Finite groups with Sylow 2-subgroups of type $A_{16}$

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A 2-group is said to be of type $X$ if it is isomorphic to a Sylow 2-subgroup of the group $X$. If $G$ is a group with a Sylow 2-subgroup $S$ of type $X$, we say that $G$ has the involution fusion pattern of $X$ if for some isomorphism $\theta$ of $S$ onto a Sylow 2-subgroup of $X$, two involutions $a, b$ of $S$ are conjugate in $G$ if and only if the involutions $\theta(a), \theta(b)$ of $\theta(S)$ are conjugate in $X$. Also we say that a group $G$ is fusion-simple if $G = O^2(G)$ and $O(G) = Z(G) = 1$.

Now we have obtained the following:

THEOREM A. Let $G$ be a fusion-simple finite group with Sylow 2-subgroups of type $A_{16}$. Then one of the following holds:

1. $G \cong A_{16}$ or $A_{17}$,
2. $G \cong A_9 \cdot E_{256}$, the split extension of an elementary abelian group $E_{256}$ of order 256 by $A_9$ with the action afforded by the 8-dimensional irreducible $GF(2)$-representation, or
3. $G$ has the involution fusion pattern of $\Omega_9(3)$.

Here $\Omega_9(3)$ denotes the orthogonal commutator group of degree 9 over the field of 3-elements and $A_m$ the alternating group on $m$-letters.

In the process of proving Theorem A we obtain the following characterization.
THEOREM B. Let $G$ be a finite group with Sylow 2-subgroups of type $A_{16}$. If $G$ has the involution fusion pattern of $A_{16}$, then $G/O(G) \cong A_{16}$ or $A_{17}$.

Proof of the Theorem A is obtained in the following way which appears to be rapidly becoming standard (cf. Gorenstein-Harada[5], [6], Solomon[9]). Let $S$ be a Sylow 2-subgroup of $G$ and $A$ be the unique elementary abelian subgroup of $S$ of order 256. At first we show that the fusion of elements of $S$ is controlled by $N_G(A)$ and $N_G(Z_2(S))$ where $Z_2(S)$ is the second center of $S$, using results of Alperin[1] and Goldschmidt[2] on conjugation family. Since $S/A$ is of type $A_8$, the structure of $N_G(A)/C_G(A)$ which is isomorphic to a subgroup of $GL(8,2)$ is determined by theorems of Harada[7] and Gorenstein-Harada[5],[6]. Then the fusion possibilities of involutions follow immediately. Here we can prove that if $A$ is strongly closed in $S$ with respect to $G$, then $G = N_G(A) \cong A_9 \cdot E_{256}$ by a recent result of Goldschmidt [4]. Characterization theorems of Gorenstein-Harada[5],[6] and Solomon[9] permit the determination of $C_G(a)/O(C_G(a))$ for all involution a in $S$. Now $O$ is an $A$-signalizer functor and a signalizer functor theorem[3] implies that $W_A = \langle O(C_G(a)) ; a \in A^\# \rangle$ has odd order. It follows that $N_G(W_A)$ is strongly embedded in $G$ provided $W_A \neq 1$. Since $G$ has more than one conjugacy class of involutions, $W_A = 1$. Therefore $O(C_G(a)) = 1$ and Kondo's characterization theorem[8] implies that $G \cong A_{16}$ or $A_{17}$. 
References


