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ON PSEUDO-DIFFERENTIAL EQUATIONS IN HYPERFUNCTION THEORY

M. SATO, T. KAWAI AND M. KASHIWARA

Since the introduction in 1969 of the sheaf $\mathcal{C}$ into hyperfunction theory (to whose sections and local sections we shall give the name 'micro-hyperfunctions' or simply 'microfunctions') the following scheme of studying linear differential equations has been established: Instead of studying equations and their hyperfunction solutions on a given manifold M,[1] study the corresponding pseudo-differential equations and their microfunction solutions on the cosphere bundle S^*M of M. Then this 'hyper-local theory' on S^*M, when projected down onto M, will give us desired informations about hyperfunction solutions of the given equation, in virtue of the fundamental exact sequence

$$0 \to \mathcal{A} \to \mathcal{B} \overset{sp}{\to} \pi_* \mathcal{C} \to 0$$

in $\mathcal{C}$-theory connecting the sheaves of analytic functions, hyperfunctions, and microfunctions. We remind the reader that π_* signifies the 0-th direct image by the canonical projection $\pi : S^*M \to M$. The image sp u of a hyperfunction $u \in \mathcal{B}(M)$ will be called the spectrum of u.[2] We also remind that, whereas $\mathcal{B}$ is a flabby sheaf on M on which linear differential operators act as local operators, $\mathcal{C}$ constitutes a flabby sheaf on S^*M on which pseudo-differential operators (in the hyperfunction-theoretic sense) act as (hyper-) local operators.

The simplest and the most immediate application of this principle will be the following results by Sato and Kawai-Kashiwara, respectively:

1. Let P be a differential operator with analytic coefficients and let u be a hyperfunction such that Pu is analytic. Then supp sp u, the support of the microfunction sp u, is contained in the characteristics of the principal symbol P_m of P, that is, in the analytic subset of S^*M defined by $P_m(x, \eta) = 0$ (Sato [1], [2], Sato-Kawai [1], Sato-Kashiwara [1], Schapira [1]). Clearly this gives a significant generalization of the fact that solutions of an elliptic equation are always analytic.

2. Let P be as above and assume that $P_m(x, \eta)$ is real and of simple characteristics. Then supp sp u consists of bicharacteristic strips; in other words, it is invariant under the Hamilton field of P_m. (Kawai [1], [2], [3]. See also Sato [2], Hörmander [3].) Thus, the observation of microfunction solutions reveals the major role of bicharacteristic strips as the true carrier of the solution of the given equation, and significantly improves the hitherto known results on propagation of singularities and regularities expressed in terms of bicharacteristic curves, which are the images of bicharacteristic strips projected onto M.

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1) In this note everything is treated in the real analytic category and the phrase 'real analytic' will be often omitted.

2) Recently Professor L. Hörmander [3] introduced the C^∞-theoretic version of supp sp u under the name 'wave front set' of u (cf. Sato [4]).

These examples may even suggest that our 'hyper-local' theory can control the global theory to a certain extent, certainly better than the usual 'local' theory can. In the present note we shall go further along this 'hyper-local' theoretic way and establish the fundamental theorems on a system of pseudo-differential equations. These theorems give the complete answer as to the structure of microfunction solutions for any system of pseudo-differential equations (of finite type as defined in Kashiwara-Kawai [1]), in the both cases of real and complex characteristics, provided that the system is of the most generic type of the kind. The results reveal that the structure of a system of pseudo-differential equations of such a generic type is extremely simple from the 'hyper-local' point of view.

Our fundamental theorem for the system of pseudo-differential equations, in the case of real characteristics, states

Theorem A. Let $\mathfrak{M} : P_\nu u = 0$ ($\nu = 1, 2, \dots$) be a system of pseudo-differential equations of finite type and finite order whose characteristics V in the cosphere bundle is real and simple in the neighborhood of (x^0, η^0). Then, in the neighborhood of (x^0, η^0),

$$\mathrm{Ext}^k_{\mathcal{P}}(\mathfrak{M}, \mathcal{C}) = 0 \quad \text{for } k > 0,$$

while for $k = 0$ the solution sheaf $\mathrm{Hom}_{\mathcal{P}}(\mathfrak{M}, \mathcal{C})$ is a sheaf supported on V which is locally constant along each bicharacteristic manifold and flabby in the transversal direction.

Here we denote by $\mathcal{P}$ the sheaf of rings of pseudo-differential operators of finite type and finite order, and we identify the system $\mathfrak{M}$ with the $\mathcal{P}$-coherent sheaf $\mathcal{P}/\mathcal{J}$, the quotient module of $\mathcal{P}$ by its left ideal $\mathcal{J}$ generated by $P_1, P_2, \dots$. Furthermore, the meaning of the above statement about the solution sheaf is the following: We have a manifold U_0, a flabby sheaf $\mathcal{C}_{U_0}$ on U_0, a neighborhood U of (x^0, η^0) in the characteristic variety V, and a smooth morphism $\rho : U \to U_0$, so that the bicharacteristic manifolds lying in U are just the fibers of ρ, and that the solution sheaf $\mathrm{Hom}_{\mathcal{P}}(\mathfrak{M}, \mathcal{C})$ restricted onto U is isomorphic to $\rho^{-1}\mathcal{C}_{U_0}$.

The vanishing of all Ext^k, $k \neq 0$, means the non-existence of obstructions to constructing a 'Poincaré complex' for the system $\mathfrak{M}$, because it implies that a $\mathcal{P}$-projective resolution of the sheaf $\mathfrak{M}$ will give rise to a corresponding injective resolution of the microfunction solution sheaf $\mathrm{Hom}_{\mathcal{P}}(\mathfrak{M}, \mathcal{C})$ by means of flabby sheaves of microfunctions.

One of the easy consequences of Theorem A is

Corollary. The support of the microfunction solution u of the equation $\mathfrak{M}$, which of course is contained in the characteristic manifold V, is actually a union of bicharacteristic manifolds. Conversely, there exists a microfunction solution of $\mathfrak{M}$ whose support coincides with a given single bicharacteristic manifold, provided that the bicharacteristic submanifold is simply connected.

This generalizes the above-cited result of Kawai-Kashiwara to the case of a system.

Theorem A is an immediate consequence of the following Theorem A'.

Theorem A'. Any system of pseudo-differential equations of finite type and finite order

$$\mathfrak{M}: P_\iota u = 0 \qquad (\iota = 1, 2, \dots),$$

whose characteristic variety is real and simple at (x^0, η^0), is 'hyper-locally' equivalent to a partial de Rham system

$$\mathfrak{M}': \frac{\partial}{\partial x'_\iota} u' = 0 \qquad (\iota = 1, 2, \dots, d),$$

d denoting the codimension of the characteristic variety in S^*M near (x^0, η^0).

The fundamental theorem for the system of pseudo-differential equations of the most generic type, in the case of complex characteristics, is the following

Theorem B. Let $\mathfrak{M}: P_\iota u = 0$ $(\iota = 1, 2, \dots)$ be a system of pseudo-differential equations of finite type and finite order. Suppose that its complex characteristic variety $V^{\mathbb{C}}$ (which lies in the complex neighborhood of S^*M) is locally defined by $f_1(x, \eta) = 0, \dots, f_d(x, \eta) = 0$ in the neighborhood of $(x^0, \eta^0) \in S^*M$, with f_ι's in the symbol ideal J of $\mathfrak{M}$, and assume that the hermitian form (called 'generalized Levi form') whose coefficients are the Poisson brackets

$$\frac{1}{2i}\{f_\iota, \bar{f}_\kappa\},$$

is non-degenerate and has the sign (d - p, p) (i.e. has d - p positive eigenvalues and p negative eigenvalues) at (x^0, η^0). Then, in the neighborhood of (x^0, η^0),

$$\mathrm{Ext}^k_{\mathcal{P}}(\mathfrak{M}, \mathcal{C}) = 0 \quad \text{for } k \neq p$$

while for k = p this is canonically isomorphic to a flabby sheaf $\mathcal{C}_W$ defined in the neighborhood of (x^0, η^0) and supported on $W \underset{\mathrm{def}}{=} V^{\mathbb{C}} \cap S^*M$.

We note that non-degeneracy of the generalized Levi form implies linear independence of $df_1, \dots, df_d, d\bar{f}_1, \dots, d\bar{f}_d$, and ω, where ω denotes the canonical 1-form $\eta_1 dx_1 + \dots + \eta_n dx_n$ on $T^*M - M$. This means that $V^{\mathbb{C}}$ is simple in the neighborhood of (x^0, η^0), that $V^{\mathbb{C}}$ and its complex conjugate $\overline{V}^{\mathbb{C}}$ intersects transversally at the complexification $W^{\mathbb{C}}$ of W, and that $\omega|_W$ does not vanish at (x^0, η^0).

On the other hand, it is a remarkable fact that the sheaf $\mathcal{C}_W$ is fully determined by W alone, and is not affected by $V^{\mathbb{C}}$ passing through W. This implies also that $\mathcal{C}_W$ is independent of p, because as will be seen from the subsequent discussions, there exist for a fixed W infinitely many $V^{\mathbb{C}}$'s with all possible values of $p = 0, \ldots, d$, each of them corresponding to some system $\mathfrak{M}$ with properties described above.

Theorem B can be derived from the following Theorem B' and Theorem B_q, which will be proved in §2. In the sequel, 'a system of pseudo-differential equations' always means 'a system of pseudo-differential equations of finite type and finite order'.

Theorem B'. Any system of pseudo-differential equations

$$\mathfrak{M} : P_\iota u = 0 \qquad (\iota = 1, 2, \ldots)$$

whose generalized Levi form is non-degenerate at a characteristic point (x^0, η^0), is 'hyper-locally' equivalent to a system of the following form, considered in the neighborhood of $x' = 0$, $\eta' = (0, \ldots, 0, 1)$.

$$\mathfrak{M}_q : \left(\frac{\partial}{\partial x'_\iota} - \frac{i}{2} \frac{\partial q(x')}{\partial x'_\iota} \frac{\partial}{\partial x'_n}\right) u' = 0 \qquad (\iota = 1, \ldots, d),$$

where $d\ (<n)$ is the codimension of the complex characteristic variety $V^{\mathbb{C}}$ in a complexification of S^*M near (x^0, η^0), and $q(x')$ is a non-degenerate real-valued quadratic form of $x'_1, \ldots, x'_d$ with the same sign as that of the generalized Levi form of $\mathfrak{M}$.

Theorem B_q. Theorem B is valid with the equation $\mathfrak{M}_q$ in Theorem B'.

Consider the equation $\mathfrak{M}_q$. (We omit primes, and we consider $\mathfrak{M}_q$ on that part of the cosphere bundle of $\mathbb{R}^n$ where $(\eta_1, \ldots, \eta_d, \eta_n) \neq (0, \ldots, 0, 0)$.) Since the equation has a covariant expression, $\mathfrak{M}_q$ can be brought to the form

$$\left(\frac{\partial}{\partial x_\iota} + i x_\iota \frac{\partial}{\partial x_n}\right) u = 0 \qquad (\iota = 1, \ldots, p)$$

$$\left(\frac{\partial}{\partial x_\iota} - i x_\iota \frac{\partial}{\partial x_n}\right) u = 0 \qquad (\iota = p+1, \ldots, d)$$

if one prefers to do so, by bringing $q(x)$ into the canonical form $-x_1^2 - \ldots - x_p^2 + x_{p+1}^2 + \ldots + x_d^2$ through a linear transformation, where $(d-p, p)$ denotes the sign of $q(x)$.

Let the first order operators $\frac{\partial}{\partial x_\iota} - \frac{i}{2} \frac{\partial q(x)}{\partial x_\iota} \frac{\partial}{\partial x_n}$ appearing in $\mathfrak{M}_q$ be denoted by P_ι, and its complex conjugate by $\overline{P}_\iota$. P_ν's are mutually commutative, and so are $\overline{P}_\nu$'s, while we have, setting $q(x) = \sum a_{\iota\kappa} x_\iota x_\kappa$ with $a_{\iota\kappa} = a_{\kappa\iota}$,

$$\frac{1}{2i}[P_\iota, \overline{P}_\kappa] = \frac{1}{2i}(P_\iota \overline{P}_\kappa - \overline{P}_\kappa P_\iota) = \frac{1}{2}\frac{\partial^2 q(x)}{\partial x_\iota \partial x_\kappa}\frac{\partial}{\partial x_n} = a_{\iota\kappa}\frac{\partial}{\partial x_n}$$

or in terms of the symbols $P_\iota(x, \eta) = \eta_\iota - \frac{i}{2}\frac{\partial q(x)}{\partial x_\iota}\eta_n$,

$$\frac{1}{2i}\{P_\iota(x,\eta), \overline{P}_\kappa(x,\eta)\} = a_{\iota\kappa}\eta_n.$$

The characteristic variety of $\mathfrak{M}_q$, which is determined by the equation $P_1(x,\eta) = 0, \ldots, P_d(x,\eta) = 0$, consists of two disjoint analytic sets W and W^a in the cosphere bundle given by

$$W = \{(x,\eta) ; x_1 = \ldots = x_d = 0,\ \eta_1 = \ldots = \eta_d = 0,\ \eta_n = 1\}$$

$$W^a = \text{antipodal of } W = \{(x, -\eta) ; (x,\eta) \in W\}$$

because we excluded that part where $\eta_1 = \ldots = \eta_d = \eta_n = 0$. The meaning of above calculation is that the generalized Levi form for the system $\mathfrak{M}_q$ is given by the coefficients $a_{\iota\kappa}$ of $q(x)$ on W, and by $-a_{\iota\kappa}$ (of $-q(x)$) on W^a. And of course we need only prove Theorem B_q at the point $x = 0$, $\eta = (0, \ldots, 0, 1)$ which is on W.

We note that the theory of 'Fourier integral operators' due to Hörmander (Hörmander [2], Egorov [1]) combined with the theory of $\mathcal{C}$ is effectively used in our reasoning. Indeed, Theorem A' is proved without any difficulty by using the classical theory of Jacobi on systems of involution and contact transformations, whereby making essential use of the hyperfunction-theoretic version of 'Fourier integral operators', and Theorem 7 of Kashiwara-Kawai [1]. We also note that our work is deeply affected by instructive articles of Lewy ([1], [2]) and recent works by I. Naruki. Theorem B' is proved also along the same lines, by making use of the following important lemmata.

Lemma 1. Given an analytic function $f(x,\eta)$ defined in the neighborhood of $(x^0, \eta^0) \in T^*M - M$, homogeneous in η, and satisfying the conditions

(1) $f(x^0, \eta^0) = 0$

(2) $\frac{1}{2\sqrt{-1}}\{f, \overline{f}\}(x^0, \eta^0) \gtrless 0$

then we can choose an analytic function $\Phi(x, \eta ; t, \overline{t})$ in (x,η) and a complex variable t and its complex conjugate $\overline{t}$, defined and real valued in the neighborhood of $(x^0, \eta^0 ; 0, 0)$ and such that $\Phi(x, \eta ; f(x,\eta), \overline{f}(x,\eta))$ is homogeneous in η, so that we have

$$\frac{1}{2\sqrt{-1}}\{f\Phi(x,\eta;f,\bar{f}),\ \bar{f}\,\Phi(x,\eta;f,\bar{f})\}=1.$$

More generally, suppose we are given an analytic function $F(x,\eta;t,\bar{t})$ defined and strictly positive valued in the neighborhood of $(x^0,\eta^0;0,0)$ and such that $F(x,\eta;f(x,\eta),\bar{f}(x,\eta))$ is homogeneous in η, then we can again choose a $\Phi(x,\eta;t,\bar{t})$ with the properties described above, so that we have

$$\{f\Phi(x,\eta;f,\bar{f}),\ \bar{f}\,\Phi(x,\eta;f,\bar{f})\}=\{f,\bar{f}\}\cdot F(x,\eta;f,\bar{f}).$$

(Proof of the generalized statement)

Abbreviating $\Phi(t,\bar{t}) \underset{\text{def}}{=} \Phi(x,\eta;t,\bar{t})$ and setting $\Psi(t,\bar{t}) \underset{\text{def}}{=} (\Phi(t,\bar{t}))^2$, the left hand side of the desired equality

$$\{f\Phi(f,\bar{f}),\ \bar{f}\,\Phi(f,\bar{f})\}=\{f,\bar{f}\}\,F(f,\bar{f})$$

is rewritten as

$$=\{f,\bar{f}\}(\Phi(f,\bar{f}))^2+f\,\Phi(f,\bar{f})\{\Phi(f,\bar{f}),\bar{f}\}+\bar{f}\Phi(f,\bar{f})\{f,\Phi(f,\bar{f})\}$$

$$=\{f,\bar{f}\}\,\Psi(f,\bar{f})+\frac{1}{2}f\{\Psi(f,\bar{f}),\bar{f}\}+\frac{1}{2}\bar{f}\{f,\Psi(f,\bar{f})\}.$$

We have, however,

$$\{\Psi(f,\bar{f}),\bar{f}\}=(\{\Psi(t,\bar{t}),\bar{f}\}+\{f,\bar{f}\}\frac{\partial\Psi(t,\bar{t})}{\partial t})_{(t,\bar{t})=(f,\bar{f})}$$

$$\{f,\Psi(f,\bar{f})\}=(\{f,\Psi(t,\bar{t})\}+\{f,\bar{f}\}\frac{\partial\Psi(t,\bar{t})}{\partial\bar{t}})_{(t,\bar{t})=(f,\bar{f})}.$$

Therefore, defining the derivations Λ and $\bar{\Lambda}$ acting on a function $g(x,p)$ in the neighborhood of (x^0,η^0) by

$$\Lambda g \underset{\text{def}}{=} \frac{\{g,\bar{f}\}}{\{f,\bar{f}\}},\qquad \bar{\Lambda}g \underset{\text{def}}{=} \frac{\{f,g\}}{\{f,\bar{f}\}}$$

and setting

$$\Psi(t,\bar{t})=\Sigma_{\mu,\nu}\psi_{\mu\nu}\frac{t^\mu\bar{t}^\nu}{\mu!\,\nu!}\quad\text{with}\quad \psi_{\mu\nu}=\psi_{\mu\nu}(x,\eta)$$

$$\Lambda\Psi(t,\bar{t}) \underset{\text{def}}{=} \Sigma_{\mu,\nu}(\Lambda\psi_{\mu\nu})\frac{t^\mu\bar{t}^\nu}{\mu!\,\nu!}$$

$$\overline{\Lambda}\Psi(t,\overline{t}) \underset{\text{def}}{=} \Sigma_{\mu,\nu} (\overline{\Lambda}\psi_{\mu\nu}) \frac{t^{\mu}\overline{t}^{\nu}}{\mu!\,\nu!}$$

we see that the requirements imposed on $\Phi(t,\overline{t})$ are certainly met if the following equation for $\Psi(t,\overline{t})$ holds:

$$\Psi(t,\overline{t}) + \frac{1}{2}t\,(\frac{\partial\Psi(t,\overline{t})}{\partial t} + \Lambda\Psi(t,\overline{t})) + \frac{1}{2}\overline{t}\,(\frac{\partial\Psi(t,\overline{t})}{\partial\overline{t}} + \overline{\Lambda}\,\Psi(t,\overline{t})) = F(t,\overline{t}).$$

Now we set $F(t,\overline{t}) = \Sigma_{\mu,\nu} c_{\mu\nu} \frac{t^{\mu}\overline{t}^{\nu}}{\mu!\,\nu!}$ with $c_{\mu\nu} = c_{\mu\nu}(x,\eta)$, $\overline{c}_{\nu\mu} = c_{\mu\nu}$, $c_{00} \underset{\neq}{>} 0$, and compare the coefficients of $t^{\mu}\overline{t}^{\nu}/\mu!\,\nu!$ on the both sides of this equation, and get

$$\psi_{\mu\nu} + \frac{1}{2}\mu\,(\psi_{\mu\nu} + \Lambda\psi_{\mu-1,\nu}) + \frac{1}{2}\nu\,(\psi_{\mu\nu} + \overline{\Lambda}\psi_{\mu,\nu-1}) = c_{\mu\nu}$$

$$\text{i.e. } (1 + \frac{1}{2}\mu + \frac{1}{2}\nu)\psi_{\mu\nu} = -\frac{1}{2}\mu\Lambda\psi_{\mu-1,\nu} - \frac{1}{2}\nu\overline{\Lambda}\psi_{\mu,\nu-1} + c_{\mu\nu}.$$

It is obvious that all $\psi_{\mu\nu}$ are determined recursively through this formula in a unique and consistent way:

$$\psi_{00} = c_{00},\ \psi_{01} = \frac{1}{3}(-\Lambda c_{00} + 2c_{10}),\ \psi_{10} = \frac{1}{3}(-\overline{\Lambda}\,c_{00} + 2c_{01}), \ldots.$$

and that the series $\Psi(t,\overline{t}) = \Sigma_{\mu,\nu} \psi_{\mu,\nu} \frac{t^{\mu}\overline{t}^{\nu}}{\mu!\,\nu!}$ thus constructed certainly converges uniformly in the neighborhood of $(\overset{0}{x},\overset{0}{\eta};0,0)$. Moreover, these $\psi_{\mu\nu}$ satisfy $\overline{\psi}_{\nu\mu} = \psi_{\mu\nu}$ and $\Psi(0,0) = \psi_{00} = c_{00} > 0$ (in the neighborhood of $(\overset{0}{x},\overset{0}{\eta})$), whence we conclude that $\Psi(t,\overline{t})$ is real valued and strictly positive valued and that $\Phi(x,\eta\,;t,\overline{t})$ with the desired property is obtained by

$$\Phi(x,\eta\,;t,\overline{t}) = \sqrt{\Psi(t,\overline{t})} \qquad \text{(q.e.d.)}$$

Lemma 2. Given two analytic functions $f(x,\eta)$ and $g(x,\eta)$, both defined in the neighborhood of $(\overset{0}{x},\overset{0}{\eta}) \in T^{*}M - M$, homogeneous in η, and satisfying the conditions

$$(1)\quad f(\overset{0}{x},\overset{0}{\eta}) = 0, \qquad g(\overset{0}{x},\overset{0}{\eta}) = 0$$

$$(2)\quad \{f,\overline{f}\} = 2\sqrt{-1}, \qquad \{f,g\} = 0,$$

then we can find another analytic function $g'(x,\eta)$ also defined near $(\overset{0}{x},\overset{0}{\eta})$, homogeneous of the same degree in η as g, and satisfying the conditions

$$g' \equiv g \,(\mathrm{mod}\ f), \quad \{f,g'\} = 0, \quad \{g',\overline{f}\} = 0.$$

Indeed, such g' is given by the following series uniformly convergent in the neighborhood of (x^0, η^0)

$$g' \underset{\text{def}}{=} g^{(0)} + \frac{1}{1!} g^{(1)} f + \frac{1}{2!} g^{(2)} f^2 + \dots ,$$

where $g^{(0)}, g^{(1)}, g^{(2)}, \dots$ are defined by

$$g^{(0)} \underset{\text{def}}{=} g, \quad g^{(\nu+1)} \underset{\text{def}}{=} \frac{-1}{2\sqrt{-1}} \{g^{(\nu)}, \bar{f}\}$$

(Proof of $\{f, g'\} = 0$)

It suffices to prove $\{f, g^{(\nu)}\} = 0$ for every ν, and this we prove by induction. If $\nu = 0$ then it is all right. So assume this is true for $g^{(\nu)}$. Then

$$\{f, g^{(\nu+1)}\} = \frac{-1}{2\sqrt{-1}} \{f, \{g^{(\nu)}, \bar{f}\}\} = \frac{-1}{2\sqrt{-1}} (\{\{f, g^{(\nu)}\}, \bar{f}\} + \{g^{(\nu)}, \{f, \bar{f}\}\})$$

and within the last expression the first term is equal to $\{0, \bar{f}\} = 0$ by the assumption of induction, while the second term is $\{g^{(\nu)}, 2\sqrt{-1}\} = 0$, whence $\{f, g^{(\nu+1)}\} = 0$.

(Proof of $\{g', \bar{f}\} = 0$)

$$\{g', \bar{f}\} = \Sigma_\nu \frac{1}{\nu!} \{g^{(\nu)} f^\nu, \bar{f}\} = \Sigma_\nu \frac{1}{\nu!} (\{g^{(\nu)}, \bar{f}\} f^\nu + g^{(\nu)} \cdot \nu f^{\nu-1} \{f, \bar{f}\})$$

$$= -2\sqrt{-1} \Sigma_\nu \frac{1}{\nu!} g^{(\nu+1)} f^\nu + 2\sqrt{-1} \Sigma_\nu \frac{1}{(\nu-1)!} g^{(\nu)} f^{\nu-1} = 0$$

(q.e.d.)

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