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Supplement to free L-spaces

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When I introduced the notion of free L-spaces in [2] the concept of canonical neighborhoods were used in the definition. However if canonical neighborhoods are weakened to semi-canonical neighborhoods, yet the theory developed in [2] seems to be almost effective for the weakened case. To note this fact will be convenient for the reader of [2].

Let X be a space, F a closed set of X, and $\mathcal{U}_F$ an anti-cover of F. An open neighborhood U of F is said to be <u>semi-canonical</u> with respect to $\mathcal{U}_F$ if $\mathrm{Cl}\,\mathcal{U}_F(X-U) \cap F = \emptyset$. Let $\mathcal{F}$ be a closed cover of X. Then $\{\mathcal{F},\ \mathcal{U}_F (F \in \mathcal{F})\}$ is said to be a <u>weak</u> L-<u>structure</u> of X if

a) $\mathcal{F}$ is σ-discrete,

b) for each $x \in X$ and each open neighborhood U of x there exist finite elements $F_1,\dots,F_n$ of $\mathcal{F}$ and their corresponding semi-canonical neighborhoods $U_1,\dots,U_n$ with respect to $\mathcal{U}_{F_1}$, ..., $\mathcal{U}_{F_n}$ respectively, such that

$$x \in \bigcap_{i=1}^{n} F_i \subset \bigcap_{i=1}^{n} U_i \subset U.$$

Let (WL) denote the class of paracompact Hausdorff spaces with weak L-structures. Then (WL) is hereditary and countably productive.

THEOREM. _If_ $X \in$ (WL), _then the following conditions are equivalent._

i) $\dim X \leq n$.

ii) X _is the image of a free_ L-_space_ Z _with_ $\dim Z \leq 0$ _under a perfect map_ f _with_ order $f \leq n + 1$.

iii) $\operatorname{Ind} X \leq n$.

iv) X _is the sum of subsets_ $X_1, \ldots, X_{n+1}$ _with_ $\dim X_i \leq 0$ _for_ $i=1,\ldots,n+1$.

v) X _has a_ σ-_closure-preserving base_ $\mathcal{U}$ _such that_ $\dim \partial U \leq n-1$ _for each_ $U \in \mathcal{U}$.

vi) X _has a stratification_ $\{U_i\}$ _such that_ $\dim \partial U_i \leq n-1$ _for each open set_ U _and each_ i.

The implication iii)$\rightarrow$v) is verified by a similar way as in [2, Theorem 2.7]. The implication v)$\rightarrow$iii) needs the following which is just a trivial modification of [1, Lemma 15.5].

LEMMA. _Let_ X _be a normal semi-stratifiable space and_ $\{F_\alpha : \alpha \in A\}$ _an order-closure-preserving closed cover of_ X. _Then_ $\operatorname{Ind} X = \sup \operatorname{Ind} F_\alpha$.

Proof. Assume that $\dim F_\alpha \leq n$ for each $\alpha \in A$. Set

$$\mathcal{H}_i = \{ H_{\alpha i} = F_\alpha \cap (X - \bigcup_{\beta<\alpha} F_\beta)_i : \alpha \in A \},\ i \in N,$$

$$H_i = \mathcal{H}_i^{\#}.$$

Then $\cup \mathcal{H}_i$ covers X. To see that $\mathcal{H}_i$ is discrete let x be an arbitrary point of X and α the minimal with $x \in F_\alpha$. Set

$$V = (X - (X - \bigcup_{\beta \leq \alpha} F_\beta)_i) - \bigcup_{\beta < \alpha} F_\beta .$$

Then V is an open neighborhood of x with $V \cap H_{\beta i} = \emptyset$ for any $\beta \neq \alpha$. Thus Ind $H_i \leq n$ which implies dim $X \leq n$ by the sum theorem.

T. Mizokami wrote to me that the equivalence of iii) and v) holds for some kind of spaces which seem to be free L-spaces. As is expected from the condition ii) we can prove that each member of (WL) is the perfect image of a free L-space. If each perfect image of a free L-space has to be a free L-space, we can conclude that (WL) coincides with the class of free L-spaces. I don't know whether this is the case or not. I don't know whether the theorem is valid for M_1 or M_3 spaces.

References

[1] K. Nagami, Dimension theory, 1970, Academic Press, New York.

[2] ———, Dimension of free L-spaces, forthcoming in Fund. Math..