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LINEAR BOUNDARY PROBLEMS OF THE ELLIPTIC AND THE EVOLUTION TYPE

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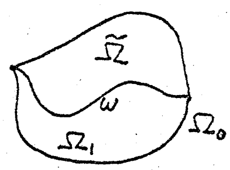
§0. Introduction

Let Ω be an open subset of $\mathbb{R}^n$, and ω an open subset of the boundary $\partial\Omega$ of Ω. We write $\tilde{\Omega} = \Omega \cup \omega$. Choose an open subset Ω_0 of $\mathbb{R}^n$ such that it contains $\tilde{\Omega}$ and $\Omega_0 \cap (\partial\Omega \setminus \omega) = \phi$. We write $\Omega_1 = \Omega_0 \setminus \tilde{\Omega}$, and $\tilde{\Omega}_1 = \Omega_1 \cup \omega$.

Now let $\mathcal{F}(\Omega_0)$ be a subspace of $\mathcal{B}(\Omega_0) = H^n_{\Omega_0}(V, \mathcal{O})$. Then we define the following two spaces:



$\mathcal{F}(\tilde{\Omega}) = \{u \in \mathcal{B}(\Omega);\ u = v/_{\Omega} \text{ for some } v \in \mathcal{F}(\Omega_0)\}$,

$\mathring{\mathcal{F}}(\tilde{\Omega}) = \{v \in \mathcal{F}(\Omega_0);\ \operatorname{supp} v \subset \tilde{\Omega}\}$.

When ω is smooth, let $R: H^{loc}_{(1)}(\tilde{\Omega}) \longrightarrow H^{loc}_{(1/2)}(\omega)$ be the trace operator, $R^m = \underbrace{\begin{pmatrix} R & & 0 \\ & \ddots & \\ 0 & & R \end{pmatrix}}_{m}: H^{loc}_{(1)}(\tilde{\Omega})^m \longrightarrow H^{loc}_{(1/2)}(\omega)^m$, and let $\frac{\partial}{\partial n}$ be the inner normal derivative at ω. In the following we fix a family of Sobolev norms on the $n-1$ dimensional manifold ω. Moreover let $B^m(\tau) = \begin{pmatrix} 1 \\ \tau \\ \vdots \\ \tau^{m-1} \end{pmatrix}$, $\tau \in \mathbb{C}$. For other notations, see the Hörmander's book, "Linear Differential Operators", Springer 1963.

In the following sections we explain some results on boundary problems for general linear differential operators. Details will appear in the forth-coming papers.

§1. Elliptic operators in non-compact manifolds

Let $\tilde{\Omega}$ be non-compact, and ω of C^∞-class. We assume that for any bounded open subset U of $\mathbb{R}^n$ the union of all compact connected components of $\tilde{\Omega}\setminus U$ is bounded. Let $P(x,D)$ be an elliptic differential operator <u>with real analytic coefficients</u> in Ω_0 of order $M = 2m$. Let p_j, $j = 1,2,\cdots,m$ be boundary differential operators with C^∞-coefficients of order m_j and of transversal order $< M$. We can write $\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_m \end{pmatrix} = B\circ R^M\circ B^M(\frac{\partial}{\partial n})$ for some under-determined system of differential operators with C^∞-coefficients $B: \mathcal{E}(\omega)^M \longrightarrow \mathcal{E}(\omega)^m$.

Now we consider the following boundary problem:

$$\begin{cases} P(x, D)U = F & \text{in } \Omega \\ p_jU = f_j & \text{in } \omega, \quad j = 1,2,\cdots,m. \end{cases} \tag{1.1}$$

<u>Theorem</u> 1.1. Under the above conditions the following three statements are equivalent. Here $s \geqq M$.

(1) The root condition and the complementing condition at ω are satisfied (cf. Agmon-Douglis-Nirenberg, Comm. Pure Appl. Math. 12, 623-727 (1959)). Moreover ω is B-convex (with respect to $\prod_{j=1}^{m} H^{loc}_{(s-m_j+1/2)}(\omega)$), i.e. for every compact set $K\subset\omega$ there exists a compact set $K'\subset\omega$ such that $u\in\prod_{j=1}^{m} H_{(-s+m_j-1/2)}\cap\mathcal{E}'(\omega)$ and $\operatorname{supp}{}^tBu\subset K$ implies $\operatorname{supp} u\subset K'$.

(2) The equation (1.1) has a solution $U\in\mathcal{E}(\tilde{\Omega})$ for every $F\in\mathcal{E}(\tilde{\Omega})$ and $f_j\in\mathcal{E}(\omega)$, $j = 1,2,\cdots,m$.

(3) The equation (1.1) has a solution $U\in H^{loc}_{(s)}(\tilde{\Omega})$ for every $F\in H^{loc}_{(s-M)}(\tilde{\Omega})$ and $f_j\in H^{loc}_{(s-m_j+1/2)}(\omega)$, $j = 1,2,\cdots,m$.

The proof depends on lemmas of the following type.

<u>Lemma</u> 1.2. The operator $\mathcal{E}(\tilde{\Omega}) \xrightarrow{P(x,\,D) \,\times\, (p_1,\,p_2,\,\cdots,\,p_m)^t} \mathcal{E}(\tilde{\Omega}) \times \mathcal{E}(\omega)^m$ is surjective

if and only if the following conditions (1) and (2) are valid. Moreover (2) and (3) are equivalent.

(1) For every $s \in \mathbb{R}$ and every compact set $K \subset \tilde{\Omega}$ there exists a compact set $K' \subset \tilde{\Omega}$ such that $U \in \mathring{\mathcal{E}}'(\tilde{\Omega})$, $u_j \in \mathcal{E}'(\omega)$, $j = 1, 2, \cdots, m$, ${}^tP(x, D)U + \sum_{j=1}^{m} {}^tp_j u_j \in H_{(s)} \cap \mathring{\mathcal{E}}'(\tilde{\Omega})$, $\mathrm{supp}({}^tP(x, D)U + \sum_{j=1}^{m} {}^tp_j u_j) \subset K$ implies $\mathrm{supp}\, U \subset K'$ and $\mathrm{supp}\, u_j \subset K' \cap \omega$, $j = 1, 2, \cdots, m$.

(2) For every $F \in \mathcal{E}(\tilde{\Omega})$, $f_j \in \mathcal{E}(\omega)$, $j = 1, 2, \cdots, m$ and for every compact set $K \subset \tilde{\Omega}$ there exists $U \in \mathcal{E}(\tilde{\Omega})$ such that $P(x, D)U = F$ in K and $p_j U = f_j$ in $K \cap \omega$, $j = 1, 2, \cdots, m$.

(3) For every $s \in \mathbb{R}$ and every compact set $K \subset \tilde{\Omega}$ there exists $t \in \mathbb{R}$ and $C > 0$ such that $U \in \mathring{\mathcal{E}}'(\tilde{\Omega})$, $u_j \in \mathcal{E}'(\omega)$, $j = 1, 2, \cdots, m$, $\mathrm{supp}\, U \subset K$, $\mathrm{supp}\, u_j \subset K \cap \omega$, ${}^tP(x, D)U + \sum_{j=1}^{m} {}^tp_j u_j \in H_{(s)} \cap \mathring{\mathcal{E}}'(\tilde{\Omega})$ implies

$$\| U \|_{(t)} + \sum_{j=1}^{m} \| u_j \|_{(t)} \leq C \cdot \| {}^tP(x, D)U + \sum_{j=1}^{m} {}^tp_j u_j \|_{(s)}.$$

Remark 1.3. Let $b_{jk} \in \mathcal{E}(\omega)$, $j = 1, 2, \cdots, m$; $k = 1, 2, \cdots, M$ and $B = (b_{jk})$. If at every $x \in \omega$ the rank of the matrix $B(x)$ is m, then ω is B-convex. For example the Dirichlet condition satisfies this.

Remark 1.4. If ω is convex flat and B is with constant coefficients of rank m, then ω is B-convex.

§2. Strictly hyperbolic operators

In this section we assume that ω is of C^∞-class and $P(x, D)$ is strictly hyperbolic with respect to $\tilde{\Omega}$ and of order m with C^∞-coefficients.

Theorem 2.1. The following four conditions are equivalent.

(1) For every $s \in \mathbb{R}$ and every compact set $K \subset \tilde{\Omega}$ there exists a compact set $K' \subset \tilde{\Omega}$ such that $u \in \mathcal{E}'(\tilde{\Omega})$, ${}^tP(x, D)u \in H_{(s)} \cap \mathcal{E}'(\tilde{\Omega})$, $\operatorname{supp} {}^tP(x, D)u \subset \Omega \cap K$ implies $\operatorname{supp} u \subset \Omega \cap K'$.

(2) The following equation (2.1) has a solution $U \in \mathring{\mathcal{E}}(\tilde{\Omega})$ for every $F \in \mathring{\mathcal{E}}(\tilde{\Omega})$.

$$P(x, D)U = F \quad \text{in} \quad \Omega_0 \tag{2.1}$$

(3) If $F \in \mathcal{E}(\Omega_0)$, $U \in \mathcal{E}(\tilde{\Omega}_1)$, and $P(x, D)U = F$ in Ω_1, then there exists $V \in \mathcal{E}(\Omega_0)$ such that $V|_{\Omega_1} = U$ and $P(x, D)V = F$ in Ω_0.

(4) The following equation (2.2) has a solution $U \in \mathcal{E}(\tilde{\Omega})$ for every $F \in \mathcal{E}(\tilde{\Omega})$ and $f_j \in \mathcal{E}(\omega)$, $j = 1, 2, \cdots, m$.

$$\begin{cases} P(x, D)U = F & \text{in} \quad \Omega \\ R \circ (\frac{\partial}{\partial n})^{j-1} U = f_j & \text{in} \quad \omega, \quad j = 1, 2, \cdots, m. \end{cases} \tag{2.2}$$

The essential part of the proof depends on the following lemma.

Lemma 2.2. The operator $P(x, D): \mathring{\mathcal{E}}(\tilde{\Omega}) \longrightarrow \mathring{\mathcal{E}}(\tilde{\Omega})$ is surjective if and only if the condition (1) of Theorem 2.1. and the following equivalent conditions hold.

(i) For every $F \in \mathring{\mathcal{E}}(\tilde{\Omega})$ and every compact set $K \subset \tilde{\Omega}$ there exists $U \in \mathring{\mathcal{E}}(\tilde{\Omega})$ such that $P(x, D)U = F$ in K.

(ii) For every $s \in \mathbb{R}$ and every compact set $K \subset \tilde{\Omega}$ there exist $t \in \mathbb{R}$ and $C > 0$ such that $U \in \mathcal{E}'(\tilde{\Omega})$, $\operatorname{supp} U \subset \Omega \cap K$, ${}^tP(x, D)U \in H_{(s)} \cap \mathcal{E}'(\tilde{\Omega})$ implies

$$\inf\{\|V\|_{(t)};\ V \in H_{(t)} \text{ and } V|_{\Omega} = U\}$$
$$\leqq C \inf\{\|W\|_{(s)};\ W \in H_{(s)} \text{ and } W|_{\Omega} = {}^tP(x, D)U\}.$$

Theorem 2.3. If $l \in \mathbb{R}$, the following conditions (1') to (3') are equivalent. If l is a nutural number, then all four conditions are equivalent.

(1') For every compact set $K \subset \tilde{\Omega}$ there exists a compact set $K' \subset \tilde{\Omega}$ such that $u \in H_{(-l-1)} \cap \mathcal{E}'(\tilde{\Omega})$ and $\operatorname{supp} {}^tPu \subset \Omega \cap K$ implies $\operatorname{supp} u \subset \Omega \cap K'$.

(2') The equation (2.1) has a solution $U \in \overset{\circ}{H}{}^{loc}_{(l+m)}(\tilde{\Omega})$ for every $F \in \overset{\circ}{H}{}^{loc}_{(l+1)}(\tilde{\Omega})$.

(3') If $F \in H^{loc}_{(l+1)}(\Omega_0)$, $U \in H^{loc}_{(l+m)}(\tilde{\Omega}_1)$, and $P(x, D)U = F$ in Ω_1, then there exists $V \in H^{loc}_{(l+m)}(\Omega_0)$ such that $V|_{\Omega_1} = U$ and $P(x, D)V = F$ in Ω_0.

(4') The equation (2.2) has a solution $U \in H^{loc}_{(l+m)}(\tilde{\Omega})$ for every $F \in H^{loc}_{(l+1)}(\tilde{\Omega})$ and $f_j \in H^{loc}_{(l+m-j+1/2)}(\omega)$, $j = 1, 2, \cdots, m$.

<u>Definition</u> 2.1. The set $\tilde{\Omega}$ is called <u>P(x, D)-proper</u> iff for every compact set $K \subset \tilde{\Omega}$ there exists a compact set $K' \subset \tilde{\Omega}$ such that $u \in \mathcal{E}'(\tilde{\Omega})$ and $\operatorname{supp} {}^tP(x, D)u \subset \Omega \cap K$ implies $\operatorname{supp} u \subset \Omega \cap K'$.

<u>Definition</u> 2.2. The set $\tilde{\Omega}$ is called <u>strongly P(x, D)-proper</u> iff $\tilde{\Omega}$ is P(x, D)-proper and for every compact set $K \subset \tilde{\Omega}$ there exists a compact set $K' \subset \tilde{\Omega}$ such that $u \in \mathcal{E}'(\tilde{\Omega})$, ${}^tP(x, D)u \in \mathcal{E}(\tilde{\Omega} \setminus K)$ implies $u \in \mathcal{E}(\tilde{\Omega} \setminus K')$.

These conditions can be characterized geometrically by the relation between the boundary $\partial\Omega \setminus \omega$ and the bicharacteristics. When ω is void, these conditions reduce to the well-known the P-convexity and the strong P-convexity conditions respectively.

<u>Theorem</u> 2.4. If $\tilde{\Omega}$ is strongly P(x, D)-proper, then the following two statements are valid.

(1'') The equation (2.1) has a solution $U \in \overset{\circ}{\mathcal{D}}{}'(\tilde{\Omega})$ for every $F \in \overset{\circ}{\mathcal{D}}{}'(\tilde{\Omega})$.

(2'') If $U \in \mathcal{D}'(\tilde{\Omega}_1)$, $F \in \mathcal{D}'(\Omega_0)$, $P(x, D)U = F$ in Ω_1, then there exists $V \in \mathcal{D}'(\Omega_0)$ such that $V|_{\Omega_1} = U$ and $P(x, D)V = F$ in Ω_0.

§3. Evolution operators with constant coefficients

Let $P(D)$ be a linear differential operator with constant coefficients. In this section we assume the existence of a closed cone Γ with its vertex at the origin, which satisfies the following conditions. For every $x \in \omega$ there exists a neighbourhood U of x such that $\Omega + \Gamma$ and $U \setminus \Omega$ do not meet. Moreover there exists a fundamental solution $E \in \mathring{\mathcal{B}}^{loc}_{\infty,\tilde{P}}(\Gamma)$ of $P(D)$, i.e. $P(D)E = \delta$. Let $1 \leq p < \infty$ and $k \in \mathcal{K}(\mathbb{R}^n)$.

Theorem 3.1. Under the above conditions, the following six statements are equivalent.

(1) The set $\tilde{\Omega}$ is $P(D)$-proper.

(2) The following equation (3.1) has a solution $U \in \mathring{\mathcal{E}}(\tilde{\Omega})$ for every $F \in \mathring{\mathcal{E}}(\tilde{\Omega})$.

$$P(D)U = F \quad \text{in} \quad \Omega_0 \tag{3.1}$$

(3) If $U \in \mathcal{E}(\tilde{\Omega}_1)$, $F \in \mathcal{E}(\Omega_0)$, and $P(D)U = F$ in Ω_1, then there exists $V \in \mathcal{E}(\Omega_0)$ such that $V|_{\Omega_1} = U$ and $P(D)V = F$ in Ω_0.

(4) The equation (3.1) has a solution $U \in \mathring{\mathcal{B}}^{loc}_{p,k\tilde{P}}(\tilde{\Omega})$ for every $F \in \mathring{\mathcal{B}}^{loc}_{p,k}(\tilde{\Omega})$.

(5) If $U \in \mathcal{B}^{loc}_{p,k\tilde{P}}(\tilde{\Omega}_1)$, $F \in \mathcal{B}^{loc}_{p,k}(\Omega_0)$ and $P(D)U = F$ in Ω_1, then there exists $V \in \mathcal{B}^{loc}_{p,k\tilde{P}}(\Omega_0)$ such that $V|_{\Omega_1} = U$ and $P(D)V = F$ in Ω_0.

(6) The equation (3.1) has a solution $U \in \mathring{\mathcal{D}}'(\tilde{\Omega})$ for every $F \in \mathring{\mathcal{E}}(\tilde{\Omega})$.

Definition 3.1. A smooth surface ω is called regular with respect to $P(D)$ of degree m iff for every normal ϑ to ω, the following relations hold: $P_m(\vartheta) \neq 0$ and $P_k^{(\alpha)}(\vartheta) = 0$, $k \neq m$ and $k - |\alpha| \geq m$. Here P_k is the homogeneous part of P of degree k and $P_k^{(\alpha)}(\xi) = (\frac{\partial}{\partial \xi})^\alpha P_k(\xi)$.

<u>Theorem</u> 3.2. If ω is of C^∞-class and is regular with respect to P(D) of degree m, then the six statements (1) to (6) in Theorem 3.1 and the following statement are equivalent.

(7) The following equation (3.2) has a solution $U \in \mathcal{E}(\tilde{\Omega})$ for every $F \in \mathcal{E}(\tilde{\Omega})$ and $f_j \in \mathcal{E}(\omega)$, $j = 1,2,\cdots,m$.

$$\begin{cases} P(D)U = F & \text{in } \Omega \\ R\circ(\frac{\partial}{\partial n})^{j-1}U = f_j & \text{in } \omega,\ j = 1,2,\cdots,m. \end{cases} \tag{3.2}$$

<u>Theorem</u> 3.3. If ω is smooth we can write

$$R\circ P(D) = \sum_{k=0}^{m} A_k\circ R\circ(\frac{\partial}{\partial n})^k \tag{3.3}$$

for some differential operators A_k: $\mathcal{E}(\omega) \longrightarrow \mathcal{E}(\omega)$, $k=0,1,\cdots,m$. We consider the case when A_m is not zero everywhere in ω. If we assume that $\tilde{\Omega}$ is P(D)-proper, then the following two statements are equivalent.

(i) The equation (3.2) has a solution $U \in \mathcal{E}(\tilde{\Omega})$ for every $F \in \mathcal{E}(\tilde{\Omega})$ and $f_j \in \mathcal{E}(\omega)$, $j = 1,2,\cdots,m$.

(ii) The equation

$$A_m u = f \quad \text{in } \omega \quad . \tag{3.4}$$

has a solution $u \in \mathcal{E}(\omega)$ for every $f \in \mathcal{E}(\omega)$.

<u>Theorem</u> 3.4. Let ω be smooth, M the order of P(D), and let the relation (3.3) be satisfied. Let $\nu \in \mathbb{N}$, $m \leqq s \leqq m+\nu$, $M-m \leqq \leqq \nu(M-m-\mu)$, and write $l = (\nu-1)(M-m)+\nu(1-\mu)$. For some positive constant C,

$$(1+|\xi|^2)^{\frac{s-\nu}{2}} \leqq C\cdot\tilde{P}(\xi), \quad \xi \in \mathbb{R}^n.$$

We assume that $\tilde{\Omega}$ is P(D)-proper, and for every $t \geqq \mu+1/2$ and $f \in H^{loc}_{(t-\mu)}(\omega)$ the equation (3.4) has a solution $u \in H^{loc}_{(t)}(\omega)$.

Then the equation (3.2) has a solution $U \in H^{loc}_{(s)}(\tilde{\Omega})$ for every $F \in H^{loc}_{(l)}(\tilde{\Omega})$ and $f_j \in H^{loc}_{(l+M-j+1/2)}(\omega)$, $j=1,2,\cdots,m$.

Theorem 3.5. Let ω be the disjoint union of ω_1, ω_2, ω_{12}, $\omega_{12} \subset \partial\omega_1 \cap \partial\omega_2$, and let ω_j, $j=1,2$ be smooth open in ω and regular with respect to P(D) of degree m. Write $\tilde{\omega}_j = \omega_j \cap \omega_{12}$, $j=1,2$. Moreover let $\tilde{\omega}_1$ and $\tilde{\omega}_2$ be regularly situated, i.e. for every compact sets $K_j \subset \tilde{\omega}_j$, $j=1,2$ there exist constants $C>0$ and $\alpha>0$ such that $d(x, K_2) \geqq C \cdot d(x, \omega_{12})^{\alpha}$, $x \in K_1$.

Then the equation

$$\begin{cases} P(D)U = F \quad \text{in } \tilde{\Omega} \\ R\circ(\frac{\partial}{\partial n})^{j-1}U = f_j \quad \text{in } \omega_1, \ j=1,2,\cdots,m \\ R\circ(\frac{\partial}{\partial n})^{j-1}U = g_j \quad \text{in } \omega_2, \ j=1,2,\cdots,m \end{cases}$$

has a solution $U \in \mathcal{E}(\tilde{\Omega})$ if and only if $F \in \mathcal{E}(\tilde{\Omega})$, $f_j \in \mathcal{E}(\tilde{\omega}_1)$, $g_j \in \mathcal{E}(\tilde{\omega}_2)$, $j=1,2,\cdots,m$ satisfy the following compatibility condition:

$$\Lambda_1(F, (f_j))(x) = \Lambda_2(F,(g_j))(x), \quad x \in \omega_{12}.$$

Here $\Lambda_1(F, (f_j))(x)$ is the C^{∞}-jet defined by

$$h_j = \begin{cases} f_{j+1}, \quad j=0,1,\cdots,m-1 \\ a^{-1}(R\circ(\frac{\partial}{\partial n})^{j-m}F - \sum_{k=0}^{m-1} A_k h_{k+j-m}), \quad j=m,m-1,\cdots, \end{cases}$$

where $R\circ P(D) = a\cdot R\circ(\frac{\partial}{\partial n})^m + \sum_{k=0}^{m-1} A_k \cdot R\circ(\frac{\partial}{\partial n})^k$, $a \neq 0$.

Theorem 3.6. If $\tilde{\Omega}$ is strongly P(D)-proper, then the following two statements hold.

(1) The equation (3.1) has a solution $U \in \mathring{\mathcal{D}}'(\tilde{\Omega})$ for every $F \in \mathring{\mathcal{D}}'(\tilde{\Omega})$.

(2) If $F \in \mathcal{D}'(\Omega_0)$, $U \in \mathcal{D}'(\tilde{\Omega}_1)$ and $P(D)U = F$ in Ω_1, then there exist $V \in \mathcal{D}'(\Omega_0)$ such that $V|_{\Omega_1} = U$ and $P(D)V = F$ in Ω_0.

§4. The case when ω is flat

Let H be a closed half space with the normal ϑ such that $\tilde{\Omega} \subset H$ and $\omega \subset \partial H$. Let P(D) be a linear differential operator with constant coefficients.

Theorem 4.1. The following condition and the five conditions (2) to (6) in Theorem 3.1 are equivalent.

(1') There exists a fundamental solution of P(D) with its support in H and $\tilde{\Omega}$ is P(D)-proper.

Theorem 4.2. We consider the equation

$$\begin{cases} P(D)U = F & \text{in } \Omega \\ R \circ B_j(D)U = f_j & \text{in } \omega,\ j = 1,2,\cdots,l, \end{cases} \tag{4.1}$$

where B_j, $j = 1,2,\cdots,l$ are linear differential operators with constant coefficients in $\mathbb{R}^n$. If $\tilde{\Omega}$ is P-proper and ω is convex, then the equation (4.1) has a solution $U \in \mathcal{E}(\tilde{\Omega})$ if and only if $F \in \mathcal{E}(\tilde{\Omega})$ and $f_j \in \mathcal{E}(\omega)$, $j = 1,2,\cdots,l$ satisfy the following condition. Let $Q(\xi)$, $Q_j(\xi)$, $j = 1,2,\cdots,l$ be polynomials in $\xi \in \mathbb{R}^n$ such that $Q_j(\xi + \tau\vartheta)$ are independent of $\tau \in \mathbb{C}$ and

$$Q(\xi)P(\xi) + \sum_{j=1}^{l} Q_j(\xi)B_j(\xi) \equiv 0.$$

Then

$$R \circ Q(D)F + \sum_{j=1}^{l} Q_j(D)f_j = 0 \quad \text{in } \omega.$$

Theorem 4.3. If P(D) is hyperbolic with respect to H, then the following statements hold.

(1) For every $F \in \mathring{\mathcal{B}}(\tilde{\Omega})$ there exists $U \in \mathring{\mathcal{B}}(\tilde{\Omega})$ such that $P(D)U = F$ in Ω_0.

(2) If $F \in \mathcal{B}(\Omega_0)$, $U \in \mathcal{B}(\Omega_1)$ and $P(D)U = F$ in Ω_1, then there exists $V \in \mathcal{B}(\Omega_0)$ such that $V|_{\Omega_1} = U$ and $P(D)V = F$ in Ω_0.

In this case we need no condition on the boundary.