpretation $(\!|\Gamma \vdash t : \tau|\!)$)</u>

$$(\!|\Gamma \vdash n : \mathbf{nat}|\!) =$$

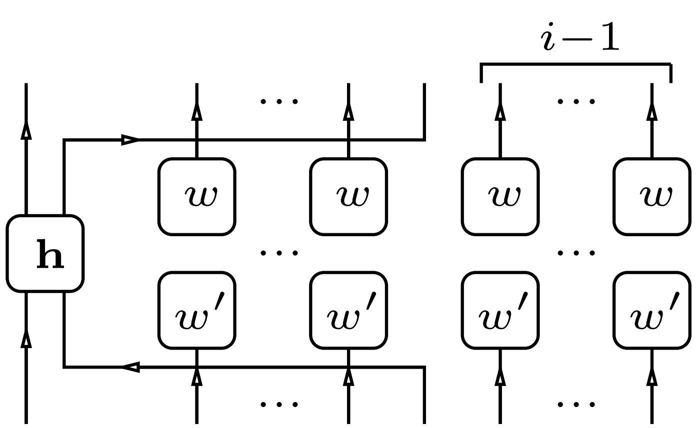


$$(\!|\Gamma, \mathtt{x} : \mathbf{nat}, \mathtt{y} : \mathbf{nat} \vdash \mathtt{x} + \mathtt{y} : \mathbf{nat}|\!) =$$



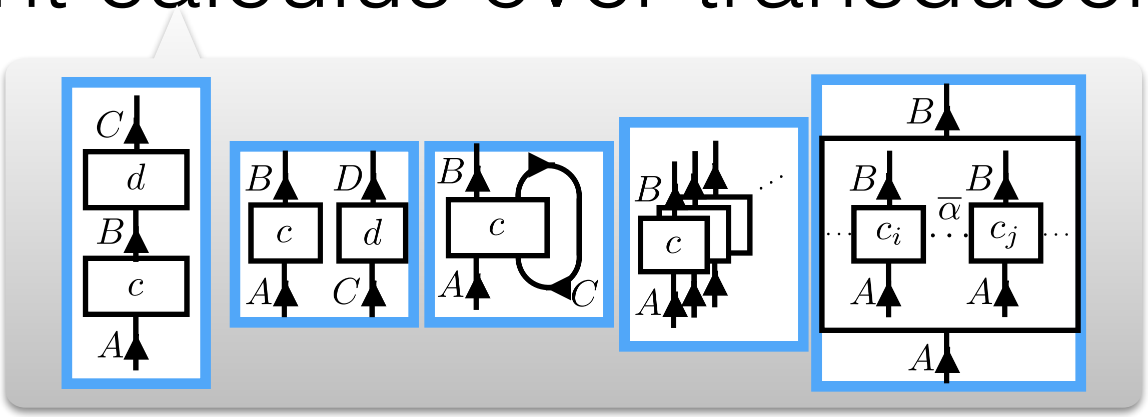
$$(\!|\Gamma \vdash \mathtt{t} + \mathtt{s} : \mathbf{nat}|\!) = (\!|\Gamma \vdash (\lambda \mathtt{xy} : \mathbf{nat}.\, \mathtt{x} + \mathtt{y})\, \mathtt{t}\, \mathtt{s} : \mathbf{nat}|\!)$$

$$(\!|\mathtt{x}_1 : \tau_1, \cdots, \mathtt{x}_n : \tau_n \vdash \mathtt{x}_i : \tau_i|\!) =$$

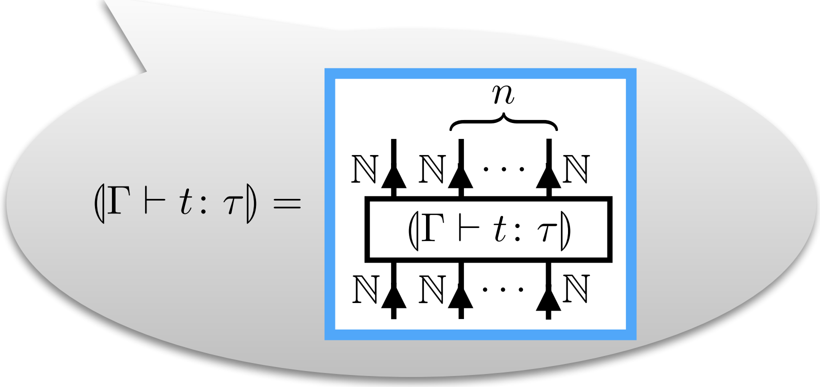


Memoryful GoI — Translation

1. introduce component calculus over transducers



2. define interpretation inductively $(\!|\Gamma \vdash t\colon \tau|\!)$



3. prove soundness of interpretation $(\!|\Gamma \vdash t\colon \tau|\!)$

Memoryful GoI — Translation

Thm. (soundness)

Theorem 6.2 (Soundness). *For closed terms* $\mathtt{t}$ *and* $\mathtt{s}$ *of type* τ*,*

- *If* $\mathtt{t} \approx \mathtt{s}$*, then* $([\llparenthesis \mathtt{t} \rrparenthesis], [\llparenthesis \mathtt{s} \rrparenthesis]) \in \Phi[\![\tau]\!]$.
- *If* $\mathtt{t} \approx \mathtt{s}$ *and* τ *is the base type* $\mathtt{nat}$*, then* $\llparenthesis \mathtt{t} \rrparenthesis \simeq^{T}_{\mathbb{N},\mathbb{N}} \llparenthesis \mathtt{s} \rrparenthesis$.

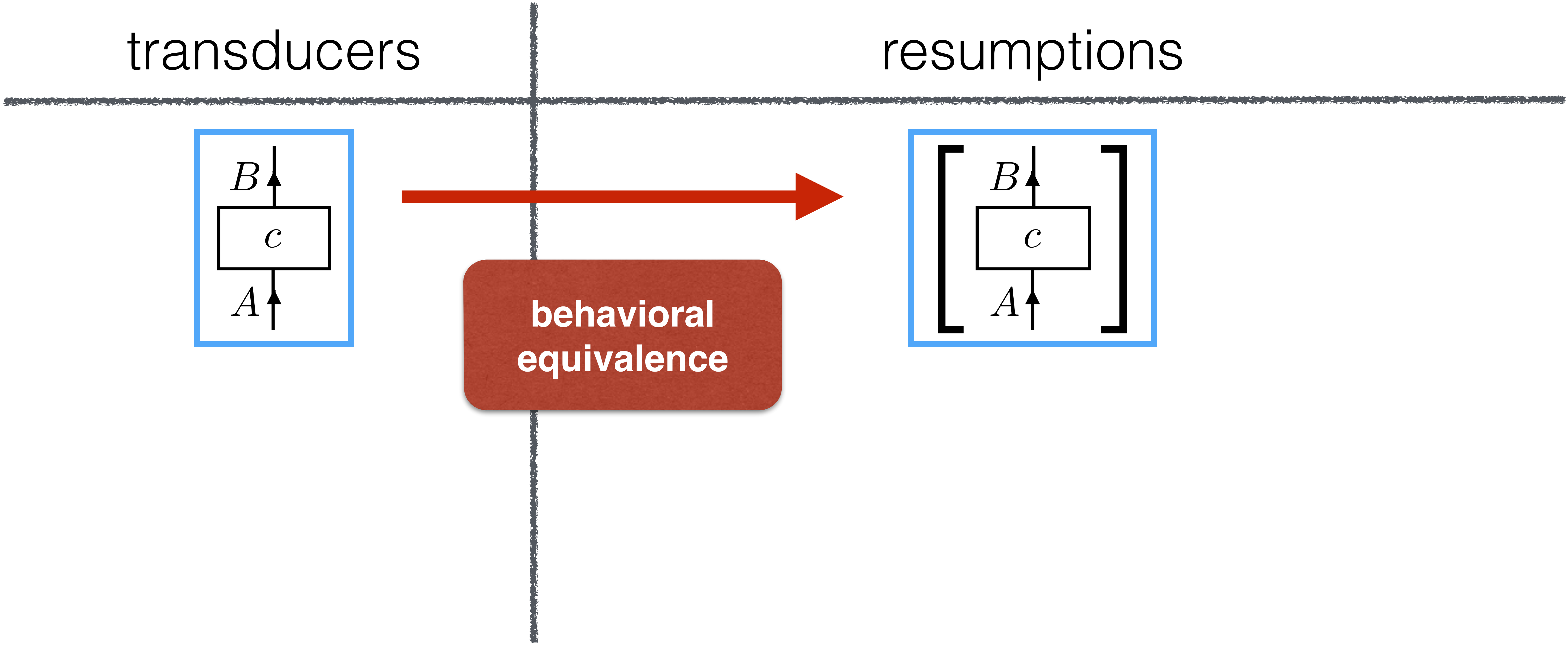
where $[\llparenthesis \mathtt{t} \rrparenthesis]$ *is the* $\mathbf{Res}(T)$*-morphism represented by* $\llparenthesis \mathtt{t} \rrparenthesis$*, and we write* $\mathtt{t} \approx \mathtt{s}$ *when the equation holds in the extension of the computational lambda calculus. For example, we have*

$$\mathtt{v}\,(3 \sqcup 5) \approx \mathtt{v}\,3 \sqcup \mathtt{v}\,5, \qquad 3 \sqcup 5 \sqcup 3 \approx 3 \sqcup 5 \approx 5 \sqcup 3$$

for any value $\mathtt{v}$ *when the extension of the computational lambda calculus has nondeterminism.*

Memoryful GoI — Translation

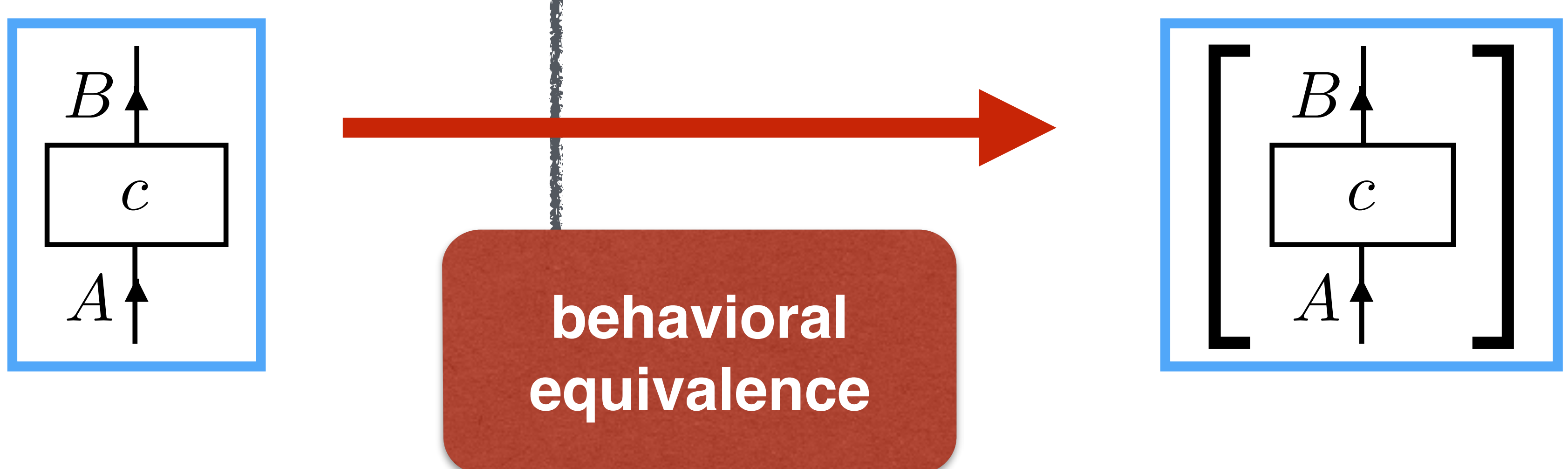
proof (soundness)



Memoryful GoI — Translation

<u>proof (soundness)</u>

transducers | resumptions

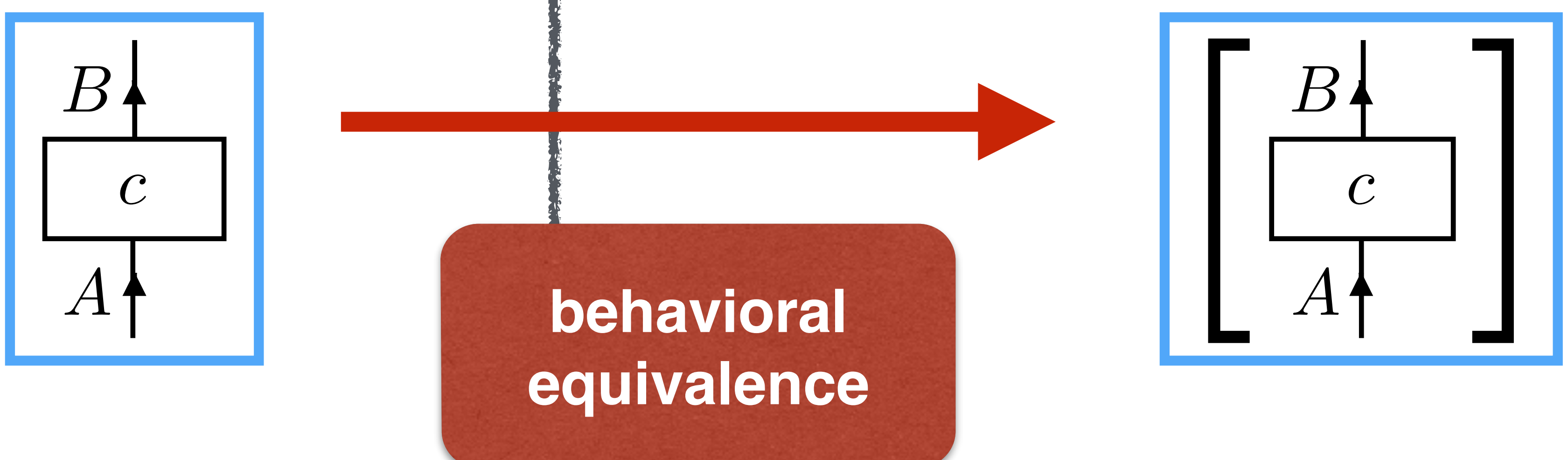


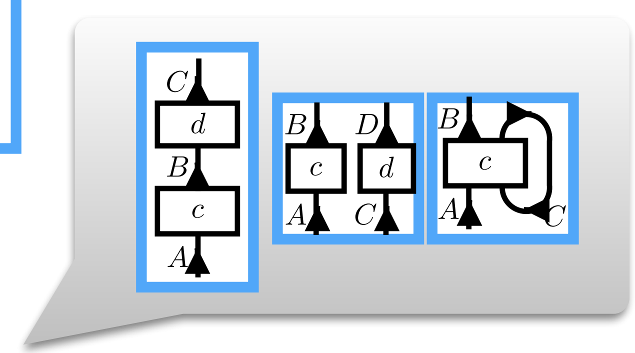
GoI situation $\left(\begin{array}{c}(\mathbf{Res}(T), \emptyset, \boxplus, \mathrm{Tr}), \\ F, J_0\phi, J_0\psi, J_0u, J_0v\end{array}\right)$

Memoryful GoI — Translation

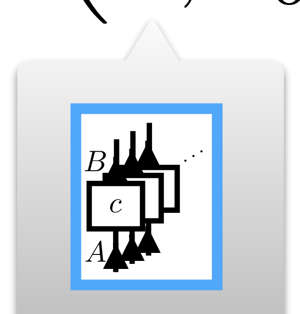
<u>proof (soundness)</u>

transducers	resumptions





GoI situation $\left(\begin{array}{c}(\mathbf{Res}(T), \emptyset, \boxplus, \mathrm{Tr}), \\ F, J_0\phi, J_0\psi, J_0u, J_0v\end{array}\right)$



$\phi : \mathbb{N} + \mathbb{N} \cong \mathbb{N} : \psi$

$u : \mathbb{N} \times \mathbb{N} \cong \mathbb{N} : v$

Memoryful GoI — Translation

proof (soundness)

resumptions	partial equivalence relations (per's) on resumptions
GoI situation $\begin{pmatrix} (\mathbf{Res}(T), \emptyset, \boxplus, \mathrm{Tr}), \\ F, J_0\phi, J_0\psi, J_0u, J_0v \end{pmatrix}$ →	cartesian closed category $\mathbf{Per}(T)$

categorical GoI [Abramsky, Haghverdi, Scott '02]
realizability

Memoryful GoI — Translation

proof (soundness)

resumptions	partial equivalence relations (per's) on resumptions
GoI situation $\left(\begin{array}{c}(\mathbf{Res}(T), \emptyset, \boxplus, \mathrm{Tr}), \\ F, J_0\phi, J_0\psi, J_0 u, J_0 v\end{array}\right)$ →	cartesian closed category $\mathbf{Per}(T)$ monad Φ on $\mathbf{Per}(T)$

categorical GoI [Abramsky, Haghverdi, Scott '02]
realizability

Memoryful GoI — Translation

<u>proof (soundness)</u>

transducers	resumptions	partial equivalence relations (per's) on resumptions
		cartesian closed category $\mathbf{Per}(T)$
		monad Φ on $\mathbf{Per}(T)$
		$\downarrow$ denotational semantics
		$[\![\vdash t\colon \tau]\!] =$ equivalence class of $\Phi[\![\tau]\!] \in \mathbf{Per}(T)$

Memoryful GoI — Translation

<u>proof (soundness)</u>

transducers	resumptions	partial equivalence relations (per's) on resumptions
		cartesian closed category $\mathbf{Per}(T)$
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		denotational semantics
$(\!\vdash t \colon \tau\!)$		$[\![\vdash t \colon \tau]\!] =$ equivalence class of $\Phi[\![\tau]\!] \in \mathbf{Per}(T)$

Memoryful GoI — Translation

proof (soundness)

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		denotational semantics
$(\!\vert \vdash t : \tau \vert\!)$ →		$[\![\vdash t : \tau]\!] =$ equivalence class of $\Phi[\![\tau]\!] \in \mathbf{Per}(T)$

Memoryful GoI — Translation

<u>proof (soundness)</u>

transducers	resumptions	partial equivalence relations (per's) on resumptions
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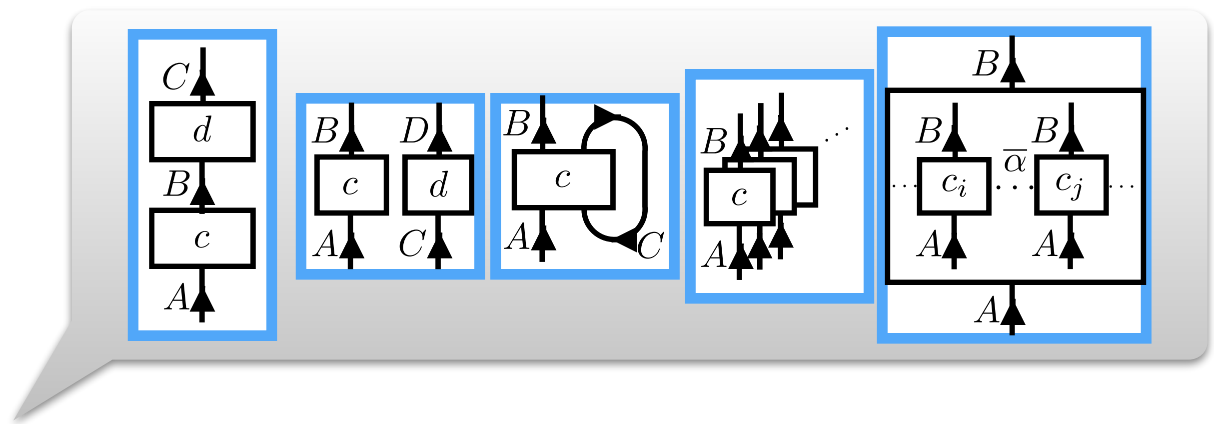
$(\!\vert \vdash t \colon \tau \vert\!)$		$[\![\vdash t \colon \tau]\!] =$ equivalence class of $\Phi[\![\tau]\!] \in \mathbf{Per}(T)$

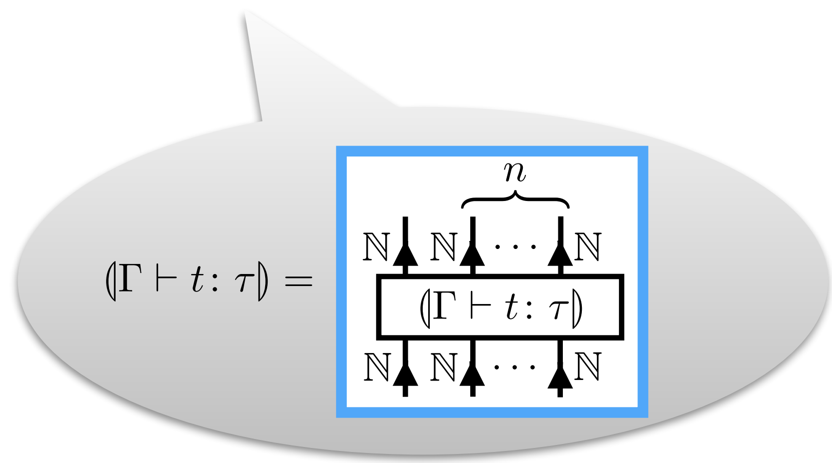
Memoryful GoI — Summary

terms

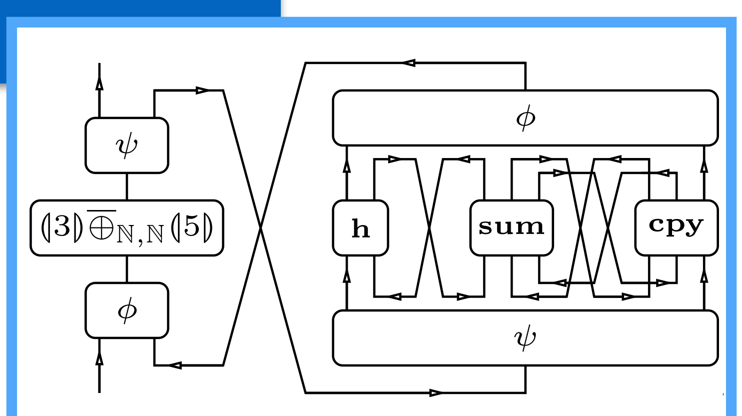
$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$



use **coalgebraic component calculus**



transducers



Our Tool *TtT*

“Terms to Transducers”

terms

memoryful GoI

transducers

λ-terms with effects

TtT Compiler

Haskell program

```
type  Td m x a b  =  (x, a) -> m (x, b)
```

TtT Simulator

simulation result

Our Tool *TtT* — Demonstration

$\boxed{3 \sqcup 5}$

```
threeOrFive =
  Oplus (Const 3) (Const 5)
```

$\boxed{(\lambda x.\, x)\; 1}$

```
idOne = Apply
  (Abst "x" $ Variable "x")
  (Const 1)
```

$\boxed{(\lambda f.\, f\, 0 + f\, 1)\; (\lambda x.\, 3 \sqcup 5)}$

```
secondNondetExample = Apply
  (Abst "f" $ sumLambda
    (Apply (Variable "f") (Const 0))
    (Apply (Variable "f") (Const 1))
  )
  (Abst "x" $ Oplus (Const 3) (Const 5))
```

Our Tool *TtT* — Demonstration

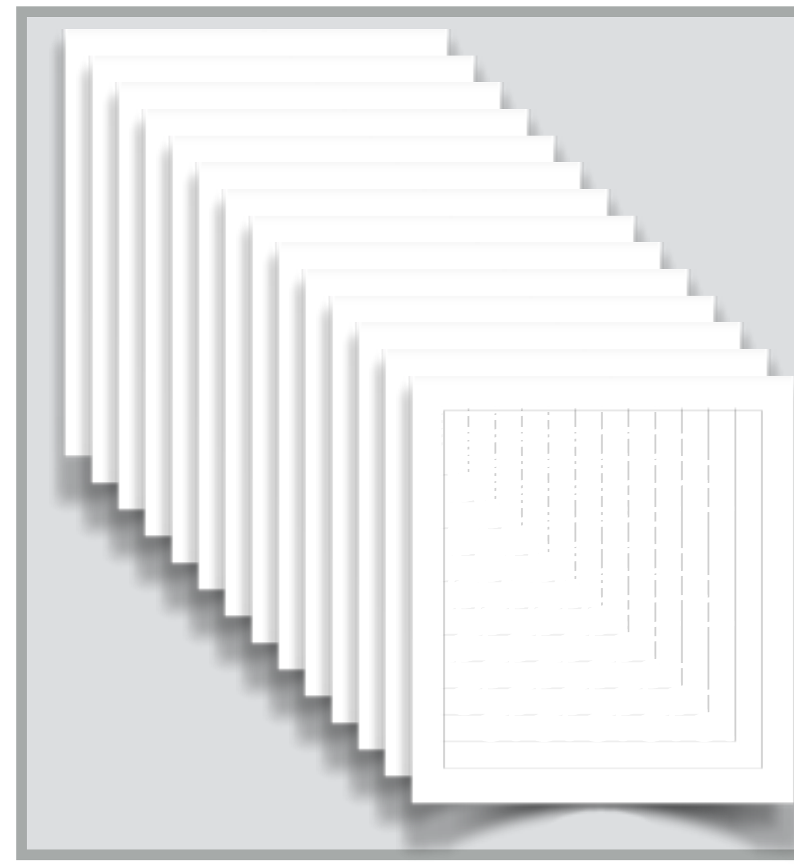
$3 \sqcup 5$

```
.    ---- dd<42,137> --->
|    Query:[ [3|_|5] @ Nothing ]
+-.    ---- dd<42,137> --->
| |    Query:[ [3]|_|5 @ * ]
| |      h; k_3; h
| |    [ [3]|_|5 @ * ]:Answer
| |    ---- dd<42,3> --->
| |  [ [3|_|5] @ Just (Left (*)) ]:Answer
| |  ---- dd<42,3> --->
| Result: 3 / State: Just (Left (*))
`-.    ---- dd<42,137> --->
  |    Query:[ 3|_|[5] @ * ]
  |      h; k_5; h
  |    [ 3|_|[5] @ * ]:Answer
  |    ---- dd<42,5> --->
  |  [ [3|_|5] @ Just (Right (*)) ]:Answer
  |  ---- dd<42,5> --->
  Result: 5 / State: Just (Right (*))
```

$(\lambda x.\, x)\ 1$

```
---- dd<42,137> --->
Query:[ [(\x.x) 1] @ ({_: *}, *) ]
  phi
  ---- gdd<42,137> --->
  Query:[ [\x.x] 1 @ {_: *} ]
    h
  [ [\x.x] 1 @ {_: *} ]:Answer
  ---- dgdd<42,137> --->
  psi; psi; phi
  ---- gdd<42,137> --->
  Query:[ (\x.x) [1] @ * ]
    h
  [ (\x.x) [1] @ * ]:Answer
  ---- dgdd<42,137> --->
  psi; e; phi; phi
  ---- dd<0,gdd<42,137>> --->
  Query:[ [\x.x] 1 @ {_: *} ]
    h; v
    0 {
      psi
      ---- dd<42,137> --->
      Query:[ (\x.[x]) 1 @ * ]
        h
      [ (\x.[x]) 1 @ * ]:Query x
      ---- <42,137> --->
      phi
    } 0
    u; h
  [ [\x.x] 1 @ {_: *} ]:Answer
  ---- dd<0,d<42,137>> --->
  psi; psi; e'; phi
  ---- dd<42,137> --->
  Query:[ (\x.x) [1] @ * ]
    h; k_1; h
  [ (\x.x) [1] @ * ]:Answer
  ---- dd<42,1> --->
  psi; e; phi; phi
  ---- dd<0,d<42,1>> --->
  Query:[ [\x.x] 1 @ {_: *} ]
    h; v
    0 {
      psi
      ---- <42,1> --->
      Answer x:[ (\x.[x]) 1 @ * ]
        h
      [ (\x.[x]) 1 @ * ]:Answer
      ---- dd<42,1> --->
      phi
    } 0
    u; h
  [ [\x.x] 1 @ {_: *} ]:Answer
  ---- dd<0,gdd<42,1>> --->
  psi; psi; e'; phi
  ---- dgdd<42,1> --->
  Query:[ (\x.x) [1] @ * ]
    h
  [ (\x.x) [1] @ * ]:Answer
  ---- gdd<42,1> --->
  psi; phi; phi
  ---- dgdd<42,1> --->
  Query:[ [\x.x] 1 @ {_: *} ]
    h
  [ [\x.x] 1 @ {_: *} ]:Answer
  ---- gdd<42,1> --->
  psi
[ [(\x.x) 1] @ ({_: *}, *) ]:Answer
---- dd<42,1> --->
Result: 1 / State: ({_: *}, *)
```

$(\lambda f.\, f\ 0 + f\ 1)\ (\lambda x.\, 3 \sqcup 5)$



(4,526 lines)

Our Tool *TtT*

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- currently no practical use

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- nevertheless worthwhile
  - helpful for studying higher-order effectful computations
    - showing dynamics of token
  - (speculative) basis of compiler for effectful computations
    - following [Mackie '95] [Pinto '01] [Ghica '07]

Our Tool *TtT*

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  - (speculative) basis of compiler for effectful computations
    - following [Mackie '95] [Pinto '01] [Ghica '07]
- fun to see GoI at work!