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Lorenz knots

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This report concerns joint work with R. F. Williams. For details, see our manuscript "Knotted periodic orbits in dynamical systems I: Lorenz knots" (preprint).

In that manuscript we study the class of knots which arise as closed orbits in the flow on R^3 determined by the system of differential equations :

$$x = -10x + 10y$$

$$y = 28x - y - xz$$

$$z = -{}^{8}\!/_{3}z + xy$$

These equation were introduced by E. N. Lorenz, and so we call our knots <u>Lorents knots.</u>

Work of Williams, Guckenheimer and others(which rests in part on numerical methods) implies the existence of a "branched 2-manifold" $H_L \subset S^3$ which has the property that the periodic orbits for the flow associated to Lorenz's equations can be collapsed into H_L in such a way that :

(1) The collapsing is 1-1 on the set of all periodic orbits.

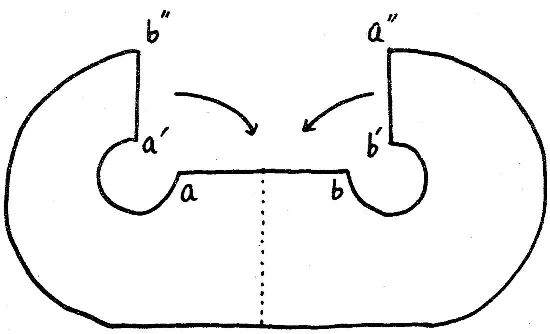
(2) The collapsing is 1-1 on each individual orbit.

(3) Every simple closed curve which can be drawn on H_L corresponds to an actual orbit.

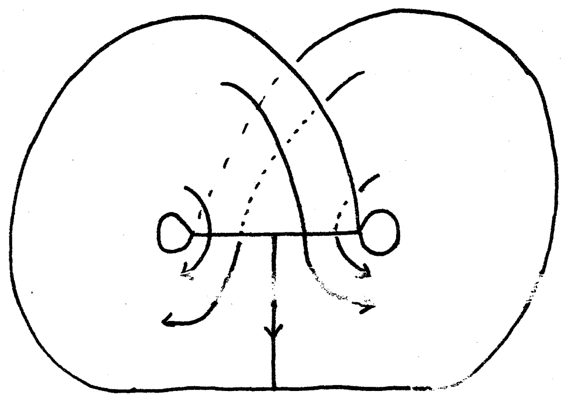
(4) Orbits on H_L are naturally oriented by an induced semi-flow on H_L.

We call H_L a "knot holder".

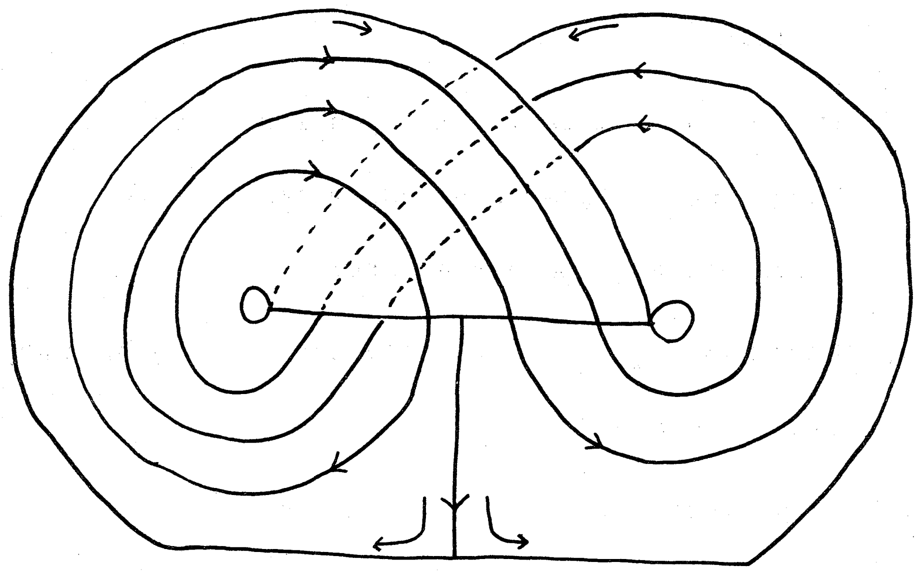
Here is a picture showing how H_L is constructed :



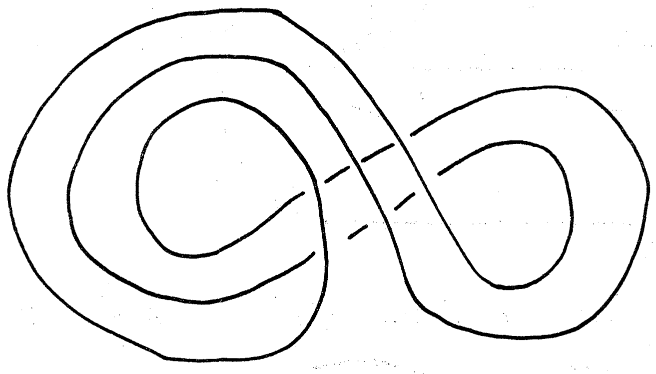
After identifying ab with a'b'' and a''b' we obtain H_L :



Here is an example of a Lorenz knot, shown embedded on H_L:



If it is removed from H_L it looks like this :



At first glance it looks unknotted, but it has the trefoil knot type. The collection of Lorenz knots contains infinitely many distinct knot types, and exhibits very interesting structure. My talk summarized what we know about this family of knots.