数理解析研究所講究録
第 416 巻 1981 年 51-59

アフィンWeyl群の商空間 (付録; E_6, E_7の不変式)

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§1. $\widetilde{W}_\ell$: an Affine Weyl 群

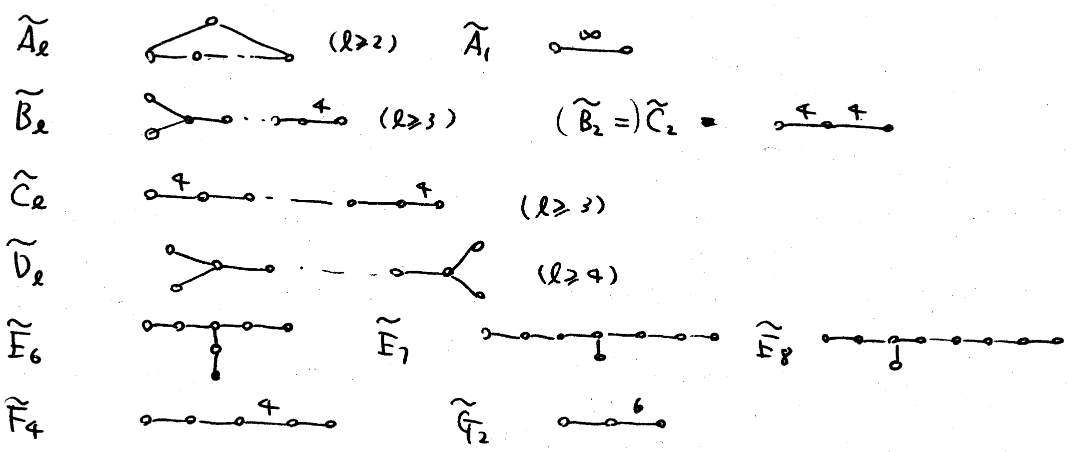
$V \simeq \mathbb{C}^\ell$: その表現空間, $\mathcal{H} = \bigcup_\alpha H_\alpha$: 鏡映面の和

$\Rightarrow$ $V/\widetilde{W}_\ell \simeq \mathbb{C}^\ell$, $\mathcal{H}/\widetilde{W}_\ell$: a hypersurface $(=: \Delta_{\widetilde{W}_\ell})$

(商空間) $\mathrm{mult}_0(\Delta_{\widetilde{W}_\ell}) = \ell+1$

$\pi_1((V-\mathcal{H})/\widetilde{W}_\ell)$ は $\widetilde{W}_\ell$ 型 Braid 群.

§2. 既約ルート系のAffine Weyl群は下記にかぎる. (Bourbaki etc)



いずれも頂点 $\ell+1$ 個のCoxeter graphにより表示してある. //

$V \simeq \sum_{i=1}^{\ell} \mathbb{C} e_i$, $V^* \simeq \sum_{i=1}^{\ell} \mathbb{C}\xi_i$ $\{\xi_i\} \leftrightarrow \{e_i\}$ dual basis

$e(\xi_j) = \exp(2\pi\sqrt{-1}\,\xi_j)$, $c(\xi_j) = \cos(2\pi\sqrt{-1}\,\xi_j)$, $s(\xi_j) = \sin(2\pi\sqrt{-1}\,\xi_j)$

とおく. e, c, s は V 上の函数である.

$\alpha = \sum a_i e_i$ のとき ~~exp~~ $e(\sum c_i \xi_j)(\alpha) = \exp(2\pi\sqrt{-1}\sum c_i a_i)$ である.

$V_{\mathbb{R}}^*$ の root 系 R に対し, $V_{\mathbb{R}}$ の超平面

$$L_{\alpha,k} = \{ x \in V \mid \langle x, \alpha \rangle = k \}$$

に関する直交鏡映 $s_{\alpha,k}$ により生成される V のアフィン変換の群が R の Affine Weyl 群 $W_a(R)$ であった. $W_a(R)$ は $W(R)$ (R の Weyl 群) の $Q(R^\vee)$ (R の双対ルート系の格子) による半直積である. R の weight の格子を $P(R)$, 基本 weight を $\varpi_1, \dots, \varpi_l$ とする. $p \in P(R)$ に対し, p を通る $W(R)$ の orbit を $W\cdot p$ とし, $W(R)$ の元による $e(p)$ の像のうち異なるものの上にわたる和を $S(e(p))$ とする.

$$S(e(p)) = \sum_{q \in W\cdot p} e(q).$$

このとき, V 上の entire fn で $W_a(R)$ 不変なものは,

$\{ S(e(\varpi_i)) \mid i = 1, \dots, l \}$ の entire fn でつくされる.

即ち, $V/W_a(R)$ の global な 座標環は $X_i = S(e(\varpi_i))$ とするとき, $\{X_1, \dots, X_l\}$ の entire fn である. $S(e(\varpi_i))$ を適当に修正することにより, 扱いやすい生成元をとれる.

§3 $W_a(R)$ 不変元の構成.

(3.1) A_l: $V_{\mathbb{R}}^* \subset \mathbb{R}^{l+1}$, 成分の和が 0. とすれば,

$$V_{\mathbb{R}}^* = \mathbb{R}^{l+1}/p_1\mathbb{R} \qquad p_1 = \xi_1 + \dots + \xi_{l+1}. \qquad R: \xi_i - \xi_j$$

$$\tilde{\varpi}_i = \xi_1 + \dots + \xi_i - \frac{i}{l+1} p_1 \equiv \xi_1 + \dots + \xi_i$$

x'_i を $\dfrac{e(\xi_1)}{e(\frac{\rho_1}{\ell+1})}, \cdots \dfrac{e(\xi_{\ell+1})}{e(\frac{\rho_1}{\ell+1})}$ の i次基本対称式とする.
(特に $x'_{\ell+1} = 1$)

$W_a(A_\ell)$ 不変式は $x_i = x'_i \bmod p_1$ $i=1,\dots,\ell$ が生成する.

$$\Delta_{\tilde{A}_\ell} = \Delta_{\ell+1}(1, x_1, x_2, \dots, x_\ell, 1) \qquad x_\ell^{\ell+1}+\cdots$$

(ここに $\Delta_k(a_0,\dots,a_k)$ は $a_0u^k + a_1u^{k-1} + \cdots + a_k$ の判別式).

$x_i + x_{\ell+1-i}$, $\frac{1}{\sqrt{-1}}(x_i - x_{\ell+1-i})$ は $V_{\mathbb{R}}$ 上 real valued.

ℓ: even $x_i + x_{\ell+1-i}$, $\frac{1}{\sqrt{-1}}(x_i - x_{\ell+1-i})$ $i=1,\dots,\frac{\ell}{2}$ ⎫ 実生成元
ℓ: odd 〃, 〃, $i=1,\dots,\frac{\ell-1}{2}$, $x_{\frac{\ell+1}{2}}$ ⎭

(3.2) B_ℓ: $V^k_{\mathbb{R}}$ の roots は $\pm\xi_i$, $\pm\xi_i \pm \xi_j$ $(i \neq j)$.

$$\tilde{\omega}_i = \xi_1 + \cdots + \xi_i \quad 1 \le i < \ell, \qquad \tilde{\omega}_\ell = \tfrac{1}{2}(\xi_1 + \cdots + \xi_\ell)$$

$W_a(B_\ell)$ 不変式は $\left\{ \begin{array}{l} \{c(\xi_j)\} \text{ の } i\text{次基本対称式 } 1 \le i < \ell \text{ と} \\ \prod_{i=1}^{\ell} c(\frac{\xi_i}{2}) \text{ となる.} \end{array} \right.$

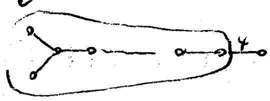
$\Leftrightarrow$ $\{c(\frac{\xi_j}{2})^2\}$ の i次 elem. sym. と $\prod_{i=1}^{\ell} c(\frac{\xi_i}{2})$.

or $\Leftrightarrow$ $\{\sum_{j=1}^{n} c(i\xi_j)\}$ $i=1,\dots,\ell-1$ と 〃.

$\left(\left(\begin{array}{l} x_i = (c(\frac{\xi_1}{2})^2, \dots, c(\frac{\xi_\ell}{2})^2) \text{ の } i\text{次 elem. sym} \\ y = \prod c(\frac{\xi_i}{2}) \end{array} \right)\right)$ とおくと

$$\Delta_{\tilde{B}_\ell} = (1 - x_1 + x_2 + \cdots + (-)^{\ell-1}x_{\ell-1} + (-)^\ell y^2)\,\Delta_\ell(1, x_1, \dots, x_{\ell-1}, y^2).$$

$= x_{\ell-1}^{\ell-1} + \cdots$



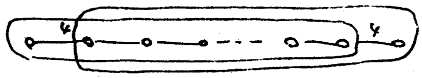
(3.3) $\tilde{C}_\ell$ $\quad V_{\mathbb{R}}^*$ の roots $\quad \pm 2\xi_i,\ \pm\xi_i \pm \xi_j\ (i \neq j)$

$$\tilde{\omega}_i = \xi_1 + \cdots + \xi_i \qquad 1 \leq i \leq \ell$$

$W_a(B_\ell)$ 不変式は $\quad x_1, \cdots, x_\ell$, $\quad x_i = (C(\xi_j)$ の i 次基本対称式$)$

$\left(\Leftrightarrow \quad \sum_{j=1}^{m} C(i\xi_j) \quad i = 1, \cdots, \ell \right)$

$$\Delta_{\tilde{C}_\ell} = (1 + x_1 + \cdots + x_{\ell-1} + x_\ell)(1 - x_1 + x_2 - \cdots + (-)^\ell x_\ell)\,\Delta_\ell(1, x_1, \cdots, x_\ell) = x_\ell^{\ell+1} + \cdots$$



(3.4) $\tilde{D}_\ell$ $\quad V_{\mathbb{R}}^*$ の roots $\quad \pm\xi_i \pm \xi_j \quad i < j$

$$\tilde{\omega}_i = \xi_1 + \cdots + \xi_i \qquad 1 \leq i \leq \ell - 2.$$

$$\tilde{\omega}_{\ell-1} = \frac{1}{2}(\xi_1 + \cdots + \xi_{\ell-1} - \xi_\ell), \qquad \tilde{\omega}_\ell = \frac{1}{2}(\xi_1 + \cdots + \xi_{\ell-1} + \xi_\ell)$$

$x_i = \left(C(\xi_j/2)^2 \text{ の } i \text{ 次基本対称式} \right)$, $\quad y = \prod_{j=1}^{\ell} c\left(\frac{\xi_j}{2}\right)$, $\quad z = \prod_{j=1}^{\ell} s\left(\frac{\xi_j}{2}\right)$

$1 \leq i \leq \ell - 2$

$$\Delta_{\tilde{D}_\ell} = \Delta_\ell(1, x_1, \cdots, x_{\ell-2}, (-)^\ell\{1 - x_1 + \cdots + (-)^{\ell-2}x_{\ell-2} + (-)^\ell y^2 - z^2\}, y^2) = x_{\ell-2}^{\ell+1} + \cdots$$

Remark. 1. $x_{\ell-2}^{\ell+1}$ は $\Delta_\ell(a_0, \cdots, a_{\ell-2}, a_{\ell-1}, a_\ell)$ での $a_{\ell-2}^{\ell-1} a_{\ell-1}^2$ より

2. $\quad y, z$ は $\quad S(\tilde{\omega}_{\ell-1})$ と $S(\tilde{\omega}_\ell)$ から一次結合でつくる.

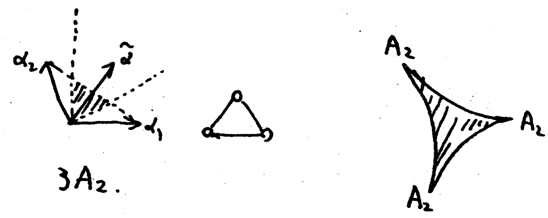
既約.

(3.5) $\tilde{G}_2$ $\quad V_{\mathbb{R}}^*$ の roots $\quad \pm\xi_i,\ \pm(\xi_i - \xi_j)\quad i < j \quad \xi_1 + \xi_2 + \xi_3 = 0.$

$\tilde{\omega}_1 = \xi_1 - \xi_2$, $\quad \tilde{\omega}_2 = -\xi_1$ $\quad \left(\begin{array}{ll} \alpha_1 = \varepsilon_1 - \varepsilon_2 & \tilde{\omega}_1 = 2\alpha_1 + \alpha_2 \\ \alpha_2 = -2\varepsilon_1 + \varepsilon_2 + \varepsilon_3 & \tilde{\omega}_2 = 3\alpha_1 + 2\alpha_2 \end{array} \right)$

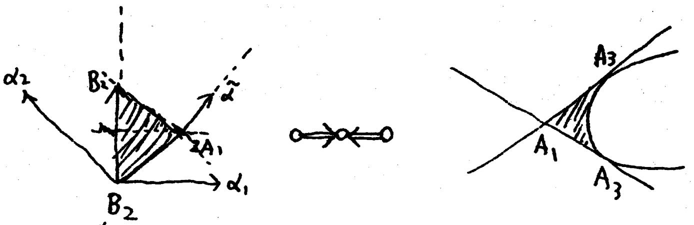
$$\left(\begin{array}{l} x_1 = \sum_{1 \leq i < j \leq 3} C(\xi_i - \xi_j) = C(\xi_1 - \xi_2) + C(2\xi_1 + \xi_2) + C(2\xi_2 + \xi_1) \\ x_2 = \sum_{i=1}^{3} C(\xi_i) = C(\xi_1) + C(\xi_2) + C(\xi_1 + \xi_2). \end{array} \right.$$

$\tilde{A}_2$



$\Delta_2(1, x_1, x_2, 1)$
$= x_1^2x_2^2 + 18x_1x_2 - 4x_1^3 - 4x_2^3 - 27$
$= (x^2+y^2)^2 - 8x^3 + 24xy^2 + 18(x^2+y^2) - 27$
$(x_1 = x+yi,\ x_2 = x-yi)$
$(3,0)$ $(-9/2, 3\sqrt{3}/2)$ $(-9/2, -3\sqrt{3}/2)$

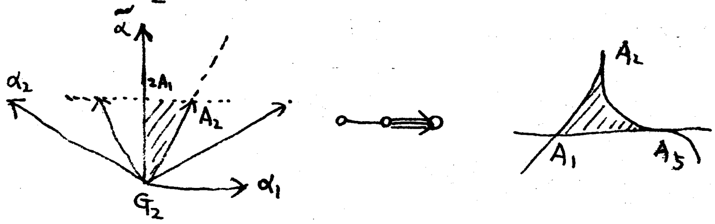
$\tilde{B}_2$



$(1-x+y^2)(x^2-4y^2)$

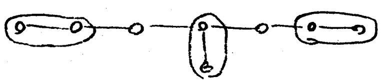
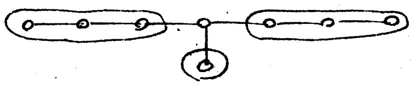
$(0,0)$, $(2,1)$ $(2,-1)$

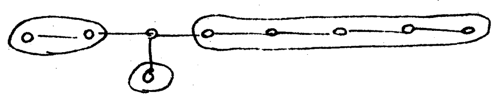
$\tilde{G}_2$



別記.

$\tilde{A}_2$, $\tilde{B}_2$ は四次曲線で，それぞれ $3A_2$，$2A_3+A_1$ 型に対応する．それは $\tilde{E}_7$ の subdiagram として実現され，実際

 ， 

$\tilde{G}_2$ は $\tilde{E}_8$ の subdiagram 

に対応する．

四次曲線と $\tilde{E}_7$ の関係は人部東介氏が調べている．
$\tilde{G}_2$ と $\tilde{E}_8$ の subdiagram も理由のある対応である．詳しくは別の機会にのべる．尚，root 系と singularity の対応は

root	$D_2=2A_1$	A_2	B_2	G_2	← これの discriminant の sing. が
sing.	A_1	A_2	A_3	A_5.	

以下は E_6, E_7 型有限 Weyl 群の不変式について記す

E_6 : $F_{E_6}(x,t) = x^4 + y^3 + t_2x^2y + t_5xy + t_6x^2 + t_8y + t_9x + t_{12}$.

(1) $m_{ij}(t) = \cancel{9}\langle dt_i, dt_j\rangle$. $i, j = 2, 5, 6, 8, 9, 12$.

$m_{2j}(t) = j\,t_j$, $m_{55}(t) = 8t_8 - 4t_6t_2 - \frac{1}{6}t_2^4$, $m_{5,6}(t) = 9t_9 + \frac{1}{2}t_5t_2^2$

$m_{5,8}(t) = -\frac{1}{2}t_9t_2 - \frac{3}{2}t_6t_5 - \frac{1}{12}t_5t_2^3$, $m_{59}(t) = 12t_{12} + \frac{1}{3}t_8t_2^2 \pm 3t_6^2 - \frac{1}{6}t_6t_2^3 + \frac{1}{6}t_5^2t_2$,

$m_{5,12}(t) = -\frac{3}{2}t_9t_6 - \frac{1}{12}t_9t_2^3 + \frac{1}{6}t_8t_5t_2$, $m_{6,6}(t) = -\frac{10}{3}t_8t_2 - \frac{2}{3}t_6t_2^2 - \frac{5}{3}t_5^2$

$m_{68}(t) = 12t_{12}\cancel{+} - \frac{4}{3}t_8t_2^2 + \frac{7}{12}t_5^2t_2$, $m_{69}(t) = -\frac{4}{3}t_9t_2^2 + \frac{7}{6}t_6t_5t_2 - \frac{13}{3}t_8t_5$

$m_{6\,12}(t) = -2t_{12}t_2^2 + \frac{7}{12}t_9t_5t_2 - \frac{8}{3}t_8^2$, $m_{88}(t) = 6t_{12}t_2 - \frac{7}{2}t_9t_5 + 4t_8t_6 - \frac{1}{24}t_5^2t_2^2$

$m_{89}(t) = -\frac{3}{2}t_9t_6 - \frac{7}{6}t_8t_5t_2 - \frac{1}{12}t_6t_5t_2^2 + \frac{5}{12}t_5^3$,

$m_{8\,12}(t) = 6t_{12}t_6 - \frac{9}{4}t_9^2 - \frac{1}{24}t_9t_5t_2^2 - \frac{4}{3}t_8^2t_2 + \frac{5}{12}t_8t_5^2$

$m_{99}(t) = -2t_{12}t_2^2 - \frac{5}{3}t_9t_5t_2 - \frac{8}{3}t_8^2 + \frac{8}{3}t_8t_6t_2 - \frac{1}{6}t_6^2t_2^2 + \frac{4}{3}t_6t_5^2$

$m_{9\,12}(t) = -3t_{12}t_5t_2 + \frac{5}{6}t_9t_8t_2 - \frac{1}{12}t_9t_6t_2^2 + \frac{5}{12}t_9t_5^2 + \frac{1}{2}t_8t_6t_5$

$m_{12\,12}(t) = -2t_{12}t_8t_2 - t_{12}t_5^2 - \frac{1}{24}t_9^2t_2^2 + \frac{11}{6}t_9t_8t_5 - \frac{4}{3}t_8^2t_6$

(想像するよりは簡単な形である。尚[1]に同じ表がある.)
← t_2の巾が少ない.

(2) Flat coördinate $s_2, s_5, s_6, s_8, s_9, s_{12}$. による t_i の表示.

$t_2 = s_2$, $t_5 = s_5$, $t_6 = s_6 - \frac{1}{24}s_2^3$, $t_8 = s_8 + \frac{1}{4}s_2s_6 - \frac{1}{576}s_2^4$

$t_9 = s_9 - \frac{1}{12}s_2^2s_5$, $t_{12} = s_{12} - \frac{1}{24}s_2^2s_8 + \frac{1}{8}s_6^2 - \frac{1}{288}s_2^3s_6 - \frac{1}{24}s_2s_5^2$

(3) $\{s_i\}$ と SYS [2] における flat generators $\{y_i\}$ との関係.

$y_2 = -s_2$, $y_5 = \frac{\sqrt{6}}{4}s_5$, $y_6 = -3s_6$, $y_8 = -3s_8$, $y_9 = \frac{3\sqrt{6}}{4}s_9$, $y_{12} = -9s_{12}$.

ambiguity は weight による $y_i \mapsto c^i y_i$ のみ.

(4) Frame の invariants による t の表示と, それらの y, s での表示.

$t_2 = -A$, $t_5 = \frac{2\sqrt{6}}{3}B$, $t_6 = -\frac{1}{3}C + \frac{1}{12}A^3$, $t_8 = -\frac{1}{3}H + \frac{1}{6}AC - \frac{1}{48}A^4$

$$t_9 = \frac{2\sqrt{6}}{9}J - \frac{\sqrt{6}}{18}A^2B, \quad t_{12} = -\frac{1}{9}K + \frac{1}{36}A^2H + \frac{1}{36}C^2 - \frac{7}{432}A^3C + \frac{2}{9}AB^2 + \frac{1}{864}A^6$$

$$A = y_2 = -s_2, \quad B = y_5 = \frac{\sqrt{6}}{4}s_5, \quad C = y_6 + \frac{1}{8}y_2^3 = -3s_6 - \frac{1}{8}s_2^3$$

$$H = y_8 + \frac{1}{4}y_2y_6 + \frac{1}{192}y_2^4 = -3s_8 + \frac{3}{4}s_2s_6 + \frac{1}{192}s_2^4, \quad J = y_9 = \frac{3\sqrt{6}}{4}s_9$$

$$K = y_{12} + \frac{1}{8}y_2^2y_8 + \frac{1}{8}y_6^2 + \frac{1}{96}y_2^3y_6 + y_2y_5^2 = -9s_{12} - \frac{3}{8}s_2^2s_8 + \frac{9}{8}s_6^2 + \frac{1}{32}s_2^3s_6 - \frac{3}{8}s_5^2.$$

(5) $\langle ds_i, ds_j\rangle$. すべてはきめていない. $\langle ds_2, ds_j\rangle = j s_j$.

$$\langle ds_5, ds_5\rangle = 8s_8 - 2s_2s_6 - \frac{1}{72}s_2^4, \quad \langle ds_5, ds_6\rangle = 9s_9 + \frac{3}{8}s_2^2s_5$$

$$\langle ds_5, ds_8\rangle = -\frac{11}{4}s_2s_9 - \frac{11}{4}s_5s_6 - \frac{1}{288}s_2^3s_5$$

$$\langle ds_5, ds_9\rangle = 12s_{12} + \frac{1}{2}s_2^2s_8 - \frac{3}{2}s_6^2 - \frac{1}{24}s_2^3s_6 + \frac{1}{2}s_2s_5^2$$

$$\langle ds_6, ds_6\rangle = -\frac{10}{3}s_2s_8 - \frac{5}{3}s_5^2 + \frac{1}{432}s_2^5$$

(6) $I_2(12)_E$ 型は $s_5 = s_6 = s_8 = s_9 = 0$ として得る. (既出)

$$F_{I_2(12)_E}(x,y) = x^4 + y^3 + s_2x^2y + -\frac{1}{24}s_2^3x^2 - \frac{1}{576}s_2^4y + s_{12} - \frac{1}{24}s_2s_5^2$$

E_7 : $F_{E_7}(x,t) = -\frac{1}{3}(x^3y+y^3) + t_2xy^2 + t_6y^2 + t_8xy + t_{10}x^2 + t_{12}y + t_{14}x + t_{18}$

$$x^2y = t_2y^2 + t_8y + 2t_{10}x + t_{14}$$

$$\frac{1}{3}x^3 + y^2 = 2t_2xy + 2t_6y + t_8x + t_{12}$$

$$\frac{1}{\mu}\mathrm{Hess} f = \frac{1}{7}(4xy^2 - x^4) = xy^2 \quad , \quad x^4 = -3xy^2$$

(1) $\langle i,j\rangle = \langle e_i, e_j\rangle \qquad e_i = \left[\frac{\partial}{\partial t_i}F_{E_7}\,d\underline{x}\right]$

$$\langle 6,12\rangle = \frac{5}{3}t_2, \quad \langle 8,10\rangle = t_2, \quad \langle 2,14\rangle = \frac{5}{3}t_2^2 = \langle 6,10\rangle = \langle 8,8\rangle$$

$$\langle 2,12\rangle = \langle 6,8\rangle = 2t_6 + \frac{5^2}{3^2}t_2^3, \quad \langle 2,10\rangle = t_8 + 2t_2t_6 + \frac{5^2}{3^2}t_2^4$$

$$\langle 6,6\rangle = \frac{2}{3}t_8 + \frac{20}{3}t_2t_6 + \frac{5^3}{3^3}t_2^4$$

$$\langle 2,8\rangle = 2t_{10} + \frac{7}{3}t_2t_8 + \frac{20}{3}t_2^2t_6 + \frac{5^3}{3^3}t_2^5$$

$$\langle 2,6\rangle = t_{12} + \frac{8}{3}t_2t_{10} + \frac{45}{9}t_2^2t_8 + 4t_6^2 + \frac{50}{3}t_2^3t_6 + \frac{5^4}{3^4}t_2^6$$

$$\langle 2,2\rangle = \frac{4}{3}t_2t_{14} + \frac{10}{3}t_2^2t_{12} + 4t_6t_{10} + \frac{80}{9}t_2^3t_{10} + \frac{2}{3}t_8^2 + \frac{28}{3}t_2t_6t_8$$
$$+ \frac{400}{27}t_2^4t_8 + 20t_2^2t_6^2 + \frac{1000}{27}t_2^5t_6 + \frac{5^5}{3^5}t_2^8$$

(2) flat coordinate s_i の t_i による表示.

$$s_2 = t_2, \quad s_6 = t_6 + \frac{4}{9}t_2^3, \quad s_8 = t_8 + \frac{4}{3}t_2t_6 + \frac{13}{27}t_2^4$$

$$s_{10} = t_{10}, \quad s_{12} = t_{12} + t_2t_{10} + \frac{5}{6}t_2^2t_8 + \frac{5}{6}t_6^2 + \frac{35}{27}t_2^3t_6 + \frac{161}{2\cdot 3^5}t_2^6$$

$$s_{14} = t_{14} + \frac{1}{3}t_2t_{12} + \frac{1}{3}t_2^2t_{10} + \frac{1}{3}t_6t_8 + \frac{10}{27}t_2^3t_8 + \frac{5}{9}t_2t_6^2 + \frac{95}{162}t_2^4t_6 + \frac{279}{3^6}t_2^7$$

$$s_{18} = t_{18} + \frac{1}{3}t_2^2t_{14} + \frac{2}{3}t_6t_{12} + \frac{11}{27}t_2^3t_{12} + t_8t_{10} + \frac{2}{3}t_2t_6t_{10} + \frac{11}{27}t_2^4t_{10} + \frac{1}{3}t_2t_8^2$$
$$+ \frac{11}{9}t_2^2t_6t_8 + \frac{44}{3^4}t_2^5t_8 + \frac{11}{27}t_6^3 + \frac{110}{81}t_2^3t_6^2 + \frac{638}{3^6}t_2^6t_6 + \cancel{\frac{10285}{2\cdot 3^9}}t_2^9$$

(3) t_i の s_i による表示.

$$t_2 = s_2, \quad t_6 = s_6 - \frac{4}{9}s_2^3, \quad t_8 = s_8 - \frac{4}{3}s_2s_6 + \frac{1}{9}s_2^4, \quad t_{10} = s_{10}$$

$$t_{12} = s_{12} - s_2s_{10} - \frac{5}{6}s_2^2s_8 - \frac{5}{6}s_6^2 + \frac{5}{9}s_2^3s_6 - \frac{1}{3^4}s_2^6,$$

$$t_{14} = s_{14} - \frac{1}{3}s_2 s_{12} + 0\cdot s_2^2 s_{10} - \frac{1}{3} s_6 s_8 + \frac{1}{18} s_2^3 s_8 + \frac{1}{6} s_2 s_6^2 - \frac{1}{54} s_2^4 s_6 - \frac{184}{36} s_2^7$$

$$t_{18} = s_{18} - \frac{1}{3}s_2^2 s_{14} - \frac{2}{3} s_6 s_{12} - s_8 s_{10} + \frac{4}{3} s_2 s_6 s_{10} - \frac{1}{9} s_2^4 s_{10} - \frac{1}{3} s_2 s_8^2 + \frac{1}{3} s_2^2 s_6 s_8$$
$$+ \frac{4}{27} s_6^3 - \frac{1}{9} s_2^3 s_6^2 + \frac{1}{2\cdot 3^4} s_2^6 s_6 + \cdots s_2^9$$

($s_2^3 s_{12}$, $s_2^5 s_8$ など)

$\mathfrak{g}$, $(\mathbb{C}^n, 0)$ での Lie alg. の germ.

$G = \exp \mathfrak{g}$ の prehomogeneity などについて1つ1つ研究しています.

[1] Guiventhal : Funct. Analy and its appl. 1980.

(2) Saito - Yano - Sekiguchi : Commun. in Alg. 1980