Corrections

19 November, 2010

Here I only correct mathematical errors of the version for St. Flour Lectures. Minor typos will be fixed when the updated version is made.

I thank E. Baur, D. Croydon, M. Felsinger, K. Kuwada, M. Kwaśnicki, G. Pete, G. Slade and O. Zeitouni for helpful comments.

- P 9 Line (-3): φ is the unique solution $\rightarrow$ φ is the unique bounded solution

- P 10 Line 2: Let φ' be another solution $\rightarrow$ Let φ' be another bounded solution

- P 11 Lemma 1.15: The lower bound $\dfrac{R_{\rm eff}(x, A\cup B)}{R_{\rm eff}(x,A)^{-1} - R_{\rm eff}(x,B)^{-1}}$ should be changed to $R_{\rm eff}(x, A\cup B)(R_{\rm eff}(x,A)^{-1} - R_{\rm eff}(x,B)^{-1})$. – The proof is right. A careless mistake when using (1.16) in the end!

- P 16 Proof of Proposition 1.25: Line (-10) to line (-4) should be changed — the proof in the version for St. Flour Lectures works only when $|V| < \infty$. Here is one way to fix the proof.

The fact that there exists a unique $u \in H^2$ that attains the infimum of (1.23) can be proved similarly to Proposition 1.13 (i). So, denoting $H^2(V) := \{u|_V : u \in H^2\}$, the map $H_V : H^2(V) \to H^2$ where $f \mapsto H_V f$ is well-defined. Let $\mathcal{H}_V := \{u \in H^2 : \mathcal{E}(u,v) = 0, \text{ for all } v \in H^2 \text{ such that } v|_V = 0\}$; a space of harmonic functions on $X \setminus V$. We claim the following:

$$\mathcal{H}_V = H_V(H^2(V)) \text{ and } R_V : \mathcal{H}_V \to H^2(V) \text{ where } R_V u = u|_V \text{ is an inverse operator of } H_V. \quad (0.1)$$

Once this is proved, then we have the linearity of H_V and furthermore we have

$$H^2 = \mathcal{H}_V \oplus \{v \in H^2 : v|_V = 0\}.$$

So let us prove (0.1). If $f \in H^2(V)$ and $u = H_V f$, then for any $v \in H^2$ with $v|_V = f$, we have

$$\mathcal{E}(\lambda(v-u)+u, \lambda(v-u)+u) \geq \mathcal{E}(u,u), \qquad \forall \lambda \in \mathbb{R},$$

because u attains the infimum in (1.23). This implies $\mathcal{E}(v-u,u) = 0$, namely $u \in \mathcal{H}_V$. Clearly $u|_V = f$, so we obtain $\mathcal{H}_V \supset H_V(H^2(V))$ and $R_V \circ H_V$ is an identity map. Next, if $u \in \mathcal{H}_V$ and $u|_V = f \in H^2(V)$, then for any $v \in H^2$ with $v|_V = f$, we have

$$\mathcal{E}(v,v) = \mathcal{E}(v-u+u, v-u+u) = \mathcal{E}(v-u,v-u) + \mathcal{E}(u,u) \geq \mathcal{E}(u,u)$$

because $\mathcal{E}(v-u,u) = 0$ (since $u \in \mathcal{H}_V$). This implies $u = H_V f$, since the infimum in (1.23) is attained uniquely by $H_V f$. So we obtain $\mathcal{H}_V \subset H_V(H^2(V))$ and $H_V \circ R_V$ is an identity map. $\square$

- P 17 Line (-14): $\mathcal{F}$ here is a domain of the Dirichlet form. For example, when $\mathcal{E}(f,f) = \frac{1}{2}\int_{\mathbb{R}^d} |\nabla f|^2 dx$, then $\mathcal{F} = W^{1,2}(\mathbb{R}^d)$, the classical Sobolev space. When we consider weighted graphs, $\mathcal{F}$ is just $\mathbb{L}^2(X,\mu)$ (or H^2 if you like).

- P26 Line 13: Add 'with $u(x) \neq u(y)$'.

- P 29 (3.9), (3.10): $X \setminus \{z\} \;\to\; B$.

- P 50 Line 9: $(\frac{p}{p_c})^r \;\to\; (\frac{p_c}{p})^r$.

- P 50 Proposition 5.12 (ii): $\Gamma(r) \le c_2/r \;\to\; \Gamma(r) \ge c_2/r$.

- P 52 Line 15: level 3^{k-1} from v $\;\to\;$ level 3^{k-1} from 0.

- P 56 Line 6: $\cdots = a_n 2^{2n} \;\to\; \cdots = a_n 2^{3n}$

- P 57 Line (-3): $D(n)$ here is the same as J_n in (5.17).

- P 61 Line 19: $\mathbb{P}$–a.s. $\;\to\;$ $\mathbb{P}$–distribution.

- P 61 Line (-11): 'and $c_1, \cdots, c_4, \alpha_1, \cdots, \alpha_3$ are positive (non-random) constants.' $\to$ '$c_1, \cdots, c_4$ are positive random constants and $\alpha_1, \alpha_2, \alpha_3$ are positive non-random constants.'

- P 66 Remark 7.10 (i): Add [Zei] (more recent survey) to the references.

- P 71 Line 5: Add 'P^0_ω-a.s.' in the end of the sentence.

- P 71 Line (-8): '$\mathbb{E}[E^0_\cdot|Y_t|^2] = \mathbb{E}[\sum_x Q_{0x}|x|^2] \le ct$ for $t \ge 1$,' $\to$ '$\mathbb{E}[E^0_\cdot|Y_1|^2] = \mathbb{E}[\sum_x Q_{0x}|x|^2] \le c$,'

- P 74 Line 8–9: Change Φ_j to φ_j to make the notation consistent and change Line 9 as follows:

$$\Pi_j = (-\chi_j) \oplus \varphi_j \in L^2_{\text{pot}} \oplus L^2_{\text{sol}}.$$

- P 75 Line 7: $L^2_{\text{pot}} \;\to\; L^2_{\text{sol}}$.

- References [56]: P. Gábor $\;\to\;$ G. Pete.

References

[Zei] O. Zeitouni. *Random walks and diffusions in random environments.* J. Phys. A: Math. Gen. **39** (2006), pp. R433–R464.