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Self-Focusing of Laser Beams in Nonlinear Media

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1. The stationary self-focusing of laser beam propagating in nonlinear media is considered. The nonlinearity of media is expressed through a dielectric function, which depends on the intensity of the beam. Since the nonlinearity makes the beam steepen, the beam is focused of itself if nonlinear effect overcomes diffractive effect. Then there exists a critical power in order to focus the beam.

If we use the quasi-steady state approximation and consider linearly polarized beam propagating in the z-direction, we obtain the following two-dimensional nonlinear Schrödinger equation[1)] for the electric field of the laser beam:

$$2iN \frac{E}{z} + \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + f(N^2|E|^2)E = 0 , \qquad (1)$$

where E denotes envelope of the electric field normalized by an amplitude of incident beam, N is a ratio of the amplitude of incident beam to that of the beam with the critical power and x(y) and z are normalized by an width of incident beam and an characteristic length representing focal distance in the media, respectively.

2. Experimentally observed facts are that, for higher power laser beam injected in carbon disulfide persistent ringed structure appeared in the beam and that, for more strong laser beam, the beam subdivided into many filaments. Townes et al.[2] have pointed out that each trapped filament was in itself approximately cylindrical and stable.

Previous work of numerical analysis has assumed that, as the beam is cylindrical symmetric, a cylindrical coordinate system is adopted to solve Eq. (1) and that motion in the θ-direction is reduced. Therefore we can not obtain behaviour of filaments generated in the beam. An additional condition, say $\partial E/\partial r|_{r=0} = 0$, is required. The condition is contradictory to the collapse of beam predicted by Zakharov et al.[3] for the cubic nonlinearity, $f \propto |E|^2$.

In order to investigate the observations and to conquer the trouble, we numerically analyze Eq. (1) without taking any conditions. We have a freedom of motion in the θ-direction, which is possible in our x-y rectangular coordinate scheme. Then the freedom is expected to give a new mechanism for trapping of the laser beam in the nonlinear media.

3. We simulate Eq. (1) under the conditions that initial light beam is the Gaussian field distribution:

$$E(x, y) = \exp\{-(x^2 + y^2)/2\}, \quad \text{at} \quad z = 0 , \qquad (2)$$

and the nonlinear dielectric function f is the cubic type; $N^2|E|^2$ for the weak intensity beam and a saturable type; $N^2|E|^2/(1 + N^2|E|^2/|E_s|^2)$ with $|E_s|^2 = 500.0$ for the strong intensity beam where dielectric saturates at a constant value $|E_s|^2$.

(2-1) The cubic type of nonlinearity

For N = 0.9, we observed that no focused peak appeared and the incident beam spreaded over by diffractive effect as shown in Fig. 1. For N = 1.5, we observed a small focused peak.

Fig. 1

After that, the beam spreaded over with a ringed structure as shown in Fig. 2. For N = 3.0, the maximum beam intensity $|E|^2_{max}$

Fig. 2

became greater than several hundreds. So we may say that the beam was collapsed.

(2-2) The saturable type of nonlinearity

We observed more interesting and dynamical structures in the central part of the beam.

For N = 3.0, we observed oscillately repeated central peaks in addition to the ringed structure as shown in Fig. 3. For

Fig. 3

N = 10.0, after the first focus of the beam, we observed complex structure with angle dependent motion which consisted of nine filaments. As shown in Fig. 4 these seem to be stable and to move in close connection with the central part.

Fig. 4

Summarizing these results, we want to notice the following points:

(i) We observed the angle dependent motion of the beam in addition to ringed structure. For the strong intensity beam with the saturable nonlinearity, we had nine peaks and each of these formed cylindrical symmetry and moved stably. These explain the experimental observations.

(ii) The central part of the beam seems to move as a whole thought it has a complex dynamical structure.

(iii) When the central part reached at its maximum or minimum, high frequency zigzag motion was appeared as shown in Fig. 5 when N = 10.0 with the saturable nonlinearity.

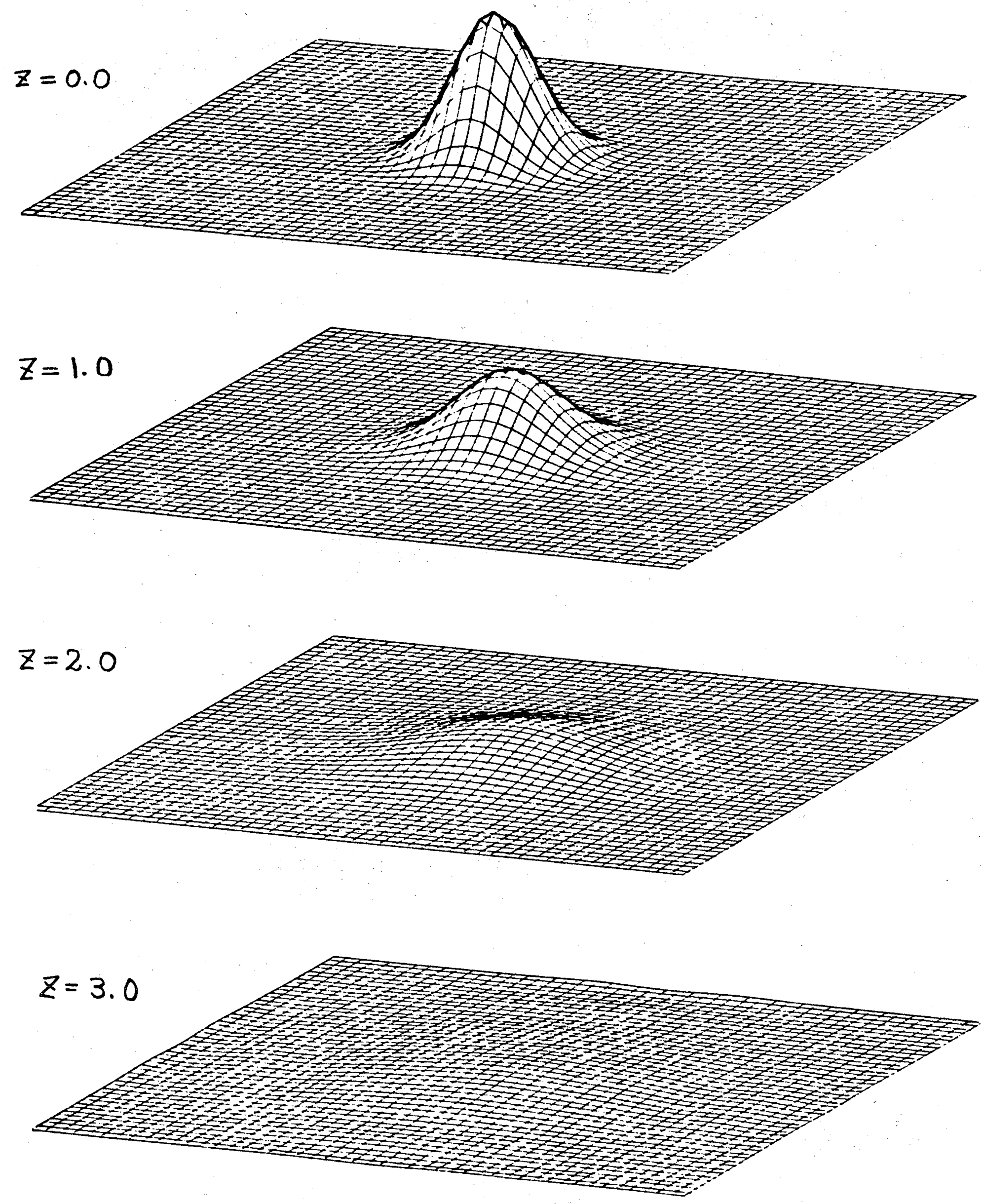
Fig. 5

References

1) Notations were followed by V. N. Lugovoi and A. M. Prokhrov; JETP Letters 7 (1968) 117.

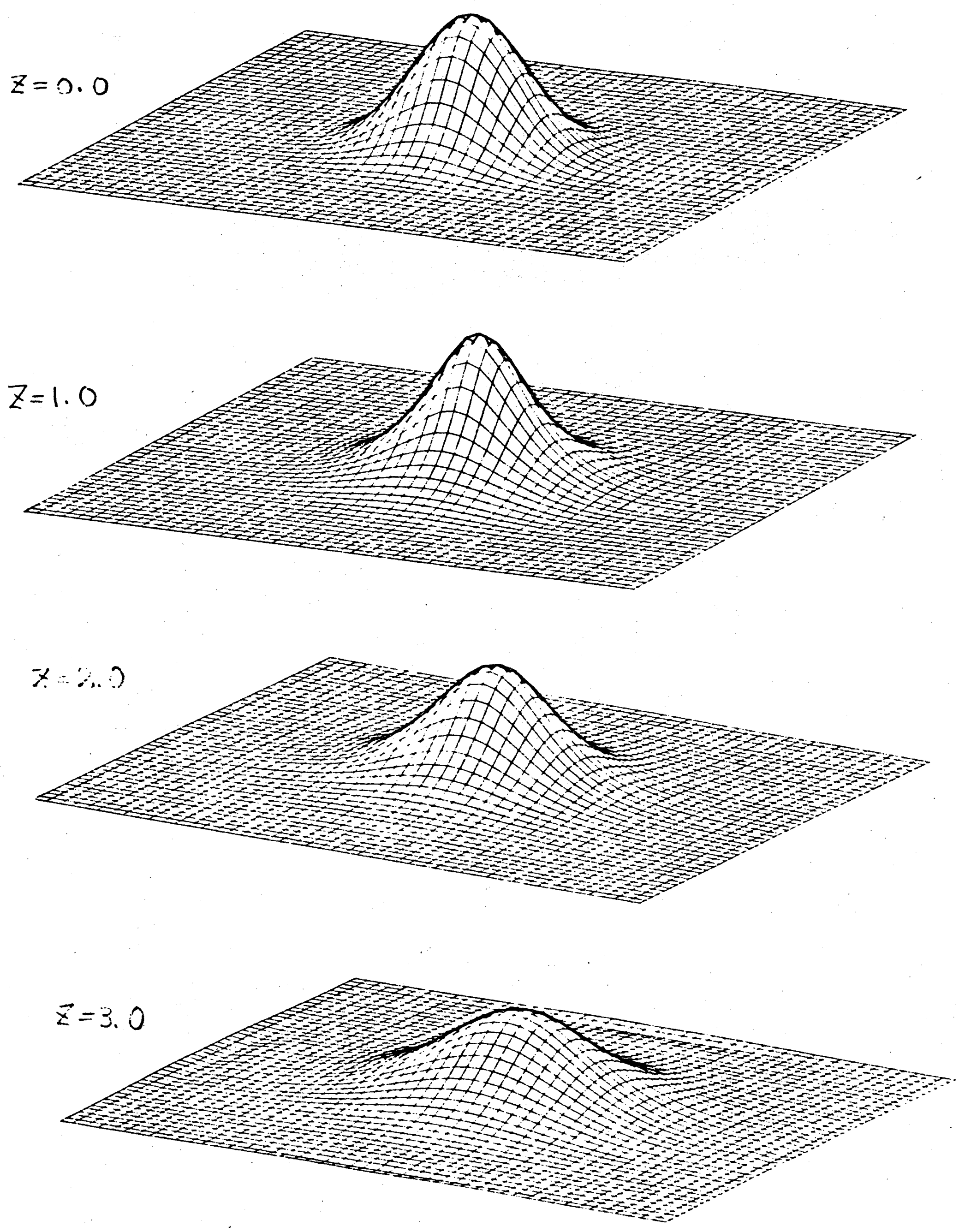
2) R. Y. Chiao et al.; I.E.E.E. QE. QE 2 (1965) 467.
A. J. Campillo, S. L. Shapiro and B. R. Suydam; Applied Physics Letters 24 (1974) 178.

3) V. E. Zakharov, V. V. Sobotev and V. S. Synakh; JETP Letters 14 (1971) 390.



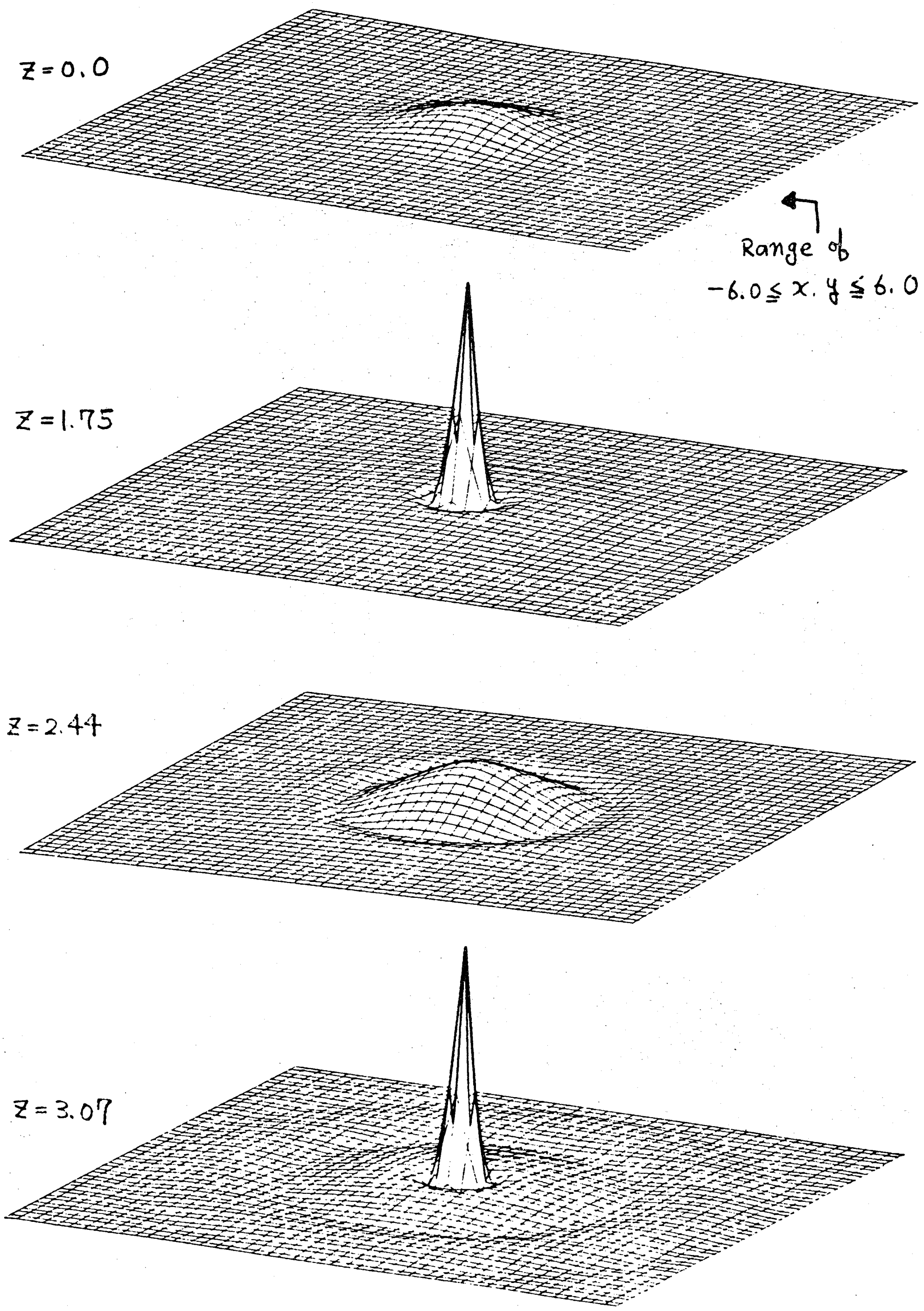
Intensity of Laser Beam $|E|^2$ for Cubic Nonlinearity of $N = 0.9$ within Range of $-6.0 \geq x, y \leq 6.0$

Fig. 1.



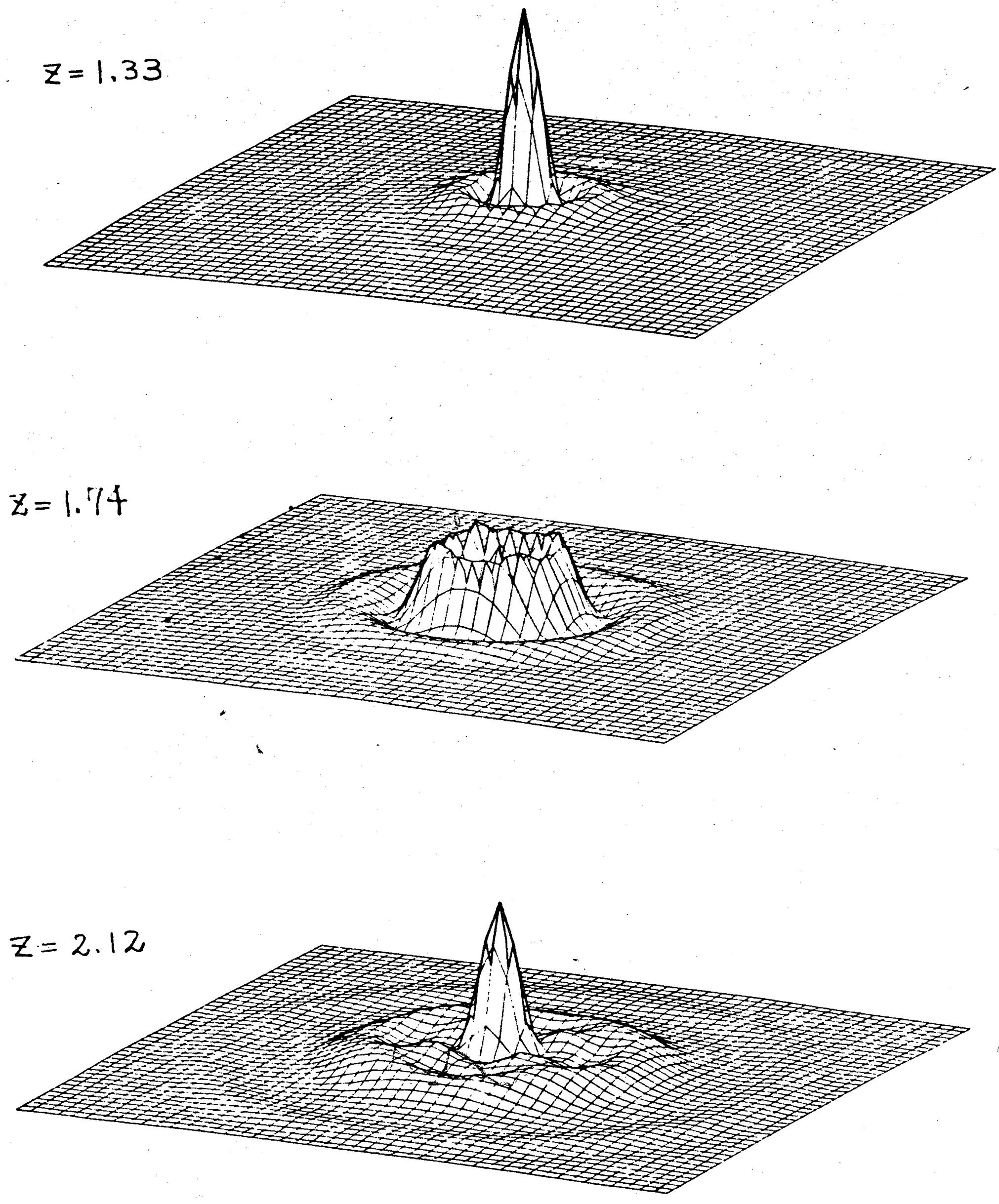
Amplitude of Laser Beam $|E|$ for Cubic Nonlinearity of $N = 1.5$ within Range of $-6.0 \leq x, y \leq 6.0$

Fig. 2.



Amplitude of Laser Beam |E| for Saturable Nonlinearity of N = 3.0 within Range of −3.0 ≤ x, y ≤ 3.0

Fig. 3



Amplitude of Laser Beam $|E|$ for Saturable Nonlinearity of $N = 10.0$ within Range of $-1.5 \leqq x, y \leqq 1.5$

Fig. 4 (1)

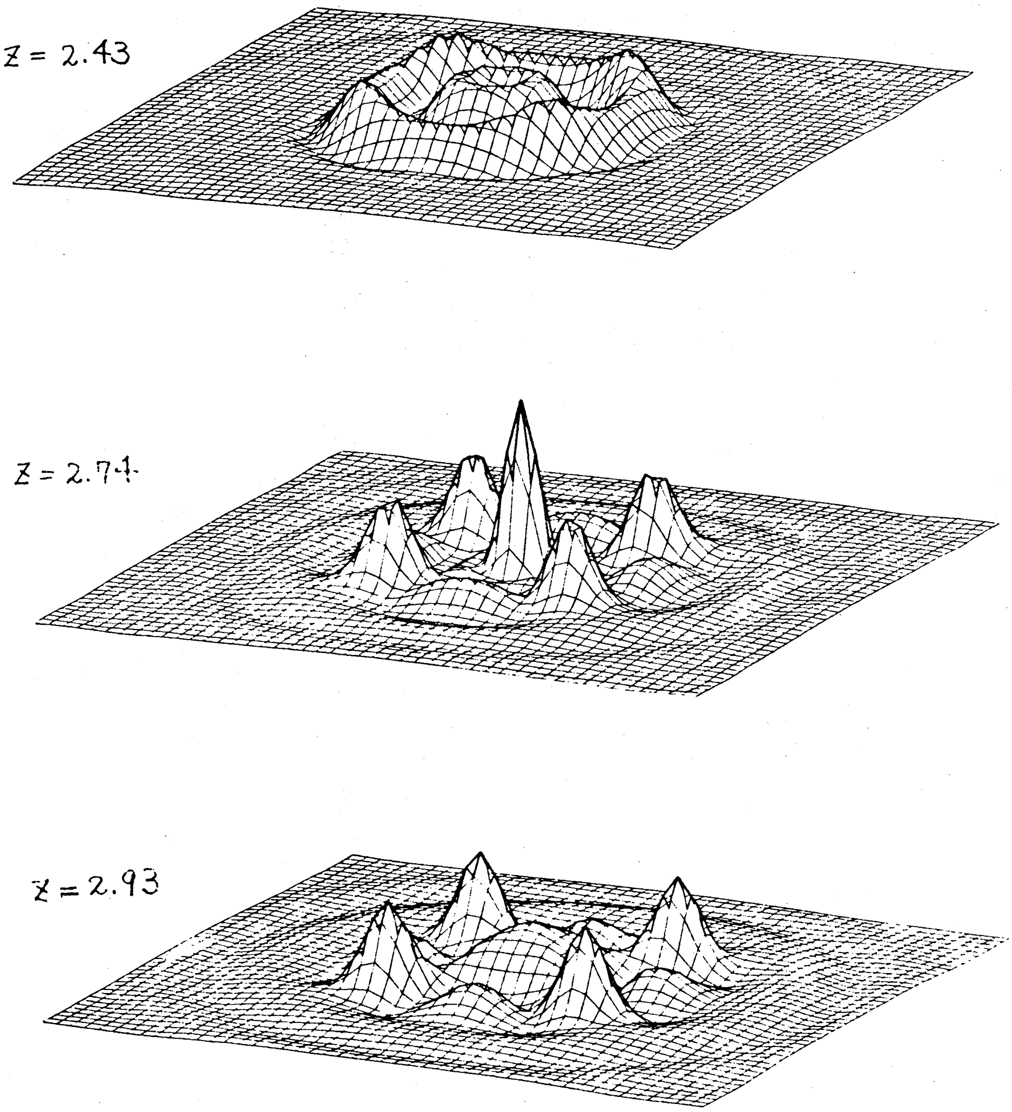
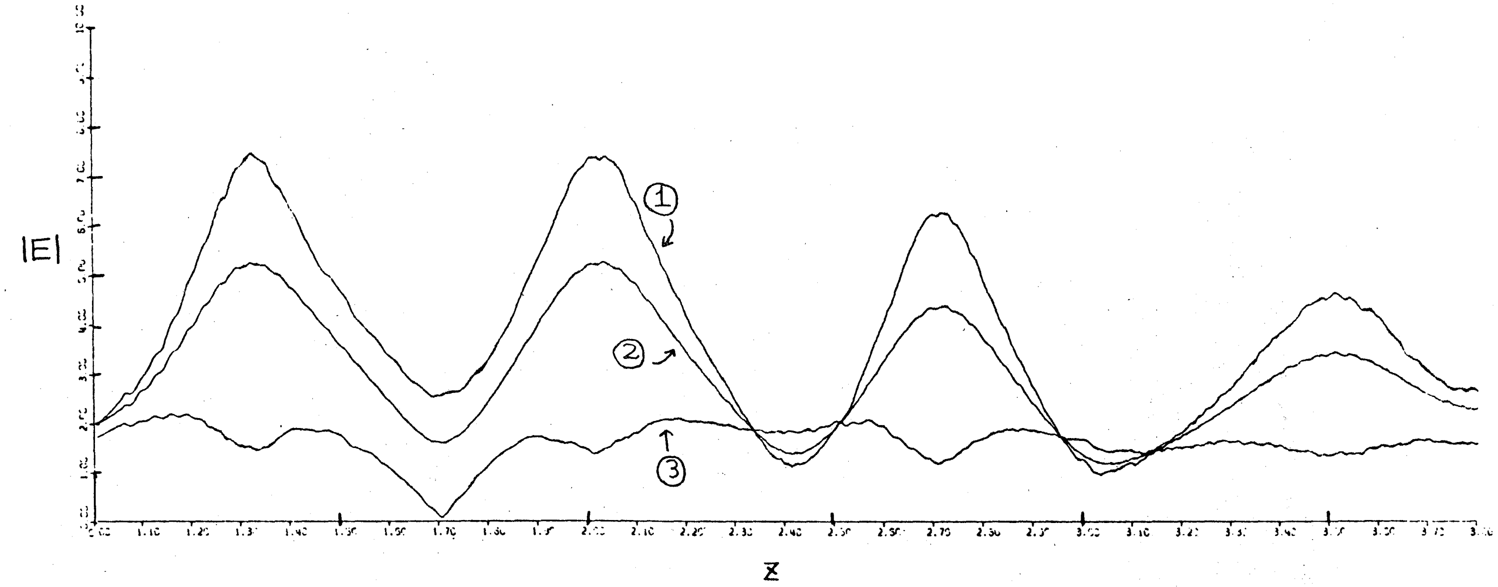


Fig. 4 (2)



Dependence of Amplitude of Laser Beam $|E|$ on z for Saturable Nonlinearity of $N = 10.0$ at Points of ①; (0, 0), ②; (0, 1/16) and ③; (0, 1/8)

Fig. 5