

The Game of "Hana"
- Representation and Analysis
using a Typed Calculus

「花」ゲーム — 型のある論理に基づく表現と解析

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ABSTRACT: The game of "Hana", a Japanese traditional card game, is treated. First, various data structures appearing in the representation of this game are axiomatized in a typed calculus which is a generalization of Scott's logic. The rule of the game itself is a rather complicated function (object formally) of a higher type representable in this calculus. An Algol-like language is introduced as the abbreviation of the terms of this particular type.

Two nontrivial theorems about this game will be proved.

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Ax.12  HANA-AWASE = begin
      M := 1;
      index' := 0;
      while M<12 do
begin
      bouquet := Deal(X);
      flower := Ordering(bouquet(YAMA), X);
      index := Max{index', if <YANAGI, ONO'TOFU>ε bouquet(table)
                    or <KIRI, HOWO> ε bouquet(table) then 2
                    else if Easbouquet(table)[Point(a) = 20]
                        then 1 else 0};
      index' := if <YANAGI, ONO'TOFU>ε bouquet(table)
                  & <KIRI, ONO'TOFU>ε bouquet(table) then 2
                  else if <MATSU, TSURU> ε bouquet(table) &
                        <SAKURA, MANMAKU> ε bouquet(table)
                  or <SAKURA, MANMAKU> ε bouquet(table) &
                        <SUSUKI, MANGETSU>ε bouquet(table)
                  then 1 else 0;
      l := m := n := 1;
      while l<5 & m<5-3 & n<3 do
begin if Active(Person(l), X)
      then begin player(n) := Person(l);
              n:=n+1
              Pay(Hand(Person(l)), table, Tax(m)× expo(2, index));
              m:=m+1 end;
      l:=l+1
end ;
      while n<3 do
begin
      player(n) := Person(l);
      n:=n+1;
      l:=l+1
end ;
      n:=1;
      while n<3 do
begin
      P:= player(n);
      if KARASU(bouquet(P)) or 6KASU(bouquet(P))
      then Show-down(bouquet(P)meet {a|Point(a)=1});

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山 柳 小野道風

桐 鳳凰 幔幕

満月 桜

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    Pay(Hand(player(n+1 mod 3)), Hand(P), TEYAKU-Bonus(bouquet(P)));
    Pay(Hand(player(n+2 mod 3)), Hand(P), TEYAKU-Bonus(bouquet(P)))
end else [];
    n:=n+1
end;
    t:=1;
    while bouquet(YAMA)≠∅ & not Battari(bouquet(Basket(player(
        t-1 mod 3)))) do
begin
    P:=player(t mod 3);
    if bouquet(Hand(P))≠∅
then begin a:=Throw-in(P, X);
        Move(a, table);
        if (bouquet(table) join bouquet(Window(player(t+1 mod 3)
            )) join bouquet(Window(player(t+2 mod 3)))) meet
            {b|Month(a)=Month(b)} ≠ ∅
then begin b:=Take-out(a, P, X);
                Move(a, Basket(P));
                Move(b, Basket(P)) end
            else [];
        end
    else [];
        c:=flower(t, X);
        Move(c, table);
        if bouquet(table) meet {d|Month(c)=Month(d)} ≠ ∅
then begin d:=Take-out(c, P, X);
                Move(c, Basket(P));
                Move(d, Basket(P)) end
            else [];
        t:=t+1
end;
    if E n ∈ {1,2,3} [Point(bouquet(Basket(player(n))))-
        {a | Flower-name(a)=YANAGI or Flower-name(a)=KIRI} ≤ 20]
    then []
else begin n:=n+1;
        while n ≤ 3 do
begin P:=player(n);

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C(n):=(DEKIYAKU-Bonus(bouquet(Basket(P)))+
      (if (n=1 or n=2)& Number(bouquet(Basket(P))) ≤ 8
        or Number(bouquet(Basket(P))) ≤ 10
        then 30 else 0)
      + Point(bouquet(Basket(P)))×exp0(2, index);
Pay(Hand(player(n+1 mod 3)), Hand(P), C(n));
Pay(Hand(player(n+2 mod 3)), Hand(P), C(n));
n:=n+1
end;
Pay(table, player( if max {C(1), C(2), C(3)} = C(1)
                    then 1 else if C(2)≤C(3) then 2 else 3,
                    Cash(table))
end;
M:=M+1

```

end
end.

Theorems (informal description).

BIKI Theorem. The third active player can eventually take in a flower which is dealt to him and chosen by him arbitrarily.

88 Theorem. The total of the points of flowers taken in by the active players is always 88 times 3.

Lemma. Every flower is eventually taken in by some of the active players.

Corollary. The game of "Hana" terminates.

References.

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