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# On Canonical Partition of Edge Set

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(Abstract)

Let E be the edge set of a nonseparable graph G. For each edge e , let t(e) be the number of trees which contain e , and $\overline{t}(e)$ be the number of trees which does not contain e . Here, let $E_+$ , $E_o$ , $E_-$ be edge sets which satisfy $t(e) < \overline{t}(e)$, $t(e) = \overline{t}(e)$, $t(e) > \overline{t}(e)$, respectively, then we have the tripartition ( $E_+$ , $E_o$ , $E_-$ ) of E. We call this tripartition the <u>canonical partition</u> of edge set E.

This paper presents properties in connection with the structure of graphs which satisfy $E = E_o$ , $E_+ = E_- = \emptyset$. G is called to have the complementary tree structure when E is partitioned into $E_1$ , $E_2$ and each $E_i$ is the edge set of a tree of G.

[Theorem 1] Let ( $\emptyset$ , E , $\emptyset$ ) be the canonical partition of E, then G have the complementary tree structure.

[Example 1] The canonical partitions of E of Graphs $L_2$, $K_4$ and $K_{3,4}$ shown in Fig. 1 are all ( $\phi$ , E , $\phi$ ) , and they also have the complementary tree structures.

[Theorem 2] Let $G_1$, $G_2$ be graphs whose canonical partitions are ( $\phi$ , $E_1$ , $\phi$ ) and ( $\phi$ , $E_2$ , $\phi$ ), respectively. If we delete any edges $e_i = (u_i, v_i) \in E_i$ (i=1,2) and connect $u_1$ and $u_2$, $v_1$ and $v_2$, respectively, then the canonical partition of the obtained graph G is ( $\phi$ , E , $\phi$ ), where $E = E_1 \cup E_2 - \{e_1, e_2\}$ . (END)

Fig. 2 explains the manner obtaining G from $G_1$ and $G_2$ by the procedure stated in Theorem 2.

The converse of this theorem is stated as follows.

[Theorem 3] Let G be the graph shown in Fig. 2(b) which is connected by two vertices u and v, and its canonical partition is ( $\phi$ , E , $\phi$ ). If we delete $E-E_i'$ and add an edge $e_i$ between u and v, then the canonical partition of the obtained graph $G_i$ is ( $\phi$ , $E_i$ , $\phi$ ), respectively, where $E_i' = E_i - \{e_i\}$ (i=1,2).

[Theorem 4] Let G be the graph whose canonical partition is ( $\phi$ , E , $\phi$ ). The canonical partition of any graph $G'$ which contains G as a partial subgraph is not ( $\phi$ , $E'$ , $\phi$ ). Also, the canonical partition of $G''$ which is contained in G as a partial subgraph is not ( $\phi$ , $E''$ , $\phi$ ). (END)

The following conjecture is of interest.

[Conjecture] The canonical partition of edge set E of three connected graph G is ( $\phi$ , E , $\phi$ ) if and only if G is $K_4$ or $K_{3,4}$.

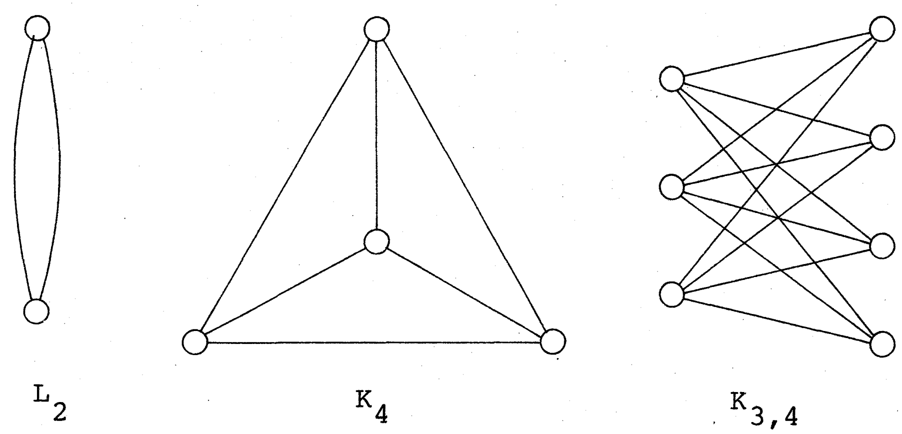


Fig. 1

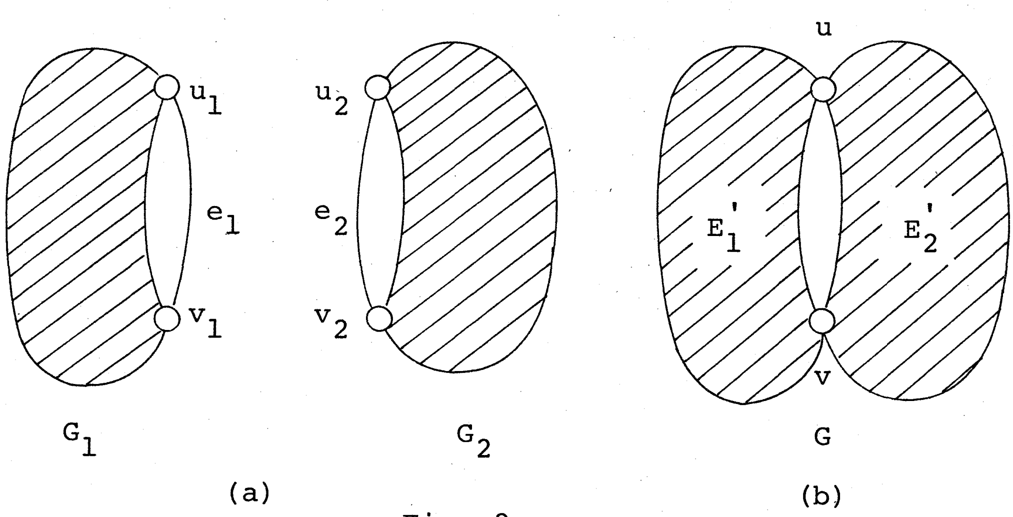


Fig. 2