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A SEMIGROUP OF ISOMORPHISM CLASSES OF SOME QUADRATIC EXTENSIONS OF RINGS

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Throughout this paper, B will mean a (non-commutative) ring with identity element 1 which has an automorphim ρ. By $B[X;\rho]$, we denote the ring of all polynomials $\sum_i X^i b_i$ ($b_i \in B$) with an indeterminate X whose multiplication is given by $bX = X\rho(b)$. Moreover, by $B[X;\rho]_2$, we denote the subset of $B[X;\rho]$ of all polynomials $f = X^2 - Xa - b$ with $fB[X;\rho] = B[X;\rho]f$. If $X^2 - Xa - b \in B[X;\rho]_2$ then $\rho(b) = b$. By $B[X;\rho]_{(2)}$, we denote the subset $\{X^2 - Xa - b \in B[X;\rho]_2 \mid \rho(a) = a\}$. Now, for $f, g \in B[X;\rho]_2$, if the factor rings $B[X;\rho]/fB[X;\rho]$ and $B[X;\rho]/gB[X;\rho]$ are B-ring isomorphic then we write $f \sim g$. Clearly the relation $\sim$ is an equivalence relation in $B[X;\rho]_2$. By $B[X;\rho]_2^{\sim}$ (resp. $B[X;\rho]_{(2)}^{\sim}$), we denote the set of equivalence classes of $B[X;\rho]_2$ (resp. $B[X;\rho]_{(2)}$) with respect to the relation $\sim$. Moreover, for $f \in B[X;\rho]_2$, if the factor ring $B[X;\rho]/fB[X;\rho]$ is separable (resp. Galois) over B then f will be called to be separable (resp. Galois). As is well known, any Galois polynomial in $B[X;\rho]_2$ is separable. By [6, Th.1], any separable polynomial of $B[X;\rho]_2$ is contained in $B[X;\rho]_{(2)}$. For $f = X^2 - Xa - b \in B[X;\rho]_2$, we denote $a^2 + 4b$ by $\delta(f)$, which will be called the discriminant of f. We shall use here the convention: $B(\rho^n) = \{u \in B \mid \alpha u = u\rho^n(\alpha)$ for all $\alpha \in B\}$ (where n is any integer). If $X^2 - Xa - b \in B[X;\rho]_2$ then $a \in B(\rho)$, $b \in B(\rho^2)$,

$\rho(b) = b$ (, and conversely). Clearly $a^2 + 4b \in B(\rho^2)$. An element a of $B(\rho^n)$ is said to be π-regular if there exists an element c in B and an integer $t \geq 0$ such that $a^t = a^{t+1}c$.

Now, in [1], K. Kitamura studied free quadratic extensions of commutative rings and its isomorphism classes. In his study, the set of polynomials of degree 2 plays an important rôle. Indeed, [1] is a study on $B[X;\rho]_2$ and $B[X;\rho]\tilde{}_2$ where B is commutative and $\rho = 1$.

In [2], K. Kishimoto studied the sets $B[X;\rho]_{(2)}$ and $B[X;\rho]\tilde{}_{(2)}$ in case $B[X;\rho]_{(2)}$ contains a Galois polynomial $X^2 - b$ (and hence $2b$ is inversible in B).

In [5], the present author studied the sets $B[X;\rho]_{(2)}$ and $B[X;\rho]\tilde{}_{(2)}$ in case $B[X;\rho]_{(2)}$ contains a Galois polynomial $X^2 - Xa - b$ (and hence the discriminant $a^2 + 4b$ is inversible in B). The study contains a generalization of [2]. Moreover, in [1], [2] and [5], it was shown that $B[X;\rho]\tilde{}_{(2)}$ forms an abelian semigroup with identity element under some composition, and the structure of this semigroup was studied to characterize the separable polynomials in $B[X;\rho]_{(2)}$.

In this paper, we shall study the separable polynomials in $B[X;\rho]_{(2)}$ and the structure of $B[X;\rho]\tilde{}_{(2)}$ in case $B[X;\rho]_{(2)}$ contains a separable polynomial whose discriminant is π-regular, and we shall show that $B[X;\rho]\tilde{}_{(2)}$ forms also an abelian semigroup with identity element under some composition such that for $C \in B[X;\rho]\tilde{}_{(2)}$ and $f \in C$, C is inversible in this semigroup if and only if f is separable. Moreover, this semigroup will be studied in various ways.

In the rest of this paper, Z will mean the center of B. Moreover, $U(B)$ denotes the set of inversible elements in B, and for any subset S of B, $U(S)$ denotes the intersection of S and $U(B)$. Clearly, $U(Z)$ coincides with the set of inversible elements in Z. Further, for any subset S of B, we use the following conventions: $S^{\rho} = \{s \in S \mid \rho(s) = s\}$; $\rho^n|S$ = the restriction of ρ^n to S (where n is any integer). By [5, (2, xvii)] and [6, Th. 1], we see that if $B[X;\rho]_2$ contains a separable polynomial then $\rho^2|Z$ is identity. As is easily seen, if an element a of $B(\rho^n)^{\rho}$ is π-regular then there exists an integer $n \geq 0$ and an idempotent ε of Z^{ρ} such that $a^nB = \varepsilon B$. This idempotent will be denoted by $e(a)$.

First, we shall prove the following

Lemma 1. Let 2 be nilpotent, and assume that $B[X;\rho]_2$ contains a separable polynomial $X^2 - b$. Then, $b \in U(B)$, and there exists an element $z \in Z$ such that $z + \rho(z) = 1$. Moreover, $B(\rho) = \{0\}$, $B(\rho^2) = bZ$, $B(\rho^2)^{\rho} = bZ^{\rho}$, and $B[X;\rho]_2 = \{X^2 - v \mid v \in B(\rho^2)^{\rho}\}$.

Proof. The first assertion is a direct consequence of [5, Lemma 2.3] and [6, Th.1]. Now, since 2 is nilpotent, there exists an integer $n > 0$ such that $2^n = 0$. Let $u \in B(\rho)$. Then, we have $u = u(z + \rho(z))^n = u(z + \rho(z))(z + \rho(z))^{n-1} = 2zu(z + \rho(z))^{n-1} = 2^n z^n u = 0$. The rest assertion will be easily seen.

Next, we shall prove the following

Lemma 2. Let ε be an idempotent in Z^{ρ} such that $\varepsilon 2^n = 2^n$ for some integer $n > 0$. Let f be an polynomial in $B[X;\rho]_2$ such that εf is Galois in $\varepsilon B[X;\rho]$ and $(1-\varepsilon)f$ is separable in $(1-\varepsilon)B[X;\rho]$. Then $\delta(f)$ is π-regular, $e(\delta(f))B \supset \varepsilon B$, and $(1-e(\delta(f)))B[X;\rho]_2 = \{(1-e(\delta(f)))(X^2 - v) \mid v \in B(\rho^2)^{\rho}\}$.

Proof. By [6, Th.2], we have $\varepsilon B = \varepsilon\delta(f)B$. Moreover, f is separable, and so, $f \in B[X;\rho]_{(2)}$. We write here $f = X^2 - Xa - b$. Then, by [5, Lemma 2.2 (2, xix)], we have $a = \delta(f)sa = \delta(f)^{n+1}s^{n+1}a$ for some s in B. Since $\varepsilon 4^n = 4^n$, it follows that $(1-\varepsilon)\delta(f)^nB = (1-\varepsilon)(ac + 4^nb^n)B = (1-\varepsilon)acB = (1-\varepsilon)\delta(f)^{n+1}B$, and whence, $\delta(f)^nB = \varepsilon\delta(f)^nB + (1-\varepsilon)\delta(f)^nB = \varepsilon\delta(f)^{n+1}B + (1-\varepsilon)\delta(f)^{n+1}B = \delta(f)^{n+1}B$. Thus $\delta(f)$ is π-regular, and $e(\delta(f))B = \delta(f)^nB \supset \varepsilon\delta(f)^nB = \varepsilon\delta(f)B = \varepsilon B$. Moreover, noting $e(\delta(f))a = a$, the other assertion will be easily seen from the result of Lemma 1.

Corollary 3. Let 2 be π-regular. If $f \in B[X;\rho]_2$ is separable then $\delta(f)$ is π-regular.

Proof. Let $f = X^2 - Xa - b$ be a separable polynomial in $B[X;\rho]_2$. Since any inversible element of B is π-regular in B, we may assume that $\delta(f)$ is not inversible in B. If $e(2) = 1$ then 2 is inversible in B, and so, $\delta(f)$ is inversible in B by [6, Th.3]. Hence $e(2) \neq 1$. First, we assume that $e(2) = 0$. Then $2^n = 0$ for some integer $n > 0$. By [5, Lemma 2.2 (2, xix)], we have $a = \delta(f)^nta = a^2r$ for some $t, r \in B$. Hence a is π-regular, and $e(a)$ is in Z^{ρ}.

Since $e(a)a$ is inversible in $e(a)B$, so is $e(a)\delta(f)$ in $e(a)B$. Hence, it follows from [6, Th.2] that $e(a)f$ is Galois in $e(a)B[X;\rho]$. Moreover, $1 - e(a) \neq 0$, and $(1-e(a))f$ is separable in $(1-e(a))B[X;\rho]$. Therefore, $\delta(f)$ is π-regular by Lemma 2. Next, we assume that $e(2) \neq 0$. Then $e(2) \in Z^{\rho}$, $e(2)B = 2^nB$, and $e(2)2^n = 2^n$ for some integer $n > 0$. Noting that $e(2)2$ is inversible in $e(2)B$, $e(2)f$ is Galois in $e(2)B[X;\rho]$ by [6, Th.2]. Moreover, $(1-e(2))f$ is separable in $(1-e(2))B[X;\rho]$. Hence by Lemma 2, $\delta(f)$ is π-regular.

Now, we shall prove the following theorem which is one of our main results.

Theorem 4. Assume that $B[X;\rho]_2$ contains a separable polynomial f whose discriminant is π-regular. Set $\varepsilon = e(\delta(f))$ and $\omega = 1 - \varepsilon$. Then, $\omega 2$ is nilpotent, and $\omega B[X;\rho]_2 = \{\omega(X^2 - v \mid v \in B(\rho^2)^{\rho}\}$. Moreover, $g = X^2 - Xu - v \in B[X;\rho]_2$, the following conditions are equivalent.

(a) g is separable.

(b) $\delta(g)$ is π-regular, $e(\delta(g)) = \varepsilon$, and $\omega B = \omega vB$.

(c) $\varepsilon B = \varepsilon\delta(g)B$, and $\omega B = \omega vB$.

Proof. Let $f = X^2 - Xa - b$. If $\varepsilon = 1$ then $\delta(f)$ is inversible in B, and whence, the assertion holds obviously. Now, we assume that $\varepsilon = 0$. Then, by [5, Lemma 2.2 (2, xix)], 2 is nilpotent and $a = 0$. Hence by Lemma 1, we ahve $B[X;\rho]_2 = \{X^2 - v \mid v \in B(\rho^2)^{\rho}\}$. Hence by [5, Lemma 2.3], it will be easily seen that (a), (b) and (c) are equivalent.

Next, we shall consider the case $\varepsilon \neq 1, 0$. Since $\varepsilon B = \delta(f)^n B$ for some integer $n > 0$, it follows that $\rho(\varepsilon) = \varepsilon$, and $4^n = \delta(f)^n r = \varepsilon\delta(f)^n r = \varepsilon 4^n$ for some r in B ([5, Lemma 2.2]). Moreover, since $\varepsilon\delta(f)$ is inversible in εB, εf is Galois in $\varepsilon B[X;\rho]$. Obviously, ωf is separable in $\omega B[X;\rho]$. Hence by Lemma 2, we have $\omega B[X;\rho]_2 = \{\omega(X^2 - v) \mid v \in B(\rho^2)^{\rho}\}$. Now, let $g = X^2 - Xu - v \in B[X;\rho]_2$. Assume (a). Then, since εg is separable in $\varepsilon B[X;\rho]$, it follows from [6, Th.2] that εg is Galois in $\varepsilon B[X;\rho]$. Moreover, ωg is separable in $\omega B[X;\rho]$. Hence by Lemma 2, $\delta(g)$ is π-regular, and $e(\delta(g))B \supset \varepsilon B = e(\delta(f))B$. By a similar way, we have $e(\delta(g))B \subset e(\delta(f))B$. This implies $e(\delta(g)) = \varepsilon$. Since $\omega g = \omega(X^2 - v)$ is separable in $\omega B[X;\rho]$, ωv is inversible in ωB by [5, Lemma 2.3], that is, $\omega B = \omega v B$. Thus we obtain (b). Assume (b). Then $\varepsilon B = e(\delta(g))B = \delta(g)^m B$ for some integer $m > 0$. This shows that $\varepsilon B = \varepsilon\delta(g)B$. Finally, assume (c). Since $\varepsilon B = \varepsilon\delta(g)B$, $\varepsilon\delta(g)$ is inversible in εB. Hence εg is Galois in $\varepsilon B[X;\rho]$ by [6, Th.2], and so, εg is separable in $\varepsilon B[X;\rho]$. Moreover, ωv is inversible in ωB. Since ωf is separable in $\omega B[X;\rho]$, there exists an element z in ωZ with $z + \rho(z) = \omega$. Hence $\omega g = \omega(X^2 - v)$ is separable in $\omega B[X;\rho]$ by [5, Lemma 2.3]. Therefore $g = \varepsilon g + \omega g$ is separable, completing the proof.

In the rest of this note, we shall deal with the set $B[X;\rho]^{\sim}_{(2)}$ (of B-ring isomorphism classes of the ring extensions $B[X;\rho]/gB[X;\rho]$ ($g \in B[X;\rho]_{(2)}$) of B).

Now, if $C \in B[X;\rho]^{\sim}_{(2)}$ and $g \in C$ then we write $C = \langle g \rangle$.

Moreover, for $g = X^2 - Xu - v$, $g_1 = X^2 - Xu_1 - v_1$ and $s \in B$, we write

$$g \times s = X^2 - Xus - vs^2$$
$$g \times g_1 = X^2 - Xuu_1 - (u^2 v_1 + vu_1^2 + 4vv_1)$$
$$g * s = X^2 - vs^2$$
$$g * g_1 = X^2 - vv_1.$$

If $B[X;\rho]_2$ contains a separable polynomial then $\rho^2|Z = 1$, and in this case, for any element α (resp. any subset S) of Z, we denote $\alpha\rho(\alpha)$ (resp. $\{\alpha\rho(\alpha) \mid \alpha \in S\}$) by $N_\rho(\alpha)$ (resp. $N_\rho(S)$).

Now, by virtue of Lemma 1, [5, Lemma 2. 10] and [3, Lemma 1. 8], we obtain the following

Lemma 5. Let 2 be nilpotent, and assume that $B[X;\rho]_2$ contains a separable polynomial $f = X^2 - b$. Let $g_1 = X^2 - v_1$ and $g_2 = X^2 - v_2 \in B[X;\rho]_{(2)}$ $(= \{X^2 - v \mid v \in B(\rho^2)^\rho\})$. Then, $g_1 \sim g_2$ if and only if $v_1 = v_2 N_\rho(\alpha)$ for some $\alpha \in U(Z)$.

From the preceding lemma and [5, Lemma 2.3], we obtain

Corollary 6. Let 2 be nilpotent, and assume that $B[X;\rho]_2$ contains a separable polynomial $f = X^2 - b$. Let $g_1 \sim g_2$ in $B[X;\rho]_{(2)}$, and $h_1 \sim h_2$ in $Z[X;\rho|Z]_{(2)}$. Then for any $g \in B[X;\rho]_{(2)}$ and $h \in Z[X;\rho|Z]_{(2)}$, there holds the following

(i) $g_1 * g * b^{-1} \sim g_2 * g * b^{-1}$ in $Z[X;\rho|Z]_{(2)}$.

(ii) $h_1 * h \sim h_2 * h$ in $Z[X;\rho|Z]_{(2)}$.

(iii) $h_1 * g \sim h_2 * g$ in $B[X;\rho]_{(2)}$.

(iv) $g_1 * g * f * b^{-1} \sim g_2 * g * f * b^{-1}$ in $B[X;\rho]_{(2)}$.

(v) $g * f * f * b^{-1} = g$, and $h * f * f * b^{-1} = h$.

(vi) g is separable in $B[X;\rho]_{(2)}$ if and only if $g * g * f * b^{-1} \sim f$ which is equivalent to that $g * g' * f * b^{-1} \sim f$ for some $g' \in B[X;\rho]_{(2)}$.

(vii) h is separable in $Z[X;\rho|Z]_{(2)}$ if and only if $h * h \sim f * f * b^{-1}$ which is equivalent to that $h * h' \sim f * f * b^{-1}$ for some $h' \in Z[X;\rho|Z]_{(2)}$.

By making use of Cor. 6, we can prove the next

Lemma 7. Let 2 be nilpotent, and assume that $B[X;\rho]_{(2)}$ contains a separable polynomial $f = X^2 - b$. Then, the set $B[X;\rho]\tilde{}_{(2)}$ (resp. $Z[X;\rho|Z]\tilde{}_{(2)}$) forms an abelian semigroup under the composition $\langle g_1\rangle\langle g_2\rangle = \langle g_1 * g_2 * f * b^{-1}\rangle$ (resp. $\langle h_1\rangle\langle h_2\rangle = \langle h_1 * h_2\rangle$) with identity element $\langle f\rangle$ (resp. $\langle f * f * b^{-1}\rangle$), and the subset $\{\langle g\rangle \in B[X;\rho]\tilde{}_{(2)} \mid g$ is separable$\}$ (resp. $\{\langle h\rangle \in Z[X;\rho|Z]\tilde{}_{(2)} \mid h$ is separable$\}$) coincides with the set of all inversible elements in the semigroup $B[X;\rho]\tilde{}_{(2)}$ (resp. $Z[X;\rho|Z]\tilde{}_{(2)}$) which is a group of exponent 2. Moreover, $B[X;\rho]\tilde{}_{(2)} \simeq Z[X;\rho|Z]\tilde{}_{(2)}$, which is isomorphic to the multiplicative semigroup $Z^\rho/N_\rho(U(Z))$.

Now, let ε be an idempotent in Z^ρ. Then $\varepsilon B = (\varepsilon B)^\rho$, $\varepsilon B(\rho) = (\varepsilon B)(\rho|\varepsilon B)$, and $\varepsilon B(\rho)^\rho = (\varepsilon B)(\rho|\varepsilon B)^\rho$. Hence we have a bijective map: $\varepsilon B[X;\rho]_{(2)} \to (\varepsilon B)[X;\rho|\varepsilon B]_{(2)}$ given by

$\varepsilon(X^2 - Xu - v) \to X^2 - X\varepsilon u - \varepsilon v$. Hence we shall identify $\varepsilon B[X;\rho]_{(2)}$ with $(\varepsilon B)[X;\rho|\varepsilon B]_{(2)}$, and by $\varepsilon B[X;\rho]^{\sim}_{(2)}$, we denote $(\varepsilon B)[X;\rho|\varepsilon B]^{\sim}_{(2)}$. We set here $\omega = 1 - \varepsilon$. Then, as is easily seen, the map:

$$B[X;\rho]_{(2)} \to \varepsilon B[X;\rho]_{(2)} \times \omega B[X;\rho]_{(2)} \text{ (direct product)}$$

given by $g \to (\varepsilon g, \omega g)$ is bijective. This induces a bijective map:

$$B[X;\rho]^{\sim}_{(2)} \to \varepsilon B[X;\rho]^{\sim}_{(2)} \times \omega B[X;\rho]^{\sim}_{(2)}$$

where $\langle g\rangle \to (\langle \varepsilon g\rangle, \langle \omega g\rangle)$. Clearly, g is separable in $B[X;\rho]$ if and only if εg and ωg are separable in $\varepsilon B[X;\rho]$ and $\omega B[X;\rho]$ respectively. If $B[X;\rho]_2$ contains a separable polynomial $f = X^2 - Xa - b$ whose discriminant is π-regular and $\varepsilon = e(\delta(f))$ $(\omega = 1 - \varepsilon)$ then $\varepsilon B[X;\rho]_2$ contains a Galois polynomial εf, $\omega 2$ is nilpotent and $\omega B[X;\rho]_2$ contains a separable polynomial $\omega f = \omega(X^2 - b)$ (Th.4, [5, Lemma 2.2], [6, Th.2]).

Now, our main results are the following theorems which can be proved by making use of the preceding remarks, Lemma 7, [5, Th.2.17], Cor. 3, [5, Lemma 2.10], [3, Lemma 1.8], [6, Th.2], [4, Th.1.2], and etc.

Theorem 8. Assume that $B[X;\rho]_2$ contains a separable polynomial $f = X^2 - Xa - b$ whose discriminant is π-regular. Set $\varepsilon = e(\delta(f))$ and $\omega = 1 - \varepsilon$. Then the set $B[X;\rho]^{\sim}_{(2)}$ (resp. $Z[X;\rho|Z]^{\sim}_{(2)}$) forms an abelian semigroup under the composition

$$\langle g_1\rangle\langle g_2\rangle = \langle \varepsilon g_1 \times \varepsilon g_2 \times \varepsilon f \times (\varepsilon\delta(f))^{-1} + \omega g_1 * \omega g_2 * \omega f * (\omega b)^{-1}\rangle$$

$$(\text{resp. } \langle h_1\rangle\langle h_2\rangle = \langle \varepsilon h_1 \times \varepsilon h_2 + \omega h_1 * \omega h_2\rangle\ )$$

with identity element

$$\langle f\rangle \quad (\text{resp. } \langle \varepsilon f \times \varepsilon f \times (\varepsilon\delta(f))^{-1} + \omega f * \omega f * (\omega b)^{-1}\rangle\ )$$

, and the subset

$$\{ \langle g\rangle \in B[X;\rho]^{\sim}_{(2)} \mid g \text{ is separable } \}$$

$$(\text{resp. } \{ \langle h\rangle \in Z[X;\rho|Z]^{\sim}_{(2)} \mid h \text{ is separable}\}\ )$$

coincides with the set of all inversible elements of $B[X;\rho]^{\sim}_{(2)}$ (resp. $Z[X;\rho|Z]^{\sim}_{(2)}$) which is a group of exponent 2. Moreover

$$B[X;\rho]^{\sim}_{(2)} \simeq Z[X;\rho|Z]^{\sim}_{(2)} \simeq \varepsilon Z[X;\rho|\varepsilon Z]^{\sim}_{(2)} \times \omega Z[X;\rho|\omega Z]^{\sim}_{(2)}$$

$$\simeq \varepsilon Z[X;\rho|\varepsilon Z]^{\sim}_{(2)} \times \omega Z^{\rho}/N_{\rho}(U(\omega Z)).$$

Theorem 9. Let 2 be π-regular and assume that $B[X;\rho]_2$ contains a separable polynomial f. Then, there exists an idempotent ε ($\omega = 1 - \varepsilon$) of Z^{ρ} such that

$$B[X;\rho]^{\sim}_{(2)} \simeq \varepsilon Z[X]^{\sim}_{2} \times \omega Z^{\rho}/N_{\rho}(U(\omega Z))$$

where if $e(2) = e(\delta(f))$ then $\varepsilon = 0$.

Corollary 10. Let $2 = 0$ and assume that $B[X;\rho]_{(2)}$ contains a separable polynomial. Let $U(B[X;\rho]^{\sim}_{(2)})$ be a group of inversible elements of $B[X;\rho]^{\sim}_{(2)}$. Then, there exists an idempotent ε ($\omega = 1 - \varepsilon$) of Z^{ρ} such that

$$U(B[X;\rho]^{\sim}_{(2)}) \simeq \varepsilon Z/\varepsilon\{z^2 - z \mid z \in Z\} \times U(\omega Z)^{\rho}/N_{\rho}(U(\omega Z))$$

where ωZ is an additive subgroup of Z, and if $B[X;\rho]_2$ contains a Galois polynomial then $\omega = 0$.

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