Corrections
19 November, 2010

Here I only correct mathematical errors of the version for St. Flour Lectures. Minor typos will be fixed when the updated version is made.
I thank E. Baur, D. Croydon, M. Felsinger, K. Kuwada, M. Kwaśnicki, G. Pete, G. Slade and O. Zeitouni for helpful comments.

- P 9 Line (-3): \( \varphi \) is the unique solution \( \rightarrow \) \( \varphi \) is the unique bounded solution

- P 10 Line 2: Let \( \varphi' \) be another solution \( \rightarrow \) Let \( \varphi' \) be another bounded solution

- P 11 Lemma 1.15: The lower bound \( \frac{R_{\text{eff}}(x, A \cup B)}{R_{\text{eff}}(x, A)^{-1} - R_{\text{eff}}(x, B)^{-1}} \) should be changed to \( R_{\text{eff}}(x, A \cup B)(R_{\text{eff}}(x, A)^{-1} - R_{\text{eff}}(x, B)^{-1}) \). The proof is right. A careless mistake when using (1.16) in the end!

- P 16 Proof of Proposition 1.25: Line (-10) to line (-4) should be changed — the proof in the version for St. Flour Lectures works only when \( |V| < \infty \). Here is one way to fix the proof.

The fact that there exists a unique \( u \in H^2 \) that attains the infimum of (1.23) can be proved similarly to Proposition 1.13 (i). So, denoting \( H^2(V) := \{ u|_V : u \in H^2 \} \), the map \( H_V : H^2(V) \to H^2 \) where \( f \mapsto H_V f \) is well-defined. Let \( \mathcal{H}_V := \{ u \in H^2 : \mathcal{E}(u, v) = 0, \text{ for all } v \in H^2 \text{ such that } v|_V = 0 \} \); a space of harmonic functions on \( X \setminus V \). We claim the following:

\[
\mathcal{H}_V = H_V(H^2(V)) \quad \text{and} \quad R_V : \mathcal{H}_V \to H^2(V) \quad \text{where } R_V u = u|_V \text{ is an inverse operator of } H_V. \quad (0.1)
\]

Once this is proved, then we have the linearity of \( H_V \) and furthermore we have

\[
H^2 = \mathcal{H}_V \oplus \{ v \in H^2 : v|_V = 0 \}.
\]

So let us prove (0.1). If \( f \in H^2(V) \) and \( u = H_V f \), then for any \( v \in H^2 \) with \( v|_V = f \), we have

\[
\mathcal{E}(\lambda(v - u) + u, \lambda(v - u) + u) \geq \mathcal{E}(u, u), \quad \forall \lambda \in \mathbb{R},
\]

because \( u \) attains the infimum in (1.23). This implies \( \mathcal{E}(v - u, u) = 0 \), namely \( u \in \mathcal{H}_V \). Clearly \( u|_V = f \), so we obtain \( \mathcal{H}_V \supset H_V(H^2(V)) \) and \( R_V \circ H_V \) is an identity map. Next, if \( u \in \mathcal{H}_V \) and \( u|_V = f \in H^2(V) \), then for any \( v \in H^2 \) with \( v|_V = f \), we have

\[
\mathcal{E}(v, v) = \mathcal{E}(v - u + u, v - u + u) = \mathcal{E}(v - u, v - u) + \mathcal{E}(u, u) \geq \mathcal{E}(u, u)
\]

because \( \mathcal{E}(v - u, u) = 0 \) (since \( u \in \mathcal{H}_V \)). This implies \( u = H_V f \), since the infimum in (1.23) is attained uniquely by \( H_V f \). So we obtain \( \mathcal{H}_V \subset H_V(H^2(V)) \) and \( H_V \circ R_V \) is an identity map. \( \square \)
here is a domain of the Dirichlet form. For example, when $\mathcal{E}(f, f) = \frac{1}{2} \int_{\mathbb{R}^d} |\nabla f|^2 dx$, then $\mathcal{F} = W^{1,2}(\mathbb{R}^d)$, the classical Sobolev space. When we consider weighted graphs, $\mathcal{F}$ is just $L^2(X, \mu)$ (or $H^2$ if you like).

- P26 Line 13: Add ‘with $u(x) \neq u(y)$’.
- P 29 (3.9), (3.10): $X \setminus \{z\} \to B$.
- P 50 Line 9: $(\frac{p}{p_e})^r \to (\frac{p_e}{p})^r$.
- P 50 Proposition 5.12 (ii): $\Gamma(r) \leq c_2/r \to \Gamma(r) \geq c_2/r$.
- P 52 Line 15: level $\delta k - 1$ from $v \to$ level $\delta k - 1$ from 0.
- P 56 Line 6: $\cdots = a_n 2^{2n} \to \cdots = a_n 2^{5n}$
- P 57 Line (-3): $D(n)$ here is the same as $J_n$ in (5.17).
- P 61 Line 19: $\mathbb{P}$-a.s. $\to \mathbb{P}$-distribution.
- P 61 Line (-11): ‘and $c_1, \cdots, c_4, \alpha_1, \cdots, \alpha_3$ are positive (non-random) constants.’ $\to$ ‘$c_1, \cdots, c_4$ are positive random constants and $\alpha_1, \alpha_2, \alpha_3$ are positive non-random constants.’
- P 66 Remark 7.10 (i): Add [Zei] (more recent survey) to the references.
- P 71 Line 5: Add ‘$\mathbb{P}$-a.s.’ in the end of the sentence.
- P 71 Line (-8): ‘$\mathbb{E}[E^0|Y_1|^2] = \mathbb{E}[\sum_x Q_{0x}|x|^2] \leq ct$ for $t \geq 1$, ’ $\to$ ‘$\mathbb{E}[E^0|Y_1|^2] = \mathbb{E}[\sum_x Q_{0x}|x|^2] \leq c$,’
- P 74 Line 8–9: Change $\Phi_j$ to $\varphi_j$ to make the notation consistent and change Line 9 as follows:

$$\Pi_j = (-\chi_j) \oplus \varphi_j \in L^2_{\text{pot}} \oplus L^2_{\text{sol}}.$$  
- P 75 Line 7: $L^2_{\text{pot}} \to L^2_{\text{sol}}$.
- References [56]: P. Gábor $\to$ G. Pete.

References