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# Two-dimensional Alternating Simple Multihead Finite Automata

## —Hierarchical Properties—

Hiroshi MATSUNO 松野浩嗣 Hiroshi TANIGUCHI 谷口弘
Katsushi INOUE 井上克司 Itsuo TAKANAMI 高浪五男
(Faculty of Engineering, Yamaguchi University)
(山口大学工学部)

## 1. Introduction

Recently, we introduced one-dimensional alternating simple multihead finite automata (ASPMHFAs) in [4], and gave several properties of these automata. This paper investigates several properties of two-dimensional alternating simple multihead finite automaton (2ASPMHFA), which can be considered as a natural extension of an ASPMHFA to two-dimensions.

Section 2 gives terminologies and notations necessary for this paper. Section 3 investigates a relationship between the accepting powers of 2 ASPMHFAs and non-alternating versions. Section 4 shows that four-way 2 ASPMHFAs are equivalent to ordinary two-dimensional alternating multihead four-way finite automata. In Section 5, we give some properties of 2 ASPMHFAs with only universal states.

## 2. Preliminaries

Definition 2.1. Let $\Sigma$ be a finite set of symbols. A two-dimensional tape over $\Sigma$ is a two-dimensional rectangular array of elements of $\Sigma$. The set of all two-dimensional tapes over $\Sigma$ is denoted by $\Sigma^{(2)+}$. Given a tape $x \in \Sigma^{(2)+}$, we let $\ell_1(x)$ be the number of rows of x and $\ell_2(x)$ be the number of columns of x. If $1 \le i \le \ell_1(x)$ and $1 \le j \le \ell_2(x)$, we let $x(i,j)$ denote the symbol in x with coordinates $(i,j)$. Furthermore, we define

$$x[(i,j),(i',j')],$$

only when $1 \le i \le i' \le \ell_1(x)$ and $1 \le j \le j' \le \ell_2(x)$, as the two-dimensional tape z satisfying the following:

(1) $\ell_1(z)=i'-i+1$ and $\ell_2(z)=j'-j+1$;

(2) for each k, $r[1 \le k \le \ell_1(z),\ 1 \le r \le \ell_2(z)]$, $z(k,r)=x(k+i-1,r+j-1)$.

(We call $x[(i,j),(i',j')]$ the "$[(i,j),(i',j')]$-segment of x".)

The reader is referred to [1,2] for formal definitions of a <u>two-dimensional multihead finite automaton</u> (2MHFA). A two-dimensional <u>simple</u> multihead finite automaton (2SPMHFA) is a 2MHFA with the restriction that one head (called the "reading head") can sense input symbols while the others (called the "counting heads") can only detect the endmarker "#". When the heads of 2MHFA (2SPMHFA) are allowed to sense the presence of other heads on the same input position, we call such 2MHFA (2SPMHFA) a " sensing" 2MHFA (2SPMHFA).

A <u>"four-way" 2MHFA (2SPMHFA)</u> is a 2MHFA (2SPMHFA) whose all heads can move in four directions, left, right, up, and down. A <u>"three-way" 2MHFA (2SPMHFA)</u> is a 2MHFA (2SPMHFA) whose all heads can move left, right, and down, but not up. In addition with these, we introduce the following two 2SPMHFA's in accordance with the variety of the moves of reading head and counting heads. A <u>"semi-four-way" 2SPMHFA</u> is a 2SPMHFA whose counting heads cannot move up.(Note that the reading head can move in four directions.) A <u>"semi-three-way" 2SPMHFA</u> is a 2SPMHFA whose reading head cannot move up.(Note that the counting heads can move in four directions.)

When an input tape x is presented to the 2MHFA (2SPMHFA) M, M starts in its initial state with each head on the upper left-hand corner of x. M accepts the input tape x if and only if it enters an accepting state at some time after each head reached the bottom boundary symbol #.

Alternating 2MHFA (2AMHFA) and alternating 2SPMHFA (2ASPMHFA) are alternating versions of 2MHFA and 2SPMHFA, respectively. That is, 2AMHFA (2ASPMHFA) is the same as an 2MHFA (2SPMHFA) except that the state set is devided into two disjoint sets, the set of universal states, and the set of existential states. Of course, each alternating automaton has the specified set of accepting states, which is a subset of the state set.

A step of a 2AMHFA (2ASPMHFA) M consists of reading a symbol from the input tape by each head, moving the heads in specified directions (note that any of the heads can remain stationary during a move), and entering a new state, in accordance with the transition function. If one of heads of M falls off the input tape, then M can make no futher move.

Definition 2.2. A configuration of a two-dimensional (sensing) alternating simple k-head finite automaton M is an element of

$$\Sigma^{(2)+} \times C_M,$$

where $C_M=Q\times((N\cup\{0\})^2)^k$ (where Q is the set of states of the finite control of M and N denotes the set of all positive integers). The first component x of a configuration $c=(x,(q,(i_R,j_R),(i_1,j_1),(i_2,j_2),\ldots,(i_{k-1},j_{k-1})))$‡ represents the input tape. The second component $(q,(i_1,j_1),(i_2,j_2),\ldots,(i_{k-1},j_{k-1}))$ ($\in C_M$) of c represents the state of the finite control, the position of reading head and the positions of k-1 counting heads, respectively. An element of $C_M$ is called "semi-configuration of M". If q is the state associated with c, then c is said to be universal

‡ We note that $0\le i_R,\ i_m\le \ell_1(x)+1,\ 0\le j_R,\ j_m\le \ell_2(x)+1$ $(1\le m\le k-1)$.

(existential, accepting) configuration if q is a universal (existential, accepting) state. The initial configuration of M on x is

$$I_M(x)=(x,(q_0,\underbrace{(1,1),(1,1),\ldots,(1,1)}_{k})),$$

where $q_0$ is the initial state of the finite control of M.

Definition 2.3. Given a two-dimensional (sensing) alternating simple multihead finite automaton M, we write $c \vdash c'$ and say that c' is a successor of c if configuration c' follows from configuration c in one step, according to the transition function of M. A computation path of M on input x is a sequence $c_0 \vdash c_1 \vdash \cdots \vdash c_n$ $(n \geq 0)$, where $c_0 = I_M(x)$. A computation tree of M is a finite, nonempty labeled tree with the properties

(1) each node $\pi$ of the tree is labeled with a configuration, $\ell(\pi)$,

(2) if $\pi$ is an internal node (a non-leaf) of the tree, $\ell(\pi)$ is universal and $\{c \mid \ell(\pi) \vdash c\}=\{c_1, c_2, \ldots, c_k\}$, then $\pi$ has exactly k children $\rho_1, \rho_2, \ldots, \rho_k$ such that $\ell(\rho_i)=c_i$,

(3) if $\pi$ is an internal node of the tree and $\ell(\pi)$ is existential, then $\pi$ has exactly one child $\rho$ such that $\ell(\pi) \vdash \ell(\rho)$.

A computation tree of M on x is a computation tree of M whose root is labeled with $I_M(x)$. An accepting computation tree of M on x is a computation tree of M on x whose leaves are all labeled with accepting configurations. We say that M accepts x if there is an accepting computation tree of M on x.

For some MHFA (SPMHFA) M, let T(M) be the set of tapes accepted by M. Deterministic 2MHFA (2SPMHFA) and nondeterministic 2MHFA (2SPMHFA) are special cases of alternating versions. That is, a nondeterministic 2MHFA (2SPMHFA) is a 2AMHFA (2ASPMHFA) which has no universal states, and a deterministic 2MHFA (2SPMHFA) is a 2AMHFA (2ASPMHFA) whose configurations each have at most one successor.

In this paper, to represent the several kinds of 2SPMHFAs (resp. 2MHFAs, sensing 2MHFAs) systematically, we use the notation XY2-kHZ (resp. X2-kHZ, XSN2-kHZ), k≥1, where,

(1) X∈{D,N,A,U},

D : deterministic

N : nondeterministic

A : alternating

U : alternating automaton with only universal states,

(2) Y∈{SP,SNSP},

SP : simple

SNSP : sensing simple,

(3) 2- : two-dimensional,

(4) kH : k-head (the number of heads are of k),

(5) Z∈{A,SA,STRA,TRA},

A : four-way

SA : semi-four-way

STRA : semi-three-way

TRA : three-way

(Of course, "SA" and "STRA" are used only for 2SPMHFA.).

For example,

DSP2-kHA : two-dimensional deterministic simple k-head four-way finite automaton

USN2-kHSTRA : two-dimensional sensing alternating k-head semi-three-way finite automaton with only universal states,

Furthermore, for each X∈{D,N,A,U}, Y∈{SP,SNSP}, k≥1, Z∈{A,SA,STRA,TRA}, and Z' $\in${A,TRA },

$\mathcal{L}$[XY2-kHZ]={T | T=T(M) for some XY2-kHZ M },

$\mathcal{L}$[X2-kHZ' ]={T | T=T(M) for some X2-kHZ' M },

$\mathcal{L}$[XSN2-kHZ' ]={T | T=T(M) for some XSN2-kHZ' M }.

In this paper, we shall concentrate on investigating the properties of 2 MHFA and 2SPMHFA whose input tapes are restricted to square ones. To represent these, we use the notations XY2-kHZ$^s$, X2-kHZ'$^s$, and XSN2-kHZ'$^s$ for each X$\in${D,N,A,U}, Y$\in${SP,SNSP}, k$\geq$1, Z$\in${A,SA,STRA,TRA}, and Z'$\in${A,TRA} . For example, an ASP2-kHA$^s$ denotes two-dimensional alternating simple k-head four-way finite automaton whose input tapes are restricted to the square ones. For each X$\in${D,N,A,U}, Y$\in${SP,SNSP}, k$\geq$1, Z$\in${A,SA,STRA,TRA}, and Z'$\in$ {A,TRA},

$\mathcal{L}$[XY2-kHZ$^s$]={T | T=T(M) for some XY2-kHZ$^s$},

$\mathcal{L}$[X2-kHZ'$^s$]={T | T=T(M) for some X2-kHZ'$^s$},

$\mathcal{L}$[XSN2-kHZ'$^s$]={T | T=T(M) for some XSN2-kHZ'$^s$}.

## 3. A Relationship between Alternating and Non-alternating Automata

This section investigates a relationship between the accepting powers of two-dimensional alternating simple multihead three-way (or semi-three-way) finite automata and non-alternating versions.

Theorem 3.1. There exists a set in $\mathcal{L}$[U2-1HTRA$^s$], but not in $\bigcup_{1\leq k<\infty}\mathcal{L}$[NSNSP2-kHSTRA$^s$].

Proof. Let $T_1=\{x\in\{0,1\}^{(2)+} \mid (\exists m\geq 2)[\ell_1(x)=\ell_2(x)=m$ & $x[(1,1),(1,m)]=x[(2,1),(2,m)]]\}$. The set $T_1$ is accepted by U2-1HTRA$^s$ M which acts as follows. Given an input x ($\ell_1(x)=\ell_2(x)=m\geq 2$), M universally (i.e., in universal states) tries to check that for each i ($1\leq i\leq m$) x(1,i)=x(2,i). That is, on first row and i-th column of x ($1\leq i\leq m$), M enters a universal state to choose one of two further actions. One action is to pick up the symbol x(1,i), to move down with the symbol in the finite control, to compare the stored symbol with the symbol x(2,i), and to enter an accepting state if both symbols are identical. The other action is to continue to move right one tape cell (in order to pick up the next symbol x(1,i+1) and compare it with the symbol x(2,i+1)). It will be obvious $T(M)=T_1$. The proof of $T_1 \notin \bigcup_{1\le k<\infty} \mathcal{L}[\text{NSNSP2-kHSTRA}^s]$ is given in the proof of Theorem 1 in [3].

Q.E.D.

Theorem 3.2. There exists a set in $\mathcal{L}[\text{N2-1HTRA}^s]$, but not in $\bigcup_{1\le k<\infty} \mathcal{L}[\text{USNSP2-kHSTRA}^s]$.

Proof. Let $T_1$ be the set described in Theorem 3.1. Then we show that $\overline{T}_1$ ‡ $\in \mathcal{L}[\text{N2-1HTRA}^s] - \bigcup_{1\le k<\infty} \mathcal{L}[\text{USNSP2-kHSTRA}^s]$. It is easily seen that $\overline{T}_1$ is accepted by N2-1HTRA$^s$. Then we show that $\overline{T}_1 \notin \bigcup_{1\le k<\infty} \mathcal{L}[\text{USNSP2-kHSTRA}^s]$. Suppose that there is a USNSP2-kHSTRA$^s$ M, $k\ge 1$, accepting $\overline{T}_1$. Let r be the numbers of state (of the finite control) of M. For each $m\ge 2$, let

$$V(m)=\{x\in\{0,1\}^{(2)+} \mid (\exists m\ge 2)[\ell_1(x)=\ell_2(x)=m] \;\&\; x[(1,1),(1,m)]=x[(2,1),(2,m)] \;\&\; x[(3,1),(m,m)]\in\{0\}^{(2)+}\}.$$

For each x in V(m), let S(x) and C(x) be sets of semi-configurations of M defined as follows.

$S(x)=\{(q,(2,j_R),(i_1,j_1),(i_2,j_2),\ldots,(i_{k-1},j_{k-1})) \mid$ there exists a computation path of M on x, $I_M(x) \overset{*}{\vdash} (x,(q,(1,j_R'),(i_1',j_1'),(i_2',j_2'),\ldots,(i_{k-1}',j_{k-1}'))) \vdash (x,(q,(2,j_R),(i_1,j_1),(i_2,j_2),\ldots,(i_{k-1},j_{k-1})))$ (that is, $(x,(q,(2,j_R),(i_1,j_1),(i_2,j_2),\ldots,(i_{k-1},j_{k-1})))$ is a computation of M just after the point where the reading head reached the second row of x)$\}$,

‡ For each set T, $\overline{T}$ denotes the complement of T.

$C(x)=\{\sigma \in S(x) \mid$ when, starting with the configuration $(x,\sigma)$, M proceeds to read the segment $x[(2,1),(m,m)]$, there exists a sequence of steps of M in which never enters an accepting state$\}$.

(Note that, for each x in V(m), C(x) is not empty, since x is not in $\overline{T}_1$, and so not accepted by M.) Then the following proposition must hold.

<u>Proposition 3.1.</u> For any two different tapes x,y in V(m), $C(x) \cap C(y)=\phi$, where $\phi$ denotes the empty set.

[For otherwise, suppose that $x \neq y$ $(x,y \in V(m))$, $C(x) \cap C(y) \neq \phi$, and $\sigma \in C(x) \cap C(y)$. Consider the tape z (with $\ell_1(z)=\ell_2(z)=m$) satisfying the following (i) and (ii).

(i) $z[(1,1),(1,m)]=x[(1,1),(1,m)]$;

(ii) $z[(2,1),(m,m)]=y[(2,1),(m,m)]$.

From (i) above and the assumption that $\sigma \in C(x)$, it follows that there exists a computation path of M on z, $I_M(z) \overset{*}{\vdash} (z,\sigma)$. Further, from (ii) above and the assumption that $\sigma \in C(y)$, it follows that when, starting with the configuration $(z,\sigma)$, M proceeds to read the segment $z[(2,1),(m,m)]$, there exists a sequence of steps of M in which M never enters an accepting state. This means that z is not accepted by M. This contradicts the fact that z is in $\overline{T}_1=T(M)$.]

Clearly, $|V(m)|^{\ddagger}=2^m$, and $p(m) \leq r(m+2)(m+2)^{2(k-1)}$, where p(m) denotes the number of possible semi-configurations of M just after the input head reached the second rows of tapes in V(m). For large m, we have $|V(m)| > p(m)$. Therefore, it follows that for large m, there must be two different words x, y in V(m) such that $C(x) \cap C(y) \neq \phi$. This contradicts Proposition 3.1. Thus, $\overline{T}_1 \notin \bigcup_{1 \leq \ell < \infty} \mathcal{L}[\text{USNSP2-kHSTRA}^s]$. This completes the proof of the theorem.

Q.E.D.

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‡ For any set S, |S| denotes the number of elements of S.

As a corollary of Theorems 3.1 and 3.2, we can get

Corollary 3.1. For each $k\geq 1$, each $Y\in\{SP, SNSP\}$, and each $Z\in\{TRA, STRA\}$,

(1) $\mathcal{L}[DY2\text{-}kHZ^s] \subsetneq \mathcal{L}[NY2\text{-}kHZ^s] \subsetneq \mathcal{L}[AY2\text{-}kHZ^s]$,

(2) $\mathcal{L}[DY2\text{-}kHZ^s] \subsetneq \mathcal{L}[UY2\text{-}kHZ^s] \subsetneq \mathcal{L}[AY2\text{-}kHZ^s]$,

(3) $\bigcup_{1\leq r<\infty}\mathcal{L}[DY2\text{-}rHZ^s] \subsetneq \bigcup_{1\leq r<\infty}\mathcal{L}[NY2\text{-}rHZ^s] \subsetneq \bigcup_{1\leq r<\infty}\mathcal{L}[AY2\text{-}rHZ^s]$,

(4) $\bigcup_{1\leq r<\infty}\mathcal{L}[DY2\text{-}rHZ^s] \subsetneq \bigcup_{1\leq r<\infty}\mathcal{L}[UY2\text{-}rHZ^s] \subsetneq \bigcup_{1\leq r<\infty}\mathcal{L}[AY2\text{-}rHZ^s]$,

(5) $\mathcal{L}[NY2\text{-}kHZ^s]$ is incomparable with $\mathcal{L}[UY2\text{-}kHZ^s]$, and

(6) $\bigcup_{1\leq r<\infty}\mathcal{L}[NY2\text{-}rHZ^s]$ is incomparable with $\bigcup_{1\leq r<\infty}\mathcal{L}[UY2\text{-}rHZ^s]$.

## 4. Simple versus Non-simple Two-dimensional Alternating Multihead Automata

The main purpose of this section is to show that four-way 2ASPMHFAs are equivalent to four-way 2AMHFAs. By using the same technique as in the proof of Lemma 2.1 in [5], we can show that the following lemma holds.

Lemma 4.1. For each $k\geq 1$ and $Z\in\{A, SA, STRA, TRA\}$,

$\mathcal{L}[ASP2\text{-}kHZ^s] = \mathcal{L}[ASNSP2\text{-}kHZ^s]$.

We can show by using Lemma 4.1 that for each $k\geq 1$, $\mathcal{L}[ASP2\text{-}kHA^s]$ is equal to $\mathcal{L}[A2\text{-}kHA^s]$.

Theorem 4.1. For each $k\geq 1$,

$\mathcal{L}[ASP2\text{-}kHA^s] = \mathcal{L}[A2\text{-}kHA^s]$.

Proof. To prove this, by Lemma 4.1 above, it is sufficient to show that $\mathcal{L}[A2\text{-}kHA^s] \subseteq \mathcal{L}[ASNSP2\text{-}kHA^s]$. Let $M_1$ be an $A2\text{-}kHA^s$, and let $H_1, H_2, \ldots, H_k$ be the input head of $M_1$. We construct an $ASNSP2\text{-}kHA^s$ $M_2$ which accepts $T(M_1)$. Let R be the reading head of $M_2$, and $C_2, C_3, \ldots, C_k$ be the counting heads of $M_2$. Suppose that an input tape x is presented to $M_2$. By letting R simulate the action of $H_1$ and by letting $C_i$ $(2\leq i\leq k)$ simulate the action of $H_i$, $M_2$ simulates one step of $M_1$ on x as follows.

(i) $M_2$ existentially guesses the symbol read by each $C_i$ $(2 \le i \le k)$. (Of course, the symbol read by R does not have to be guessed.)

(ii) $M_2$ then enters a universal state to choose one of the following actions.

(a) Directly simulate one step of $M_1$ on x by using the symbol guessed above and the symbol read by R, and then continue to simulate the next step of $M_1$ on x.

(b) Check whether the guess in (i) was correct by moving R to the position of $C_i$ $(2 \le i \le k)$ and by reading the symbol under $C_i$. (This action is possible because of the sensing function of $M_2$.) Enter an accepting state if and only if the guess was correct.

It will be obvious that $M_2$ exactly accepts the set $T(M_1)$. Q.E.D.

It is unknown whether or not $\mathcal{L}[\text{ASP2-kHTRA}^s] = \mathcal{L}[\text{A2-kHTRA}^s]$, for each $k \ge 1$ , but in the case of three-way 2ASPMHFAs with only universal states, we can get the following results.

Corollary 4.1. For each $k \ge 2$,

(1) $\mathcal{L}[\text{USP2-kHTRA}^s] \subsetneq \mathcal{L}[\text{U2-kHTRA}^s]$, and

(2) $\mathcal{L}[\text{USNSP2-kHTRA}^s] \subsetneq \mathcal{L}[\text{USN2-kHTRA}^s]$.

Proof. Let $T_1$ be the set described in the proof of Theorem 3.1. As shown in the proof of Theorem 3.2, $\overline{T}_1 \notin \bigcup_{1 \le k < \infty} \mathcal{L}[\text{USNSP2-kHSTRA}^s]$. On the other hand, it is easily seen that $\overline{T}_1 \in \mathcal{L}[\text{U2-2HTRA}^s]$ (in fact, $\overline{T}_1$ is accepted by $\text{D2-2HTRA}^s$). From these facts, it follows that the corollary holds. Q.E.D.

## 5. Some Properties of 2ASPMHFA with only Universal States

In this section, we give some properties of 2ASPMHFAs with only universal states (2USPMHFA). We first investigate the relationship among the accepting powers of 2USPMHFAs with different move directions of heads.

Theorem 5.1. For each k≥1 and each Y∈{SP,SNSP},

(1) $\mathcal{L}[\text{UY2-kHTRA}^s] \subsetneq \mathcal{L}[\text{UY2-kHSA}^s]$,

(2) $\bigcup_{1\le r<\infty}\mathcal{L}[\text{UY2-rHTRA}^s] \subsetneq \bigcup_{1\le r<\infty}\mathcal{L}[\text{UY2-rHSA}^s]$,

(3) $\mathcal{L}[\text{UY2-kHSTRA}^s] \subsetneq \mathcal{L}[\text{UY2-kHA}^s]$, and

(4) $\bigcup_{1\le r<\infty}\mathcal{L}[\text{UY2-rHSTRA}^s] \subsetneq \bigcup_{1\le r<\infty}\mathcal{L}[\text{UY2-rHA}^s]$.

Proof. Let $T_1$ be the set described in Theorem 3.1. To prove the theorem, it is sufficient to show that $\overline{T}_1 \in \mathcal{L}[\text{D2-1HA}^s] - \bigcup_{1\le k<\infty}\mathcal{L}[\text{USNSP2-kHSTRA}^s]$. It is easily seen that $\overline{T}_1 \in \mathcal{L}[\text{D2-1HA}^s]$. On the other hand, it is shown in the proof of Theorem 3.2 that $\overline{T}_1 \notin \bigcup_{1\le k<\infty}\mathcal{L}[\text{USNSP2-kHSTRA}^s]$. This completes the proof of the theorem. Q.E.D.

It is unknown whether or not $\mathcal{L}[\text{UY2-kHTRA}^s] \subsetneq \mathcal{L}[\text{UY2-kHSTRA}^s]$ and $\mathcal{L}[\text{UY2-kHSA}^s] \subsetneq \mathcal{L}[\text{UY2-kHA}^s]$ for each Y∈{SP,SNSP} and each k≥1.

We next examine hierarchies based on the number of counting heads.

Theorem 5.2. For each Y∈{SP,SNSP}, k≥2, and each Z∈{STRA,TRA},

$\mathcal{L}[\text{UY2-kHZ}^s] \subsetneq \mathcal{L}[\text{UY2-(k+1)HZ}^s]$.

Proof. For each r≥1 and each m (≥r), let

$R_r(m)=\{x\in\{0,1\}^{(2)+} \mid (\ell_1(x)=1)\ \&\ (\ell_2(x)=m)\ \&\ (x \text{ has exactly } r \text{ "1"s})\}$,

and for each r≥1, let

$A(r)=\{x\in\{0,1\}^{(2)+} \mid (\exists m\ge 2)\ [\ell_1(x)=\ell_2(x)=m]\ \&\ x[(1,1),(1,m)]=x[(2,1),(2,m)]\in R_r(m)\}$.

To prove the theorem, it is sufficient to show that for each k≥2, $\overline{A(2k)} \in \mathcal{L}[\text{DSP2-(k+1)HTRA}^s] - \mathcal{L}[\text{USNSP2-kHSTRA}^s]$. We omit the proof of $\overline{A(2k)} \in \mathcal{L}[\text{DSP2-(k+1)HTRA}^s]$, since it is similar to the proof of Theorem 3 in [3]. Now suppose that there is a USNSP2-kHSTRA M accepting $\overline{A(2k)}$. Then, by using counting arguments similar to those in the proofs of Theorem 3.1

(of this paper) and Theorem 3 in [3], we can find a tape x in $\overline{A(2k)}$ such that there is a sequence of steps of M on x in which M never enters an accepting state (thus x is not accepted by M). This is a contradiction. This completes the proof of the theorem. Q.E.D.

## 6. Conclusions

In addition to the above results, we have got several properties about the classes of the sets recognized by leaf-size bounded 2ASPMHFAs. Leaf-size [6], in a sense, reflects the minimal number of processors which run in parallel in recognizing a given input. These results will be reported elsewhere.

We conclude this paper by giving several interesting open problems.

(1) For each $k \geq 1$, $\mathcal{L}[\text{ASP2-kHTRA}^s] = \mathcal{L}[\text{A2-kHTRA}^s]$ ?

(2) For each $k \geq 2$, $\mathcal{L}[\text{ASP2-kHTRA}^s] \subsetneq \mathcal{L}[\text{ASP2-kHSTRA}^s] \subsetneq \mathcal{L}[\text{ASP2-kHA}^s]$ ?

(3) For each $Y \in \{\text{SP}, \text{SNSP}\}$, each $k \geq 1$, and each $Z \in \{\text{A}, \text{SA}, \text{STRA}, \text{TRA}\}$, $\mathcal{L}[\text{AY2-kHZ}^s] \subsetneq \mathcal{L}[\text{AY2-(k+1)HZ}^s]$ ?

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