RIMS Workshop on

Mathematical Analysis of Viscous Incompressible Fluid

Organizers Toshiaki Hishida (Nagoya University)
Yoshihiro Shibata (Waseda University)

November 30–December 2, 2022

Venue: RIMS, Kyoto University, Room No. 420

Program

Wednday, November 30

14:00–14:50 Yoshihiro Shibata (Waseda University)
L_1 maximal regularity for the Stokes equations with free boundary conditions in the half space

15:00–15:50 Takahiro Okabe (Osaka University)
Forced rapidly dissipative Navier-Stokes flows

16:00–16:50 Tomoyuki Nakatsuka (Matsuyama University)
On solvability of the time-periodic problem for the Navier-Stokes equation

Thursday, December 1

9:40–10:30 Giovanni P. Galdi (University of Pittsburgh)
Hopf bifurcation for Navier-Stokes flow past a rotating obstacle

10:40–11:30 Naoto Kajiwara (Gifu University)
Maximal regularity for the Stokes equations with various boundary conditions in the half space

11:40–12:30 Tomoki Takahashi (Nagoya University)
Anisotropically weighted L^q-L^r estimates of the Oseen semigroup in exterior domains, with applications to the Navier-Stokes flow past a rigid body

14:00–14:50 Xiao Ren (Fudan University)
Recent results on the stationary Navier-Stokes equations in 2D exterior domain

15:00–15:50 Kenta Ishimoto (Kyoto University)
Problems in microswimmer hydrodynamics

16:00–16:50 Reinhard Racke (Universität Konstanz)
Stability of relaxed Navier-Stokes equations

Friday, December 2

10:00–10:50 Yoshihiro Ueda (Kobe University)
Recent progress in the stability theory for the symmetric hyperbolic system with general relaxation

11:00–11:50 Miho Murata (Shizuoka University/Tohoku University)
Global well posedness for a Q-tensor model of nematic liquid crystals

This workshop is held by RIMS
https://www.kurims.kyoto-u.ac.jp/kyoten/en/workshop.html

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https://www.waseda.jp/fsci/mathphys/en

Abstracts

Giovanni P. Galdi (University of Pittsburgh) Dec. 1 (Thu), 9:40

Hopf bifurcation for Navier-Stokes flow past a rotating obstacle

We consider a body $\mathcal{B}$, sufficiently smooth but of arbitrary shape, completely immersed in a viscous liquid and rotating with a constant angular velocity, ω_*. The flow of the liquid is characterized by a constant and non-zero velocity field $\mathbf{v}_\infty$ at large distances from $\mathcal{B}$, with $\mathbf{v}_\infty$ parallel to ω_*. We then provide sufficient conditions for the existence of Hopf-type time-periodic bifurcation from a steady-state branch of solutions. Besides some technical requirements, the main assumptions regard a suitable monotonic correlation between ω_* and the frequency of the bifurcating solution, along with a generalized transversality condition.

Kenta Ishimoto (Kyoto University) Dec. 1 (Thu), 15:00

Problems in microswimmer hydrodynamics

In this talk, I will overview the microswimmer hydrodynamics and discuss possible mathematical problems to be solved in the field. Microswimmers are microscopic self-propelled objects in fluid such as bacteria, plankton and chemically-activated colloids. Due to the small Reynolds number around these tiny objects, the fluid flow obeys the Stokes equation. Even though the fluid equation is a simple, linear PDE, the swimmer dynamics are rich and diverse owing to deforming and moving boundaries. I will also present some of my recent works, including hydrodynamic 'shape' theory and swimming of odd materials.

Naoto Kajiwara (Gifu University) Dec. 1 (Thu), 10:40

Maximal regularity for the Stokes equations with various boundary conditions in the half space

We prove resolvent L_p estimates and maximal L_p-L_q regularity estimates for the Stokes equations with Dirichlet, Neumann and Robin boundary conditions in the half space. Each solution is constructed by a Fourier multiplier of x'-direction and an integral of x_N-direction. After we show a sufficient condition of L_p-boundedness or L_p-L_q-boundedness for such Fourier integral operators, we apply it to prove the theorems. The assumptions are

only bounded, holomorphic, and the symbols are dominated by a homogeneous function of order -1 for x_N-directions. We give a new simple approach to get two estimates in the half space.

Miho Murata (Shizuoka University/Tohoku University) Dec. 2 (Fri), 11:00

Global well posedness for a Q-tensor model of nematic liquid crystals

We consider the model for a viscous incompressible liquid crystal flow proposed by Beris and Edwards in 1994. The model is coupled system by the Navier-Stokes equations with a parabolic-type equation describing the evolution of the director fields $\mathbb{Q}$, which is called $\mathbb{Q}$-tensor. The aim of this talk is to prove the existence and uniqueness of the strong solution to a $\mathbb{Q}$-tensor model of nematic liquid crystals in $\mathbb{R}^N$, $N \geq 3$ with the help of the maximal L_p -L_q regularity and the L_p -L_q decay estimates to the linearized equations. In this talk, we introduce a general framework to prove the global well posedness for small initial data of quasilinear parabolic equations in unbounded domains; additionally, we discuss mathematical analysis for our linearized problem. This is a joint work with Professor Yoshihiro Shibata (Waseda University).

Tomoyuki Nakatsuka (Matsuyama University) Nov. 30 (Wed), 16:00

On solvability of the time-periodic problem for the Navier-Stokes equation

In this talk, we consider the existence of time-periodic solutions to the Navier-Stokes equation in the whole space. We decompose periodic solutions into steady and purely periodic parts, and we analyze the equations they should satisfy. Based on the analysis of the purely periodic solutions represented by the Fourier transform to the Stokes equation, their additional property can be obtained and we use it to construct a time-periodic solution of the Navier-Stokes equation.

Takahiro Okabe (Osaka University) Nov. 30 (Wed), 15:00

Forced rapidly dissipative Navier-Stokes flows

We consider the energy decay problem of the incompressible Navier-Stokes flows on the whole space. By Kajikiya and Miyakawa(1986), Wiegner(1987), the optimal decay rate of the flow, especially, of the nonlinear term of the solution has been investigated. Indeed,

Fujigaki and Miyakawa(2001) gave the asymptotic expansion with the concrete expression of the first order term of the nonlinear term whose decay rate gives a threshold. Then, Miyakawa and Schonbek(2001) obtained the necessary and sufficient conditions on the coefficients of the leading order terms for the rapid decay where we need the all information of the flow on the space-time region. So, in general, the rapid decay is hardly obtained. Recently, Brandolese and Okabe(2021) derived a rapid decay by the action of an external force. In this talk, we give a refinement of the procedure of the previous results and reconsider the asymptotic expansion of the nonlinear term with the weaker condition on the initial data. This talk is based on the joint work with Lorenzo Brandolese (Universite Lyon 1).

Reinhard Racke (Universität Konstanz) Dec. 1 (Thu), 16:00

Stability of relaxed Navier-Stokes equations

We investigate the formation of singularities in one-dimensional hyperbolic compressible Navier-Stokes equations, a model proposing a relaxation leading to a hyperbolization through a nonlinear Cattaneo law for heat conduction as well as through the constitutive Maxwell type relations for the stress tensor. It is shown that there are in general no global C^1 solutions for the studied system, for some large initial data. This is in contrast to the global large well-posedness for the non-relaxed, classical system. Relations to incompressible Navier-Stokes equations, and possible higher-dimensional situations are also addressed.

Xiao Ren (Fudan University) Dec. 1 (Thu), 14:00

Recent results on the stationary Navier-Stokes equations in 2D exterior domain

In this talk, we first recall the history of the 2D Navier-Stokes exterior problem, with an emphasis on the classical flow-around-obstacle problem. Then we discuss some recent progress: (1)The works of Korobkov, Pileckas and Russo on the convergence of arbitrary D-solutions and the nontriviality of Leray's solution. (2)The uniqueness of Finn-Smith solutions and the justification of Leray's method for small Reynolds numbers. (3)Existence and uniqueness results for the forced problem in the whole plane. Finally, we present some open problems for future study. The talk is based on joint works with Mikhail Korobkov and Julien Guillod.

Yoshihiro Shibata (Waseda University) Nov. 30 (Wed), 14:00

L_1 maximal regularity for the Stokes equations with free boundary conditions in the half space

I will talk about the L_1 maximal regularity for the Stokes equations with free boundary conditions in the half space. This is a joint work with Keiichi Watanabe (Waseda Univ.). Our approach is to use several resolvent estimates. Our basic idea is seemed to be similar to Da-Prato and Grisvard theory. But, we can treat the boundary value problem, although their method can be applied to only zero boundary conditions. And the Stokes problem with non-homogeneous free boundary conditions can not be reduced to the zero boundary conditions in the optimal framework unlike the heat equations with non-homogeneous Neumann conditions, because of the divergence conditions and the non-locality of the pressure term.

Tomoki Takahashi (Nagoya University) Dec. 1 (Thu), 11:40

Anisotropically weighted L^q-L^r estimates of the Oseen semigroup in exterior domains, with applications to the Navier-Stokes flow past a rigid body

We consider the spatial-temporal behavior of the Navier-Stokes flow past a rigid body in $\mathbb{R}^3$. In a series of his celebrated papers, Finn succeeded in constructing a stationary solution u_s, that exhibits a paraboloidal wake region behind the body, in particular, $u_s(x) = O\big((1+|x|)^{-1}(1+|x|-x_1)^{-1}\big)$ when the translational velocity of the body is parallel to the x_1-direction. This talk develops analysis in Lebesgue spaces with the anisotropic weight $(1+|x|)^{\alpha}(1+|x|-x_1)^{\beta}$. We first derive anisotropically weighted L^q-L^r estimates of the Oseen semigroup in exterior domains. As applications of those estimates, we study the stability/attainability of the Navier-Stokes flow in anisotropically weighted L^q spaces to deduce the temporal rate with the wake of nonstationary solutions.

Yoshihiro Ueda (Kobe University) Dec. 2 (Fri), 10:00

Recent progress in the stability theory for the symmetric hyperbolic system with general relaxation

In this talk, we study the dissipative structure for the linear symmetric hyperbolic system with general relaxation. If the relaxation matrix of the system has symmetric properties, Shizuta and Kawashima(1985) introduced the suitable stability condition, and Umeda, Kawashima and Shizuta(1984) analyzed the dissipative structure. On the other hand, Ueda, Duan and Kawashima(2012,2018) focused on the system with non-symmetric relaxation and got partial results. Furthermore, they argued the new dissipative structure

called the regularity-loss type. In this situation, this talk aims to extend the stability theory introduced by Shizuta and Kawashima(1985) and Umeda, Kawashima and Shizuta(1984) to our general system. Furthermore, we will consider the optimality of the dissipative structure. If we have time, I would like to discuss some physical models for its application and new dissipative structures.