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A complement to the theory of G-CW complexes

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Let G be a topological group. By a G-space X we mean a topological space X together with a continuous G-action on X. If a G-equivariant map $f: X \to Y$ between G-spaces induces isomorphisms $f_*: \pi_n(X^H, x) \to \pi_n(Y^H, f(x))$ for every $n \geq 0$, every (closed) subgroup H of G and $x \in X^H = \{x \in X;\ gx = x \text{ for every } g \in H\}$, then f is called a weak G-homotopy equivalence.

We have defined the notion of (Hausdorff) G-CW complexes and have shown that any weak G-homotopy equivalence between G-CW complexes is a G-homotopy equivalence. The purpose of this talk is to show a canonical construction of a pair of a G-CW complex K_X and a weak G-homotopy equivalence $\rho: K_X \to X$ for a (Hausdorff) G-space X [Theorem 1]. As an application we prove that the singular G-(co)homology theory on X defined by Illman coincides with the cellular G-(co)homology theory on K_X [Theorem 2].

The construction of K_X is as follows. Take the geometric realization $|S(X)|$ of the singular complex of X. The underlying discrete group of G operates on $|S(X)|$. Define $|S_G(X)|$ to be the G-space with the strongest topology so that the G-action be continuous. Since $\pi_n(|S_G(X)|^H) \to \pi_n(X^H)$ are onto homomorphisms, we have only to kill the kernels by attaching many many G-cells.

Remark. We need to define the notion of a non-Hausdorff G-CW complex when X is not a Hausdorff space.

Question. Is the induced map $\rho/G : K_X/G \to X/G$ a weak homotopy equivalence for our (K_X, ρ)?

References

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