数理解析研究所講究録
第 456 巻 1982 年 160-161

Problems

[1] Let $f(s)$ be a Dirichlet series absolutely convergent in some half-plane $\sigma > \sigma_a \geq 0$ and satisfy the functional equation

$$f(s) = f(-s)$$

($f(s)$ is defined in some manner in the strip $0 \leq \sigma \leq \sigma_a$). Can one prove that $f(s)$ equals identically a constant under some suitable conditions?

[2] Let $\Phi(z) = \sum_{n=1}^{\infty} a_n \exp(2\pi i n z)$ be a cusp form of weight k, and let $L_\Phi(s,\chi)$ be the Dirichlet-Hecke series associated with Φ. Does there exists a Dirichlet character χ for which $L_\Phi(\frac{k}{2},\chi)$ does not vanish? There are many known results about the value $L_\Phi(\frac{k}{2} + it,\chi)$ when $|t|$ is large (see, e.g. A. Good, J. Number Theory 13 (1980), 18-65.

[3] We may stress again the question of determining $g(4)$ in 华林 problem. It should be noted that R. Balasubramanian has recently proved (Hardy-Ramanujan J. 2 (1979), 1-34) that

$$(19 \leq)\ g(4) \leq 21,$$

where the first inequality has been known.

[4] Let $\sigma_r(n) = \sum_{d|n} d^r$ and for $b > 0$, $\rho > 0$ put

$$P^\rho(x,r,b) := \frac{1}{\Gamma(\rho + 1)} \sum_{n\leq x}{}' (x^b - n^b)^\rho \sigma_{-r}(n) - S^\rho((\pi x)^b, r, b),$$

where the prime on the suumation sign means that if $\rho = 0$ and

$n = x$, then the term $\sigma_{-r}(n)$ is to be halved, and

$$S^{\rho}(x,r,b) = \sum_{\xi} \operatorname*{Res}_{s=\xi} \frac{\Gamma(s)\pi^{-bs}\zeta(bs)\zeta(bs+r)}{\Gamma(s+\rho+1)} x^{s+r} .$$

Also for $a \in \mathbb{R}$, $k \in \mathbb{N}$ define

$$G_{a,k}(x) := \sum_{n \leq \sqrt{x}} n^{a} B_{k}\left(\frac{x}{n} - \left[\frac{x}{n}\right]\right),$$

where $B_k(x)$ is the k-th Bernoulli polynomial. Is there any explicit relation between $P^{\rho}(x,r,b)$ and $G_{a,k}(x)$? (There is an asymptotic relation between $P^{1}(x,r,r)$ and $G_{1-r,2}(x)$ if $-\frac{1}{2} \leq r < 3$, $r \neq 0, 1$.)

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