数理解析研究所講究録404

実験配置の理論と応用

京都大学数理解析研究所

1980年11月

RIMS *Kokyuroku* 404

Theory of Experimental Layouts and Its Applications

November, 1980

Research Institute for Mathematical Sciences

Kyoto University, Kyoto, Japan

実験配置の理論と応用

研究会報告集

1980年7月14日～7月16日

研究代表者　景山三平

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SYMPOSIUM ON THEORY OF EXPERIMENTAL LAYOUTS AND ITS APPLICATIONS

Place : Research Institute for Mathematical Sciences, Kyoto University, Kyoto, Japan

Date : July 14 - 16, 1980

Organizer : Sanpei Kageyama, Department of Mathematics, Faculty of School Education, Hiroshima University

PROGRAM AND ABSTRACT

1. S. Kageyama
(Hiroshima University)
On 5-designs

Abstract: We show that an inequality $b \geqq v(v-1)$ holds for a 5-(v,k,λ_5) design. Furthermore, a Steiner system $S(5,6,12)$ is shown to be the unique 5-design with $b = v(v-1)$, up to complementation.

2. M. Yoshizawa
(Keio University)
Block intersection numbers of block designs

Abstract: The following results are given:

Theorem 1. For each $n \geqq 1$ and $\lambda \geqq 1$,

(a) there exist at most finitely many block-schematic t-(v,k,λ) designs with $k-t=n$ and $t \geqq 3$, and

(b) if also $\lambda \geqq 2$, there exist at most finitely many block-schematic t-(v,k,λ) designs with $k-t=n$ and $t \geqq 2$.

Theorem 2. A Steiner system $S(t,t+1,v)$ is block-schematic if and only if one of the following holds: (i) $t=2$, (ii) $t=3$, $v=8$, (iii) $t=4$, $v=11$, (iv) $t=5$, $v=12$.

3. N. Ito and H. Kimura
(Univ. of Illinois and Hokkaido Univ.)

Hadamard matrices with 2-transitive automorphism groups

Abstract: We consider Hadamard matrices with 2-transitive automorphism groups not containing regular normal subgroups. Under some assumptions we have some results. For example (1) the degrees of Hadamard matrices are square; (2) automorphism groups are non-solvable and; (3) they are not 3-transitive.

4. K. Takeuchi
(University of Tokyo)

Randomization design Revisited

Abstract: Randomization Design, in which factor levels are randomized, was discussed by several authors including Taguchi, Satterthwaite, some twenty years ago, but has been since nearly forgotten. In 1958 Kiefer proved that randomly balanced extremely unbalanced designs are optimum in terms of the local power of the test of null hypothesis, which fact has never been further analyzed. The author once wrote a series of papers on this topic, and now wants to revitalize interests in this problem, and discusses its basic features taking the simplest case of comparing means of several normal populations for illustration.

5. K. Suda

(National Gunma Technical College)

An automatical design and analysis system for orthogonal experiments

Abstract: The use of fractional factorial designs has now become widely accepted as an efficient way to carry out experiments in Quality Control. However, one of the main difficulties with the fractional factorial designs involving many different factors is how to construct an orthogonal design which can estimate various effects without being confounded. In such situations, we developed the program that can automatically construct an orthogonal design and compute the estimates of the main effects and interactions for any given model for micro-computer. We show this automatical design and analysis system has wide applications for improving product quality.

6. T. Shirakura

(Kobe University)

Norm of alias matrices for $(\ell+1)$-factor interactions in balanced fractional 2^m factorial designs of resolution $2\ell+1$

Abstract: Consider a balanced fractional 2^m factorial design of resolution $2\ell+1$ derived from a balanced array of strength $2\ell+1$. Consider the norm $||A|| = \{tr(A'A)\}^{1/2}$ of alias matrix A for $(\ell+1)$-factor interactions in this design. This norm can be used as a measure for selecting a design. In this paper, an explicit expression for $||A||$ is given by using algebraic structures of a balanced design. By

this expression, designs of resolution $V(\ell=2)$ which minimize $||A||$ are presented for any fixed assemblies satisfying (i) $m = 5$, $16 \leqq N \leqq 32$, (ii) $m = 6$, $22 \leqq N \leqq 32$, and (iii) $m = 7$, $29 \leqq N \leqq 50$.

7. R. Nishii

(Hiroshima University)

On fractional factorial designs with orthogonal structure

Abstract: Fold-over designs have been discussed as they give the orthogonality between estimate of odd parameter and one of even parameter. Here level-symmetric designs are defined, which are given by generalization of the concept of fold-over designs. Those designs are proved to have the structure that any odd parameter and any even parameter can be estimated uncorrelatedly, and it is proved that designs with this structure must be level-symmetric designs.

8. M. Kuwada

(Maritime Safety Academy)

On an alias relation to 3-factor interactions in balanced fractional 3^m factorial designs derivable from balanced arrays of strength 5

Abstract: Consider a balanced fractional 3^m factorial design T derivable from a balanced array of strength 5. By use of the multidimensional relationship and its algebra, we will present an explicit expression for the norm of A_T, i.e., $||A_T|| = \{tr(A_T'A_T)\}^{1/2}$, where A_T is the alias matrix.

9. Y. Ohashi

(University of Tokyo)

An application of "sub-sampling" to the robust estimation of σ (σ^2)

Abstract: In this paper, a procedure utilizing BIBD is proposed for the robust estimation of $\sigma(\sigma^2)$ from a normal sample possibly with a few outliers. An original sample is divided into m subsamples; m sample variances are calculated and from them a robust estimate $\tilde{\sigma}_1^2$ (here, a Winsorized mean) is obtained. Although $\tilde{\sigma}_1^2$ is robust, its efficiency is considerably low as compared with the usual estimator, that is, the sample variance, so "sub-sampling" is repeated r times, giving $\tilde{\sigma}_2^2, \cdots, \tilde{\sigma}_r^2$ and the final estimate $\tilde{\sigma}^2 := (\sum_i \tilde{\sigma}_i^2)/r$. It is shown that much higher efficiency is achieved by the systematic repetition scheme utilizing BIBD than by the random repetition or other non-systematic schemes. Means and MSE's of $\tilde{\sigma}^2$ and $\tilde{\sigma} = \text{const} \times \sqrt{\tilde{\sigma}^2}$ are numerically compared with those of familiar alternatives such as linear estimators under the "slippage" model. The above procedure is applicable to the estimation of error variance in a linear model, and an application to "jack-knife" is suggested.

10. I. Takahashi

(University of Tsukuba)

The least Hamming distance method applied to a file construction

Abstract: R.C.Bose and others introduced balanced files based on k-dimensional subspaces S with strength t in $GF(q)^m$. We propose new filing scheme based on a subspace S fitted on

the set R of given records. The criterion of the fitness is the least Hamming distance and fitting algorith is essentially the same as decoding method of Reed Muller codes.

11. M. Yamada

(Tokyo Woman's Christian College)

On the Williamson matrices of Turyn's type

Abstract:

In 1972 Turyn found an infinite family of Williamson matrices. Namely if $q = 2n-1$ is a prime power $\equiv 1 \pmod 4$, then there exists a Williamson matrix of order $4n$. Whiteman gave a new proof by using the trace from $GF(q^2)$ to $GF(q)$ in 1973. In this paper we interpret this theorem in terms of the theory of the Gauss sum over a finite field.

We let: $E = GF(q^2)$, $F = GF(q)$, $q = p^t \equiv 1 \pmod 4$ and $n = (q+1)/2$,

S_E = trace from E, S_F = trace from F, $S_{E/F}$ = trace from E to F,

ξ : a generator of E^*, ζ_n : an n-th root of unity, $\zeta_p = e^{2\pi i/p}$.

χ_4 : the character of E such that $\chi_4(\xi) = i$,

χ_n : the character of E such that $\chi_n(\xi) = \zeta_n$ and $\chi_n^{n'} = 1$,

$\chi = \chi_4 \cdot \chi_n$ = the character of E which gives the Legendre symbol when restricted to F, $u_r = \zeta_n^r + \zeta_n^{-r}$ $(r = 1, \ldots, (n-1)/2)$,

$\tau_E(\chi) = \sum_{\alpha \in E} \chi(\alpha) \zeta_p^{S_E \alpha}$ = the Gauss sum over E,

$\tau_F(\chi) = \sum_{\alpha \in F} \chi(\alpha) \zeta_p^{S_F \alpha}$ = the Gauss sum over F.

Put $\theta_\chi = \tau_E(\chi)/\tau_F(\chi)$. Then we have

$$\begin{aligned}\theta_\chi &= \sum_{\alpha \bmod^* F^*} \chi(\alpha)\bar{\chi}(S_{E/F}\alpha) \\ &= \sum_{r=0}^{n-1} (-1)^r \left\{ \chi(S_{E/F}\xi^{2r}) + i^n \chi(S_{E/F}\xi^{2r+n}) \right\} \zeta_n^{2r} \\ &= (-1)^{(n+1)/2}(-1+i)\left\{ \frac{1+i}{2} + \sum_{r=1}^{n-1} \beta_r \zeta_n^{2r} \right\},\end{aligned}$$

where $\beta_r = \dfrac{(-1)^{(n+1)/2 + r}}{-1+i}\left\{\chi(S_{E/F}\xi^{2r}) + i^n\chi(S_{E/F}\xi^{2r+n})\right\}$ $(r = 1,\dots,n-1)$.
We know that β_r is $+1$, -1, $+i$, or $-i$, and $\beta_{n-r} = \beta_r$. Further if we put $K_\chi = \dfrac{\theta_\chi}{(-1)^{(n+1)/2}(-1+i)} = \dfrac{1+i}{2} + \sum_{r=1}^{n-1} \beta_r \zeta_n^{2r}$ then $2K_\chi 2\bar{K}_\chi = 2q$.
Therefore let A_+, A_-, B_+, B_- be a partition of $\Omega = \{1,\dots,(n-1)/2\}$ for $\beta_r = 1, -1, i, -i$ respectively, then the Williamson equation

$$\begin{aligned} 4n &= 2 + 2K_\chi 2\bar{K}_\chi \\ &= 1^2 + 1^2 + \Big(1+2\sum_{r\in A_+} u_r - 2\sum_{r\in A_-} u_r\Big)^2 + \Big(1+2\sum_{r\in B_+} u_r - 2\sum_{r\in B_-} u_r\Big)^2, \end{aligned}$$

is true for every n-th root of unity.

We have several results on θ_χ. By the Davenport-Hasse theorem we have

$$\theta_\chi^2 = J(\chi, \chi^q),$$

where $J(\chi,\chi^q)$ is the Jacobi sum. And by Stickelberger's theorem we have the factorization

$$\theta_\chi \sim \mathfrak{P}^{\theta}, \quad \theta = \frac{t}{f} \sum_{c\in Z^*(4n'),\ B_1(\langle -\frac{c}{4} - \frac{c}{n'}\rangle) > 0} \sigma_c^{-1},$$

where $\mathfrak{P}$ is the prime ideal in the cyclotomic field $Q(\zeta_{4n'})$ such that $\mathfrak{P}\,|\,p$, f is the smallest positive integer which satisfies $p^f \equiv 1 \pmod{4n'}$, $Z^*(4n')$ is the multiplicative group of $\mathbb{Z}/4n'\mathbb{Z}$, σ_c an automorphism $\zeta_{4n'} \to \zeta_{4n'}^c$ of $Z^*(4n')$, and $B_1(x) = x - 1/2$ is the Bernoulli polynomial of degree 1.

It is not settled whether we can get a new family of Williamson matrices by developing our interpretation.

12. K. Sawada

(Nagoya Institute of Technology)

The Williamson matrices of a special form

Abstract: We consider Williamson equations of the following type:

$$1^2 + 1^2 + (1+2\sum_{\nu\in A_+} u_\nu - 2\sum_{\nu\in A_-} u_\nu)^2 + (1+2\sum_{\nu\in B_+} u_\nu - 2\sum_{\nu\in B_-} u_\nu)^2 = 4n,$$

where A_+, A_-, B_+, B_- is a partition of $\{1,2,\cdots,\frac{n-1}{2}\}$, corresponding to the decomposition $4n = 1^2+1^2+(1+4W_1)^2+(1+4W_2)^2$. An infinite class of the Williamson matrices found by Turyn belongs to this class. In this paper it is shown that $\#A_+$, $\#A_-$, $\#B_+$, $\#B_-$ are explicitly determined in terms of W_1 and W_2. We have found that there are no Williamson matrices of the above form, except for those due to Turyn, for $n \leqq 37$ and for $n = 61$.

13. J. Kinoshita

(Hokkaido University)

Networks and Matroids

Abstract: This paper explains a max-flow and min-cut theorem concerning networks with matroid restrictions in capacity.

14. K. Ushio

(Niihama Technical College)

On bipartite decomposition of a complete bipartite graph

Abstract: A complete bipartite graph K_{n_1,n_2} $(n_1 \leqq n_2)$ is said to have a bipartite decomposition if it can be decomposed into a union of line-disjoint subgraphs each

isomorphic to a complete bipartite graph K_{k_1,k_2} $(k_1 \leqq k_2)$. In this paper, a necessary condition for a complete bipartite graph K_{n_1,n_2} $(n_1 \leqq n_2)$ to have a bipartite decomposition is given. And several theorems which state that the necessary condition is also sufficient in many cases are given.

15. S. Tazawa

(Hiroshima College of Economics)

On claw-decomposition of a complete multi-partite graph $K_m(n_1, n_2, \cdots, n_m)$

Abstract:

Let $V_1, V_2, \cdots, V_m$ be point sets with $n_1, n_2, \cdots, n_m$ points each. A graph is said to be complete m-partite graph, denoted by $K_m(n_1, n_2, \cdots, n_m)$, if no line joins two points in the same point set and if each point in V_i is adjacent to all points of sets other than V_i. A complete bipartite graph $K_2(1, c)$ is, in particular, called a claw of degree c. In this paper two theorems are given, provided $n_1, n_2, \cdots, n_m$ are positive integers satisfying $\sum_{i=1}^{m-1} \sum_{j=i+1}^{m} n_i n_j / c$ = integer and $n_1 \leq n_2 \leq \cdots \leq n_m$: (a) The case $N-n_m < c$, where $N = \sum_{i=1}^{m} n_i$. $K_m(n_1, n_2, \cdots, n_m)$ is decomposed into a union of line-disjoint claws of degree c each if and only if $(N-n_m)\left\lceil \frac{n_m}{c} \right\rceil \leq \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} n_i n_j / c \leq \sum_{i=1}^{m-1} n_i \left\lfloor \frac{N-n_i}{c} \right\rfloor$. Here $\lfloor r \rfloor$ is the greatest integer not exceeding r and $\lceil r \rceil$ is the smallest integer not less than r. (b) The case $N-n_m \geq c$. If $K_m(n_1, n_2, \cdots, n_m)$ is decomposed into a union of line-disjoint claws of degree c each, then $\sum_{i=1}^{m-1} \sum_{j=i+1}^{m} n_i n_j \geq N-n_m$.