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D-変形について

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§1. Regular Polygram

<u>定義</u> 以下の(1),(2)を満たす diagram Σ を <u>DS-diagram</u>, または <u>regular polygram</u> という:

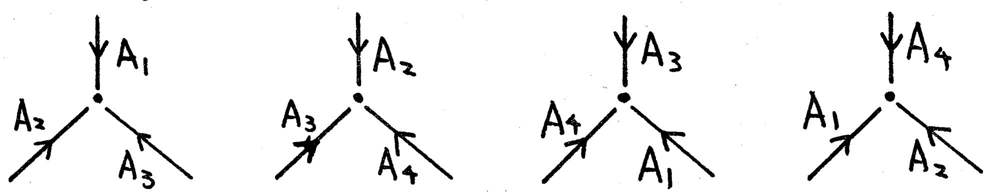
(1) $\sigma \in \Sigma$ が 2-gram $\Rightarrow$ $\mu(\sigma) = 2$

$A \in \Sigma$ が 1-gram $\Rightarrow$ $\mu(A) = 3$

$v \in \Sigma$ が 0-gram $\Rightarrow$ $\mu(v) = 4$

但し μ は m-gram の重複度を表す.

(2) 0-gram v の近傍(4個所)の様子は



$A_i \neq A_j$ if $i \neq j$ (向きも込めての意味なので たとえば $A_2 = A_1^{-1}$ ということはあり得る)

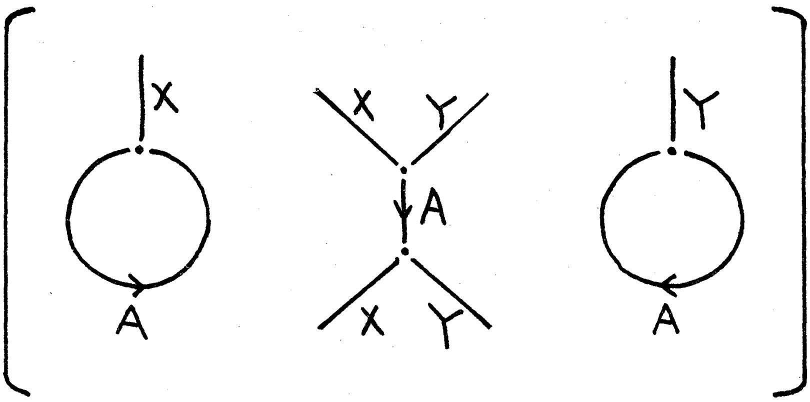
となっている.

<u>Prop</u> DS-diagram は polygram である.

註) DS-diagram を regular polygram と呼ぶことが多いであろう.

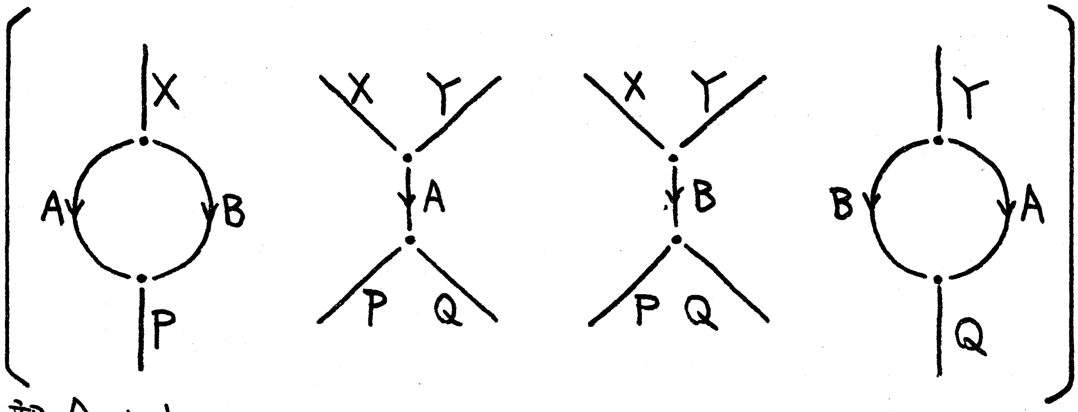
regular polygram Σ について考える. J を Σ の monogram, $A, B, \cdots \in \Sigma$ を 1-gram とする.

(i) $\dot{J} = A$ (i.e. J は 1辺形) ならば Σ は必然的に



の部分をもつ.

(ii) $\dot{J} = AB^{-1}$ (i.e. J は 2辺形) ならば Σ は必然的に

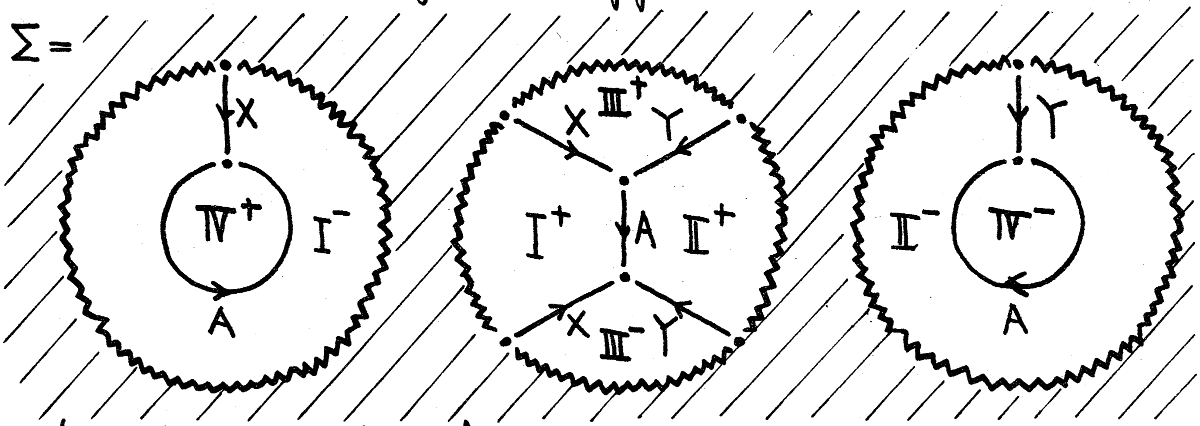


の部分をもつ.

§2. D_1-変形は δ-変形.

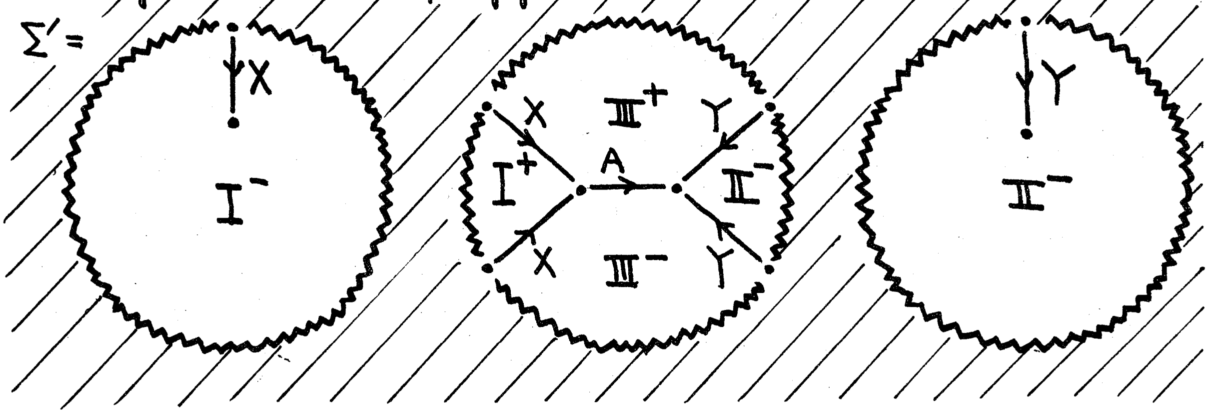
<u>定義</u> Σ を regular polygram とする.

$\Sigma =$



を (regular でない) polygram:

$\Sigma' =$

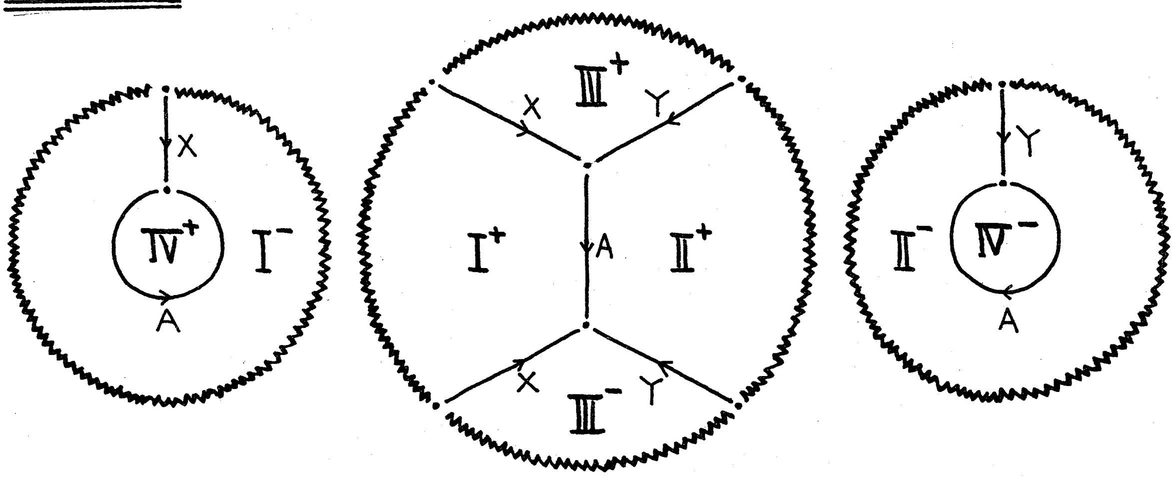


に変える変形を <u>D_1-変形</u> という.

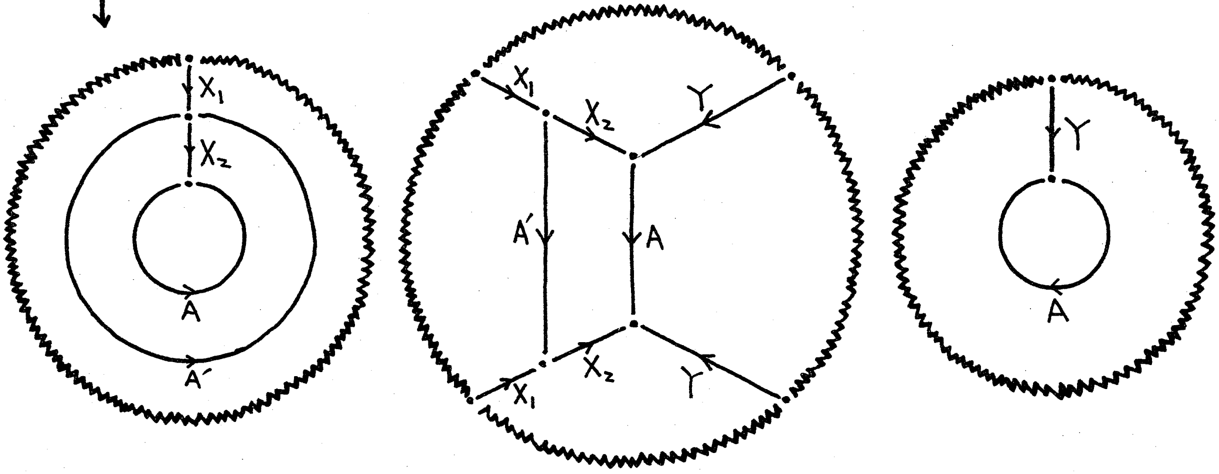
<u>Prop.</u> D_1-変形は δ-変形 (すなわち基本変形の合成) である.

このProp. は以下の図で示される.

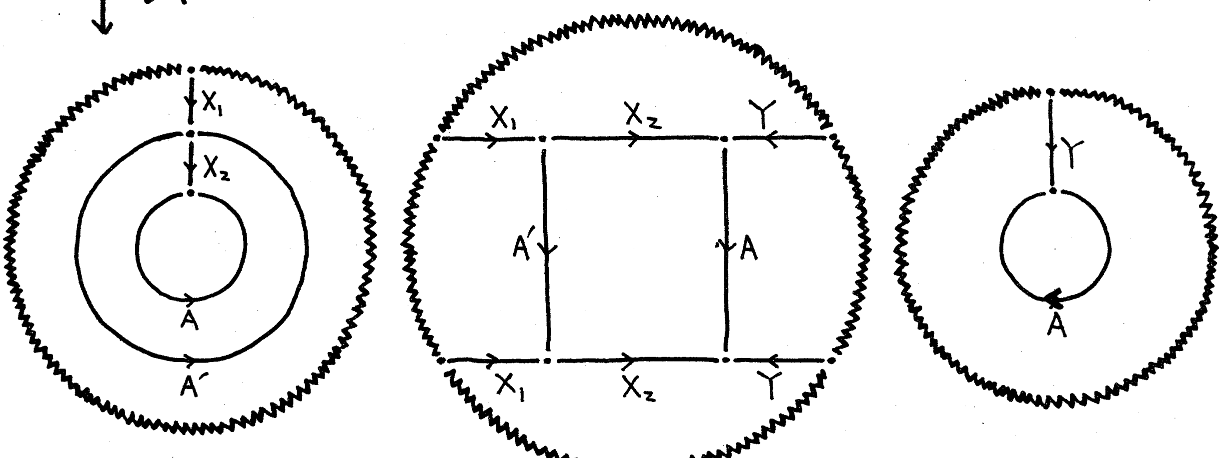
START



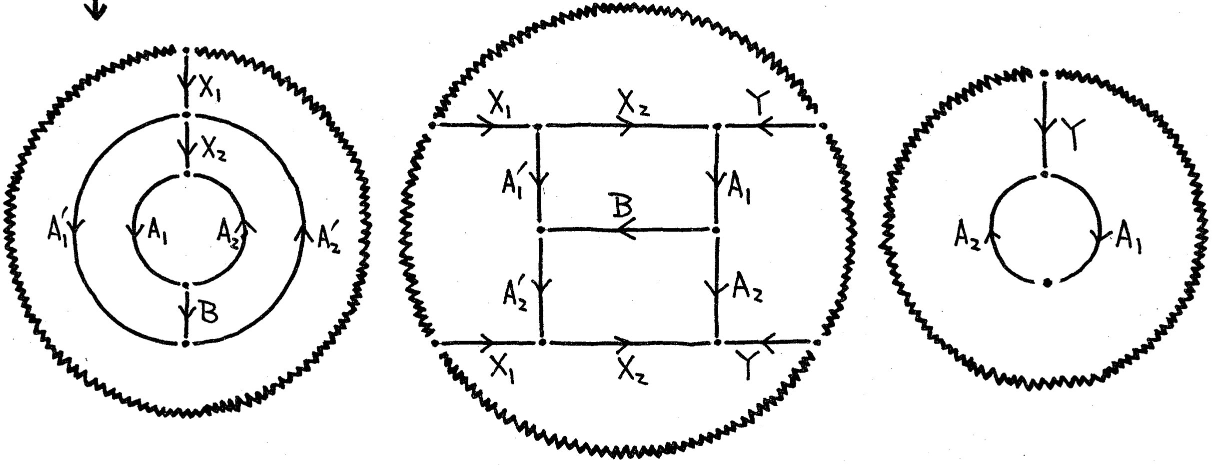
$\downarrow$ $\mathcal{S}$: $X \to X_1X_2$, A'の添加.



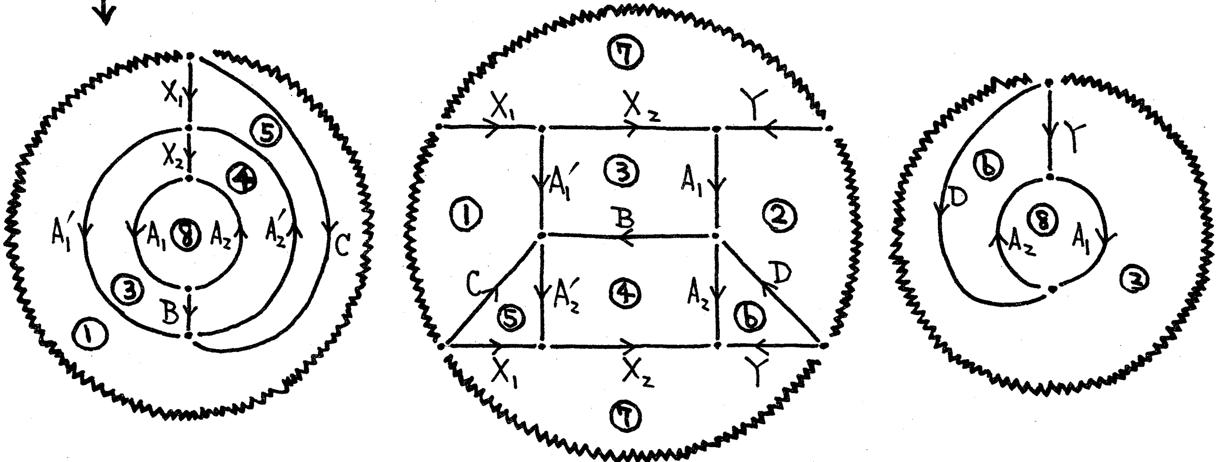
$\downarrow$ $\mathcal{A}$



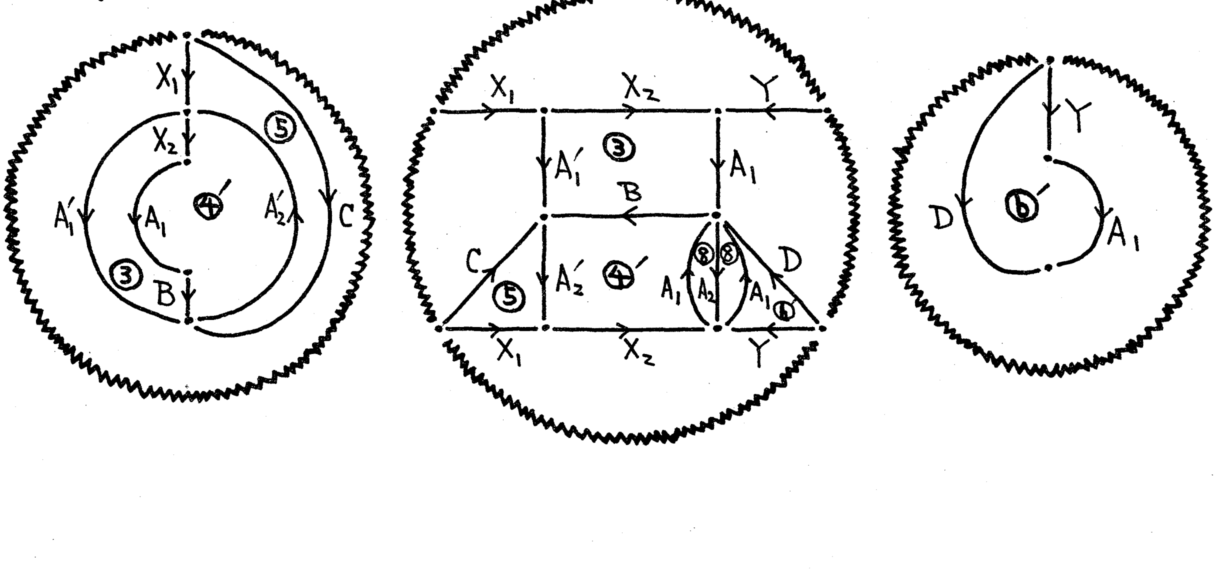
$\downarrow$ δ : $A \to A_1 A_2$, $A' \to A'_1 A'_2$, B の添加.



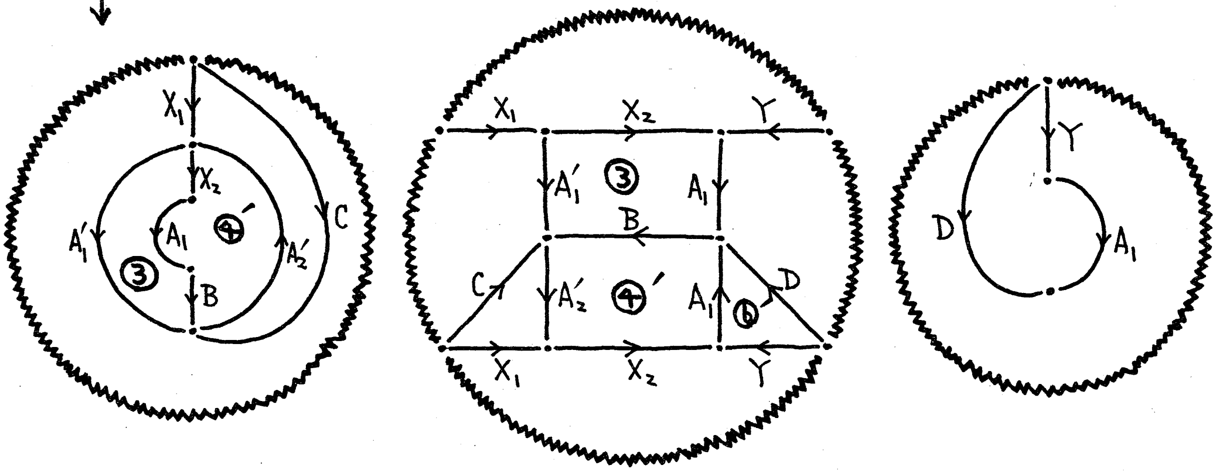
$\downarrow$ δ : C, D の添加.



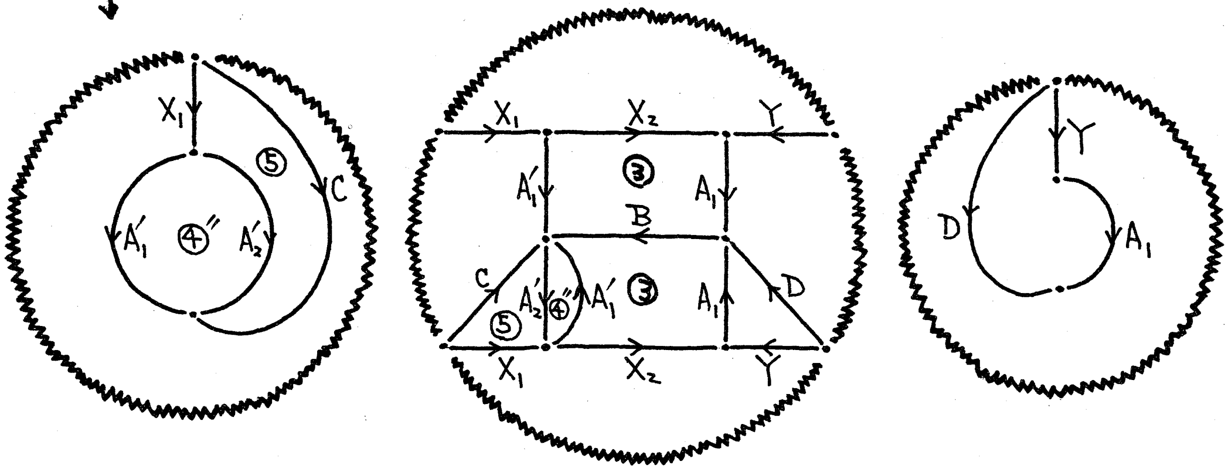
$\downarrow$ g : $\begin{cases} (⑧, ④) \to (⑧, ④') \\ (⑧, ⑥) \to (⑧, ⑥') \end{cases}$



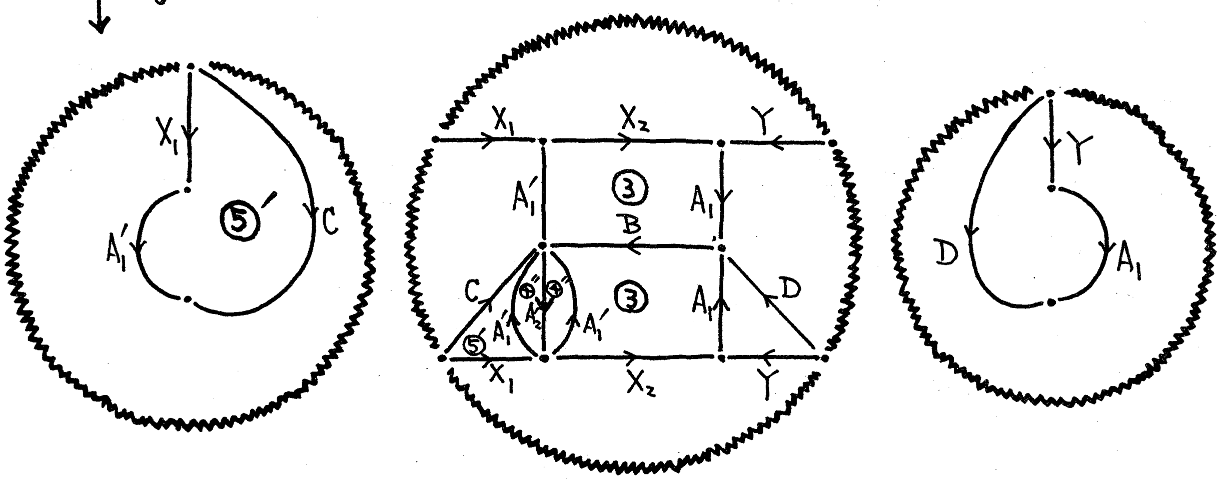
↓ C : ⑧の消去



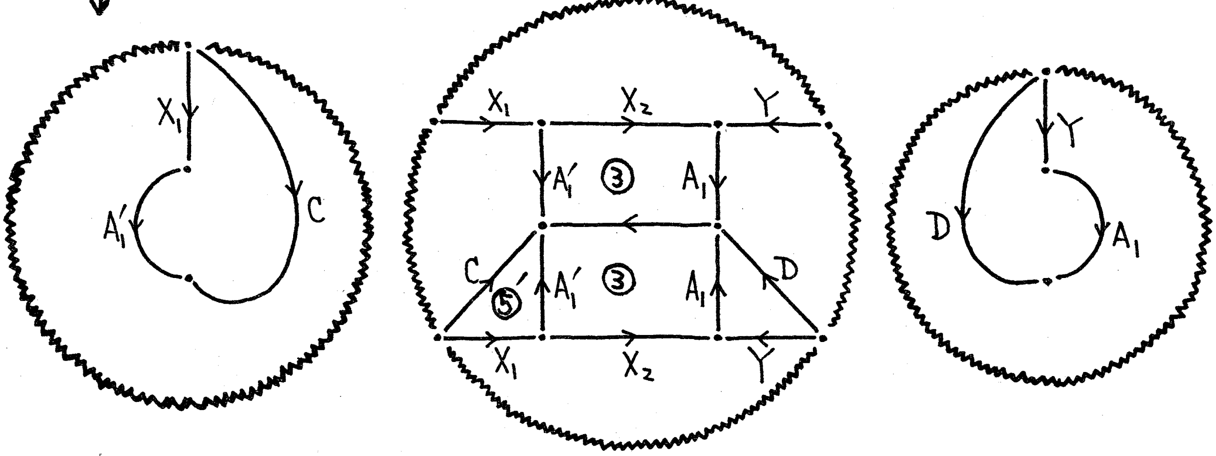
↓ O_J : (③, ④′) → (③, ④″)



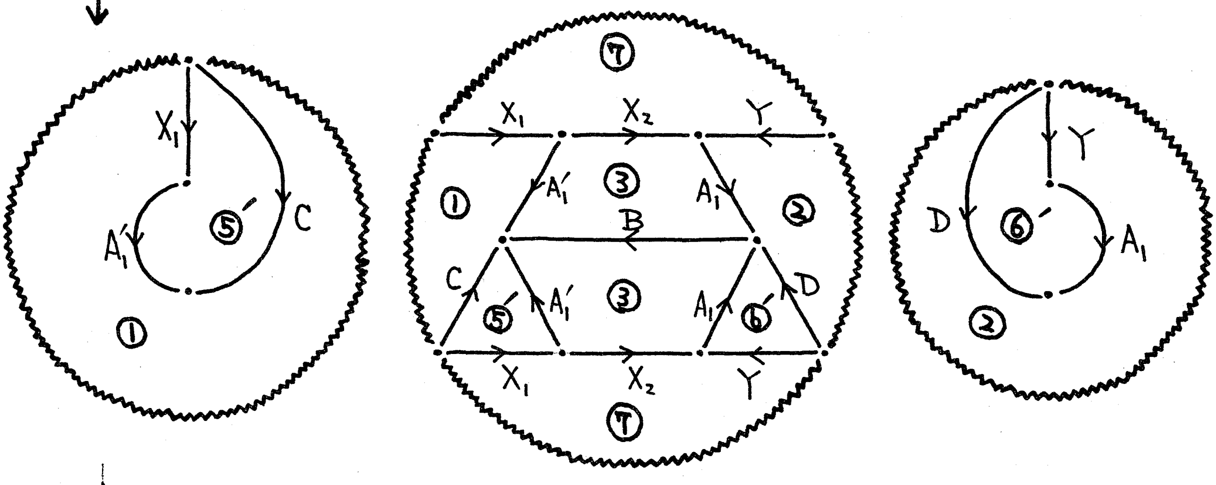
↓ O_J : (④″, ⑤) → (④″, ⑤′)



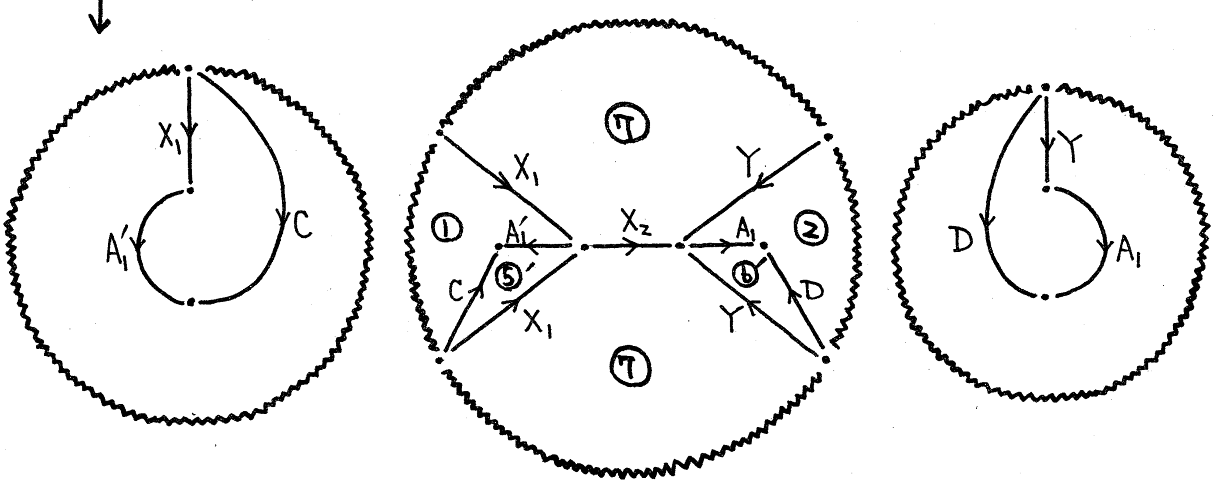
C : ④″ の消去



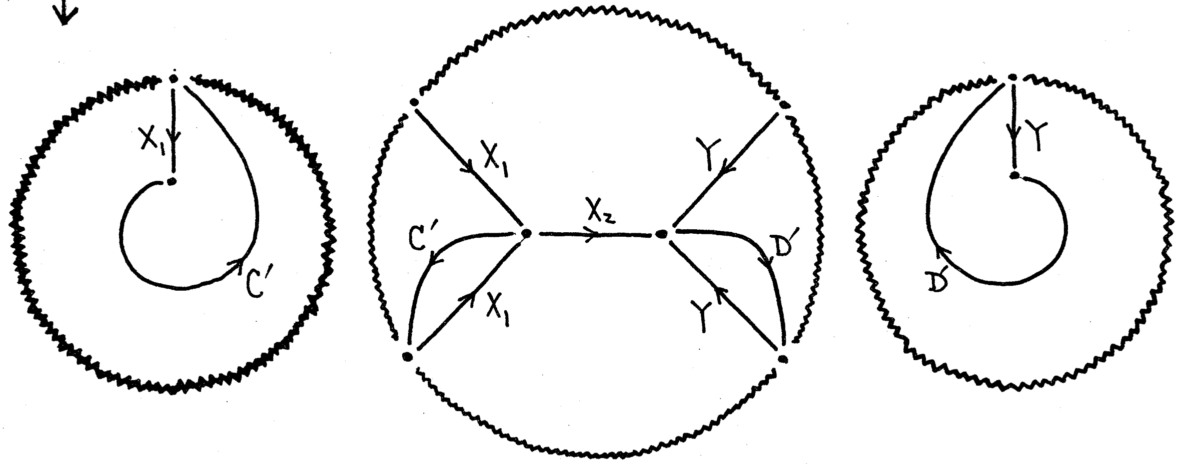
A



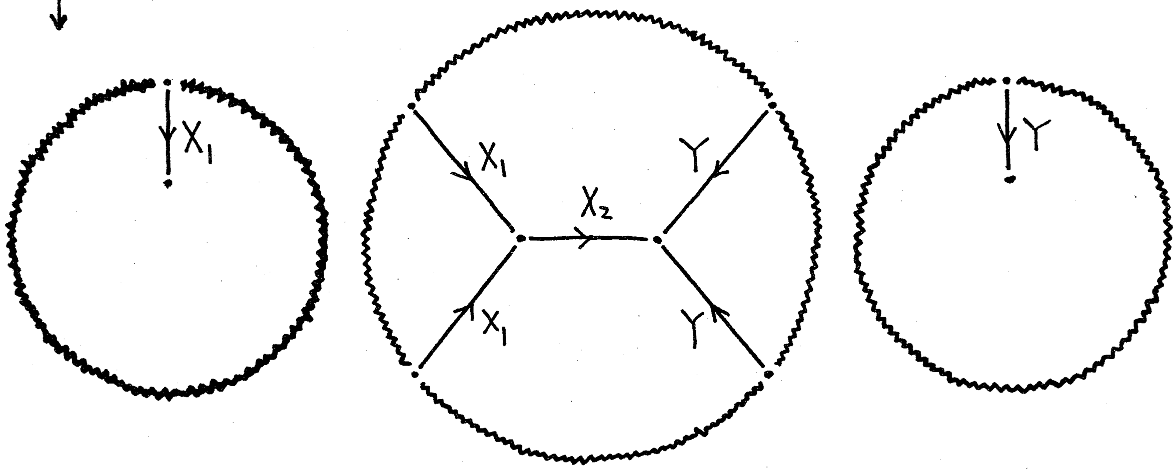
C : ③ の消去.



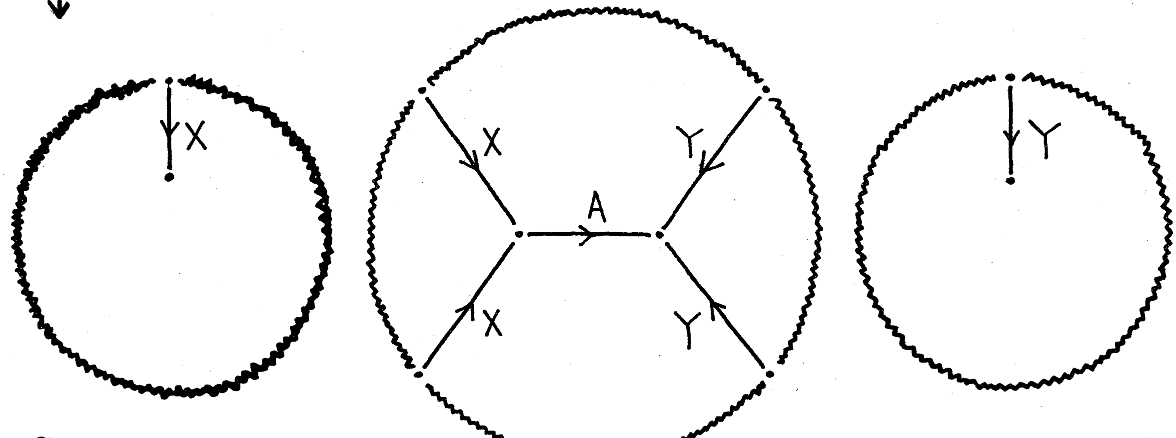
↓ $\mathcal{S}$: $A_1' C^{-1} \to C'$, $A_1 D^{-1} \to D'$



↓ $\mathcal{S}$: C', D' の除去.



↓ $\mathcal{I}$: $X_1 \to X$, $X_2 \to A$.

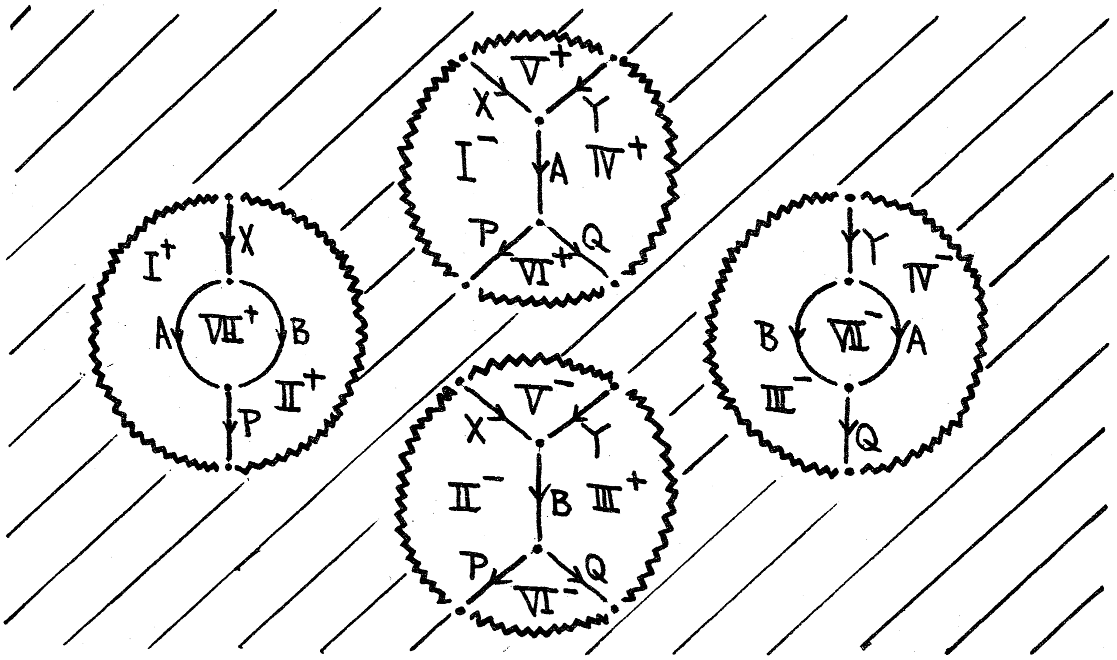


GOAL

§3. D_2-変形は δ-変形

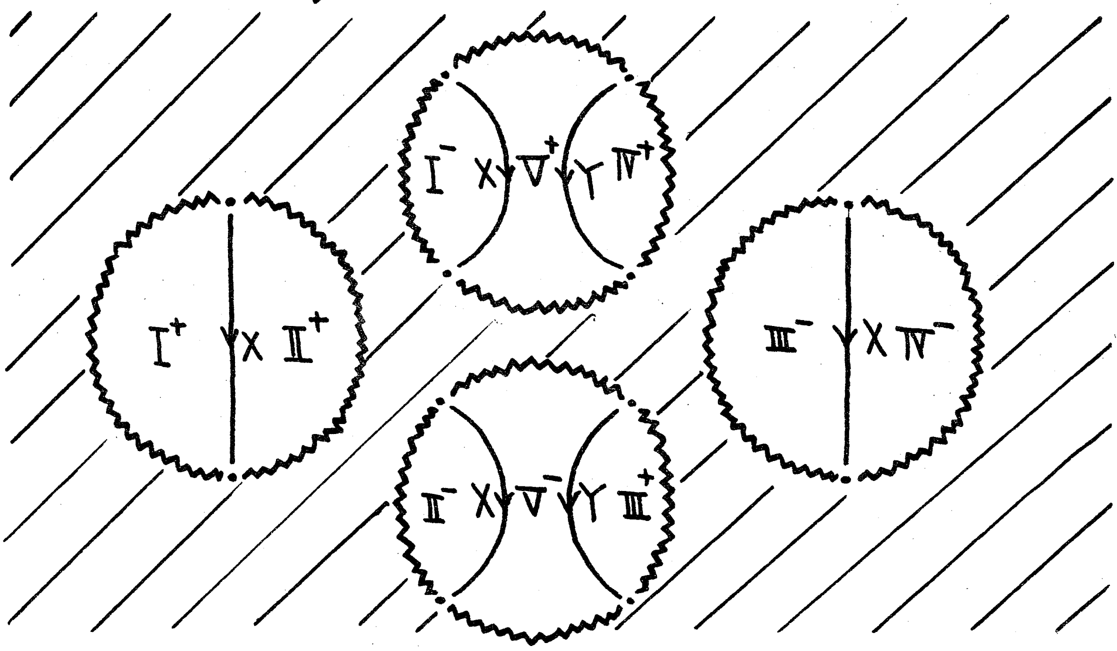
<u>定義</u> regular polygram Σ を regular polygram Σ' に変える次のような変形を <u>D_2-変形</u> という:

$\Sigma =$

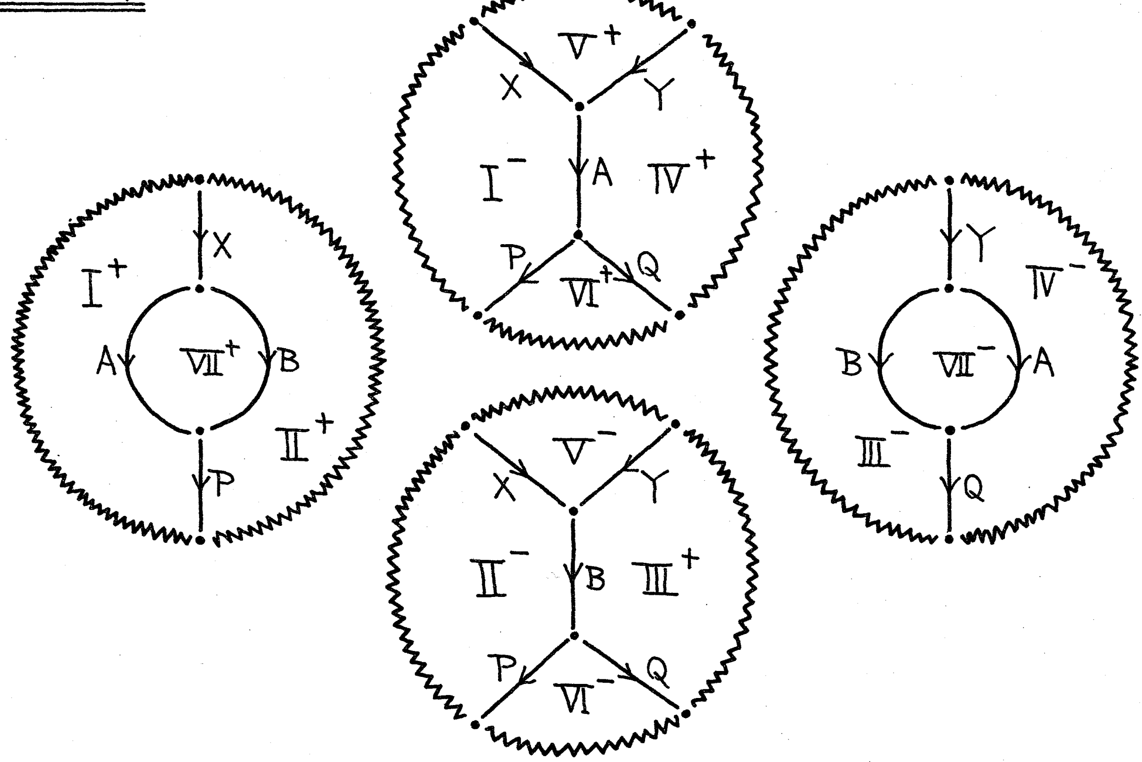


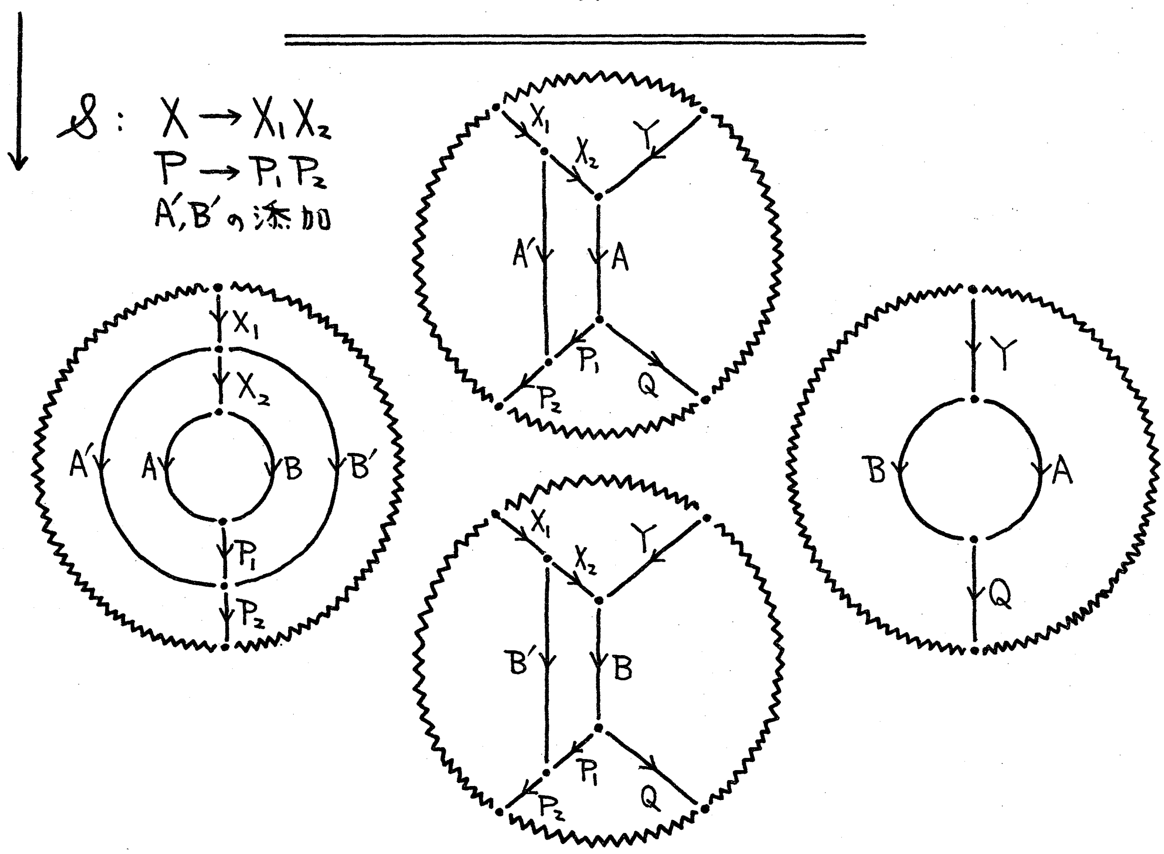
↓ D_2-変形

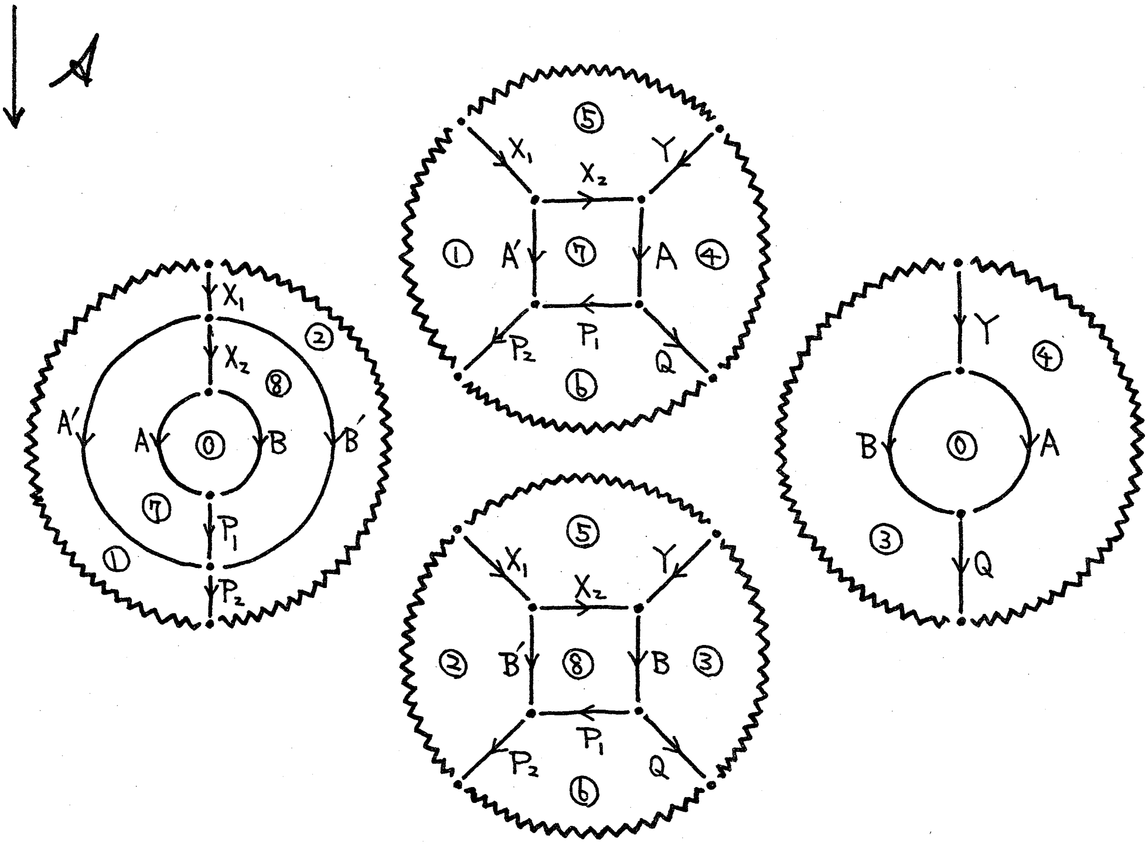
$\Sigma' =$

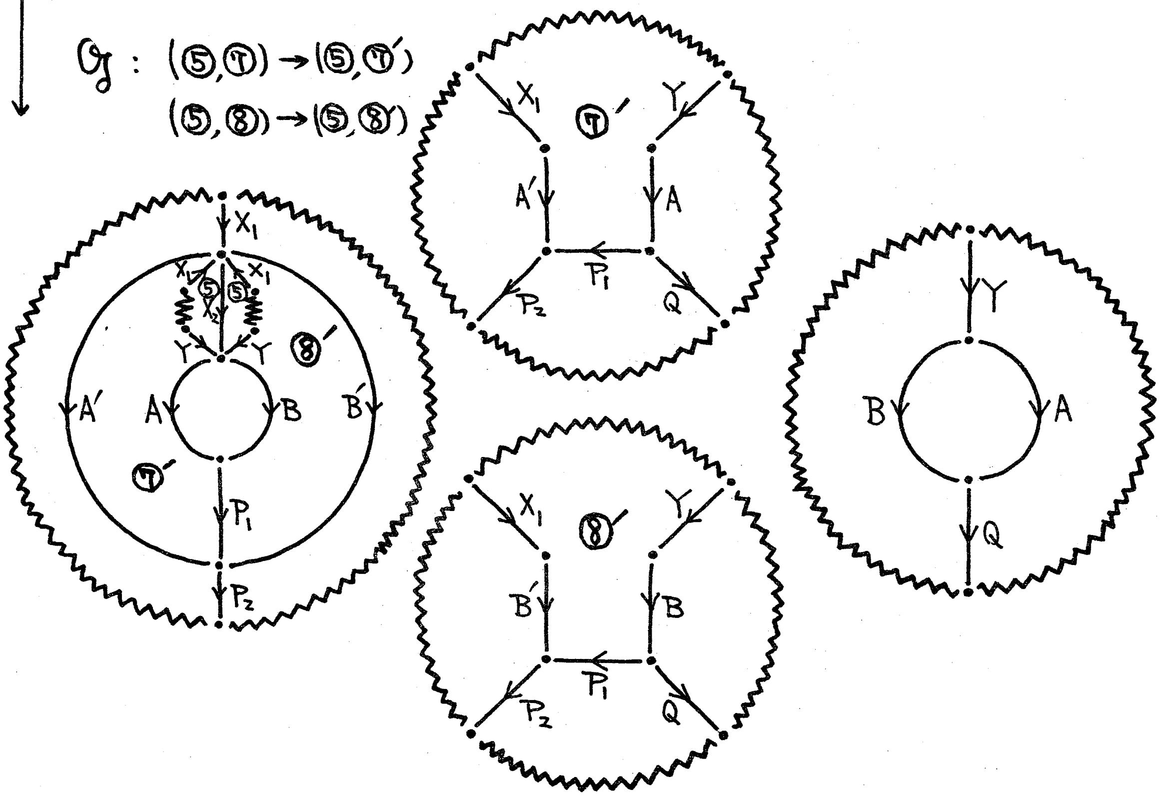


START

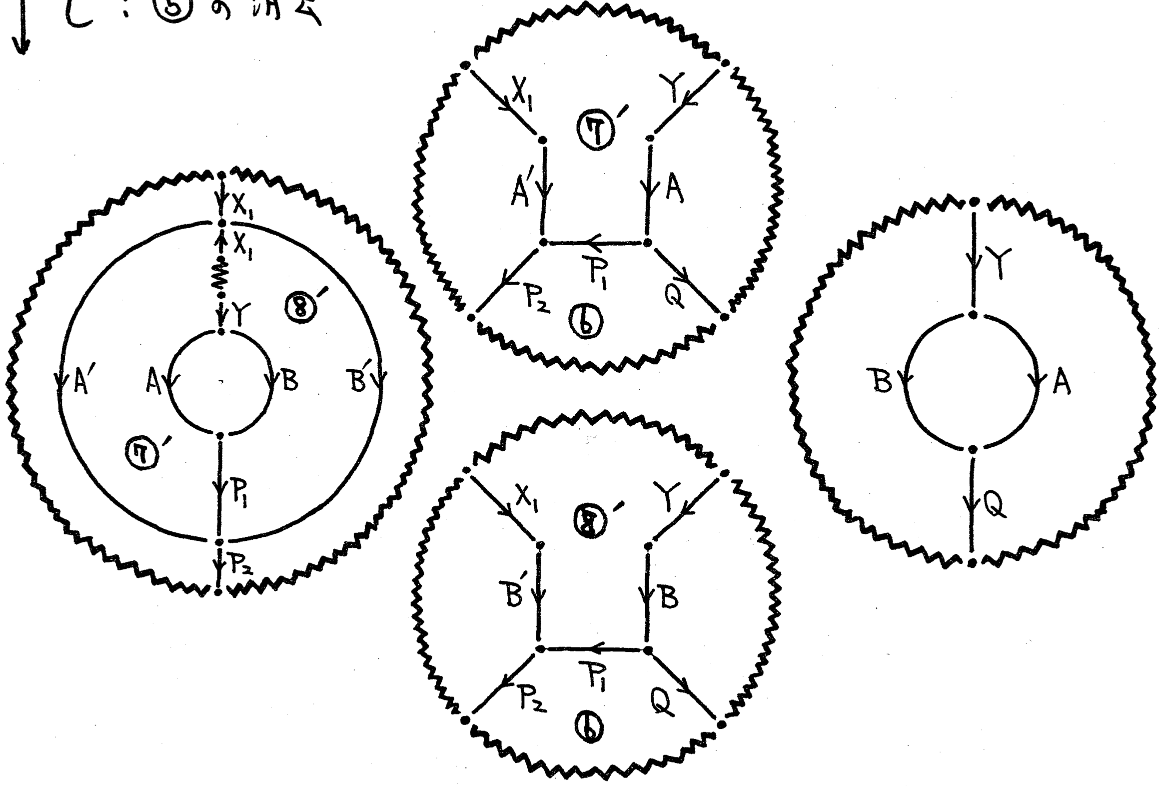

X
Y
A
B
P
Q
Ⅰ+
Ⅱ+
Ⅶ+
Ⅴ+
Ⅰ−
Ⅳ+
Ⅵ+
Ⅳ−
Ⅶ−
Ⅲ−
Ⅴ−
Ⅱ−
Ⅲ+
Ⅵ−


δ : X → X1X2
P → P1P2
A′,B′の添加
X1
X2
P1
P2
A
B
A′
B′
Y
Q

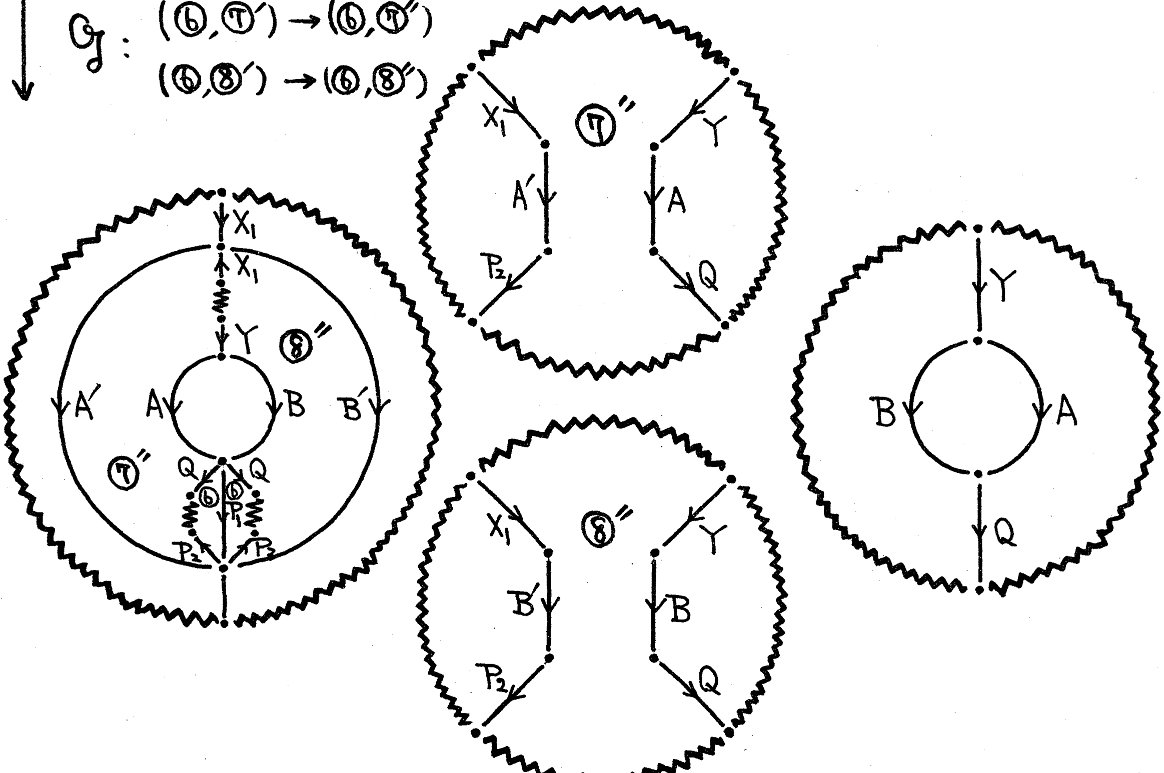

A
⑤
X_1
Y
X_2
①
A'
⑦
A
④
P_2
P_1
Q
⑥
X_1
X_2
②
⑧
A'
A
⓪
B
B'
⑦
P_1
①
P_2
Y
④
B
⓪
A
③
Q
⑤
X_1
Y
X_2
②
B'
⑧
B
③
P_1
P_2
Q
⑥


$\mathcal{O}_J$: (⑤,⑦) → (⑤,⑦′)
(⑤,⑧) → (⑤,⑧′)
X_1
⑦′
Y
A'
A
P_1
P_2
Q
X_1
X_1
X_1
⑤
⑤
X_2
Y
Y
⑧′
A'
A
B
B'
⑦′
P_1
P_2
Y
B
A
Q
X_1
⑧′
Y
B'
B
P_1
P_2
Q

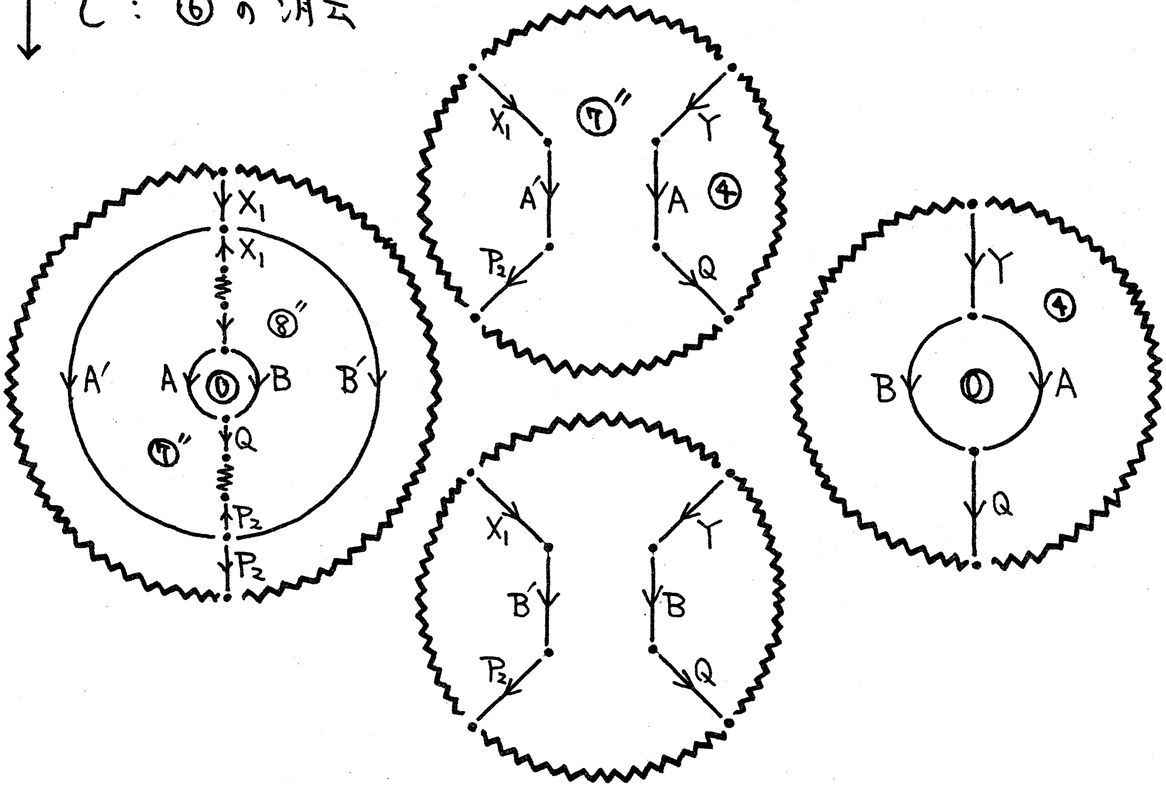
C : ⑤の消去



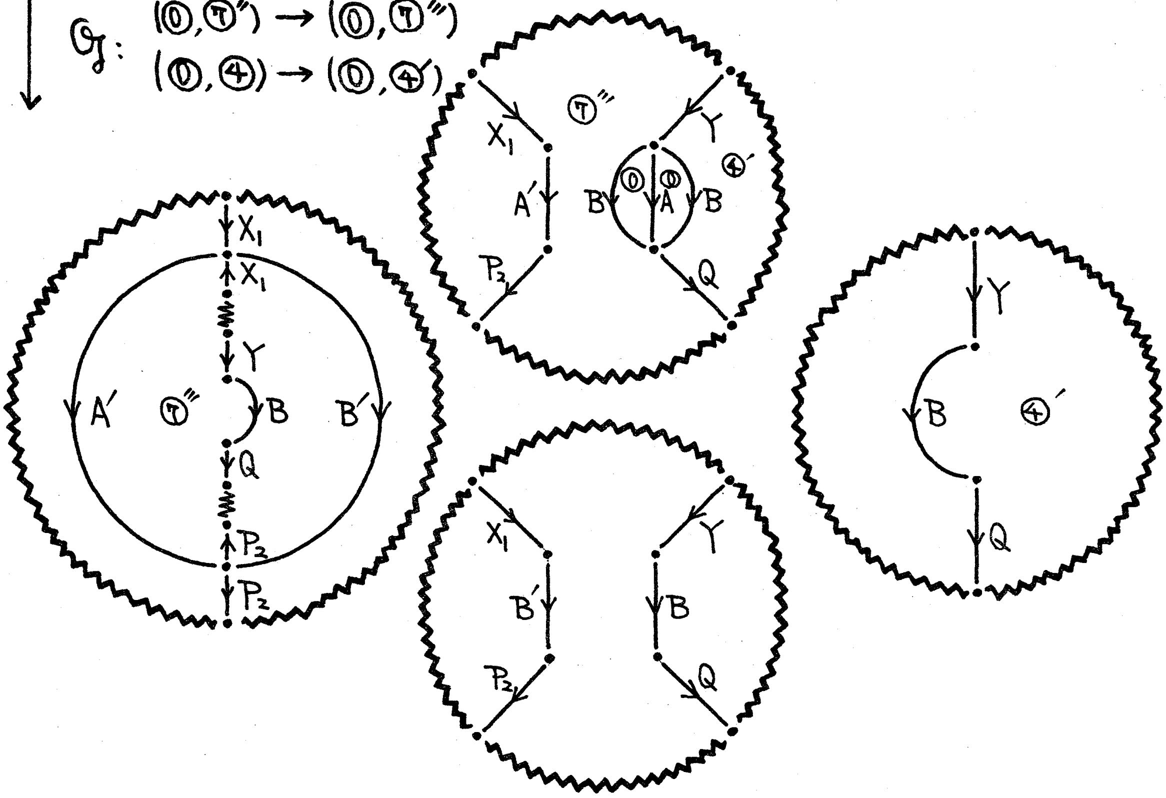
G : (⑥,⑦') → (⑥,⑦'')
(⑥,⑧') → (⑥,⑧'')



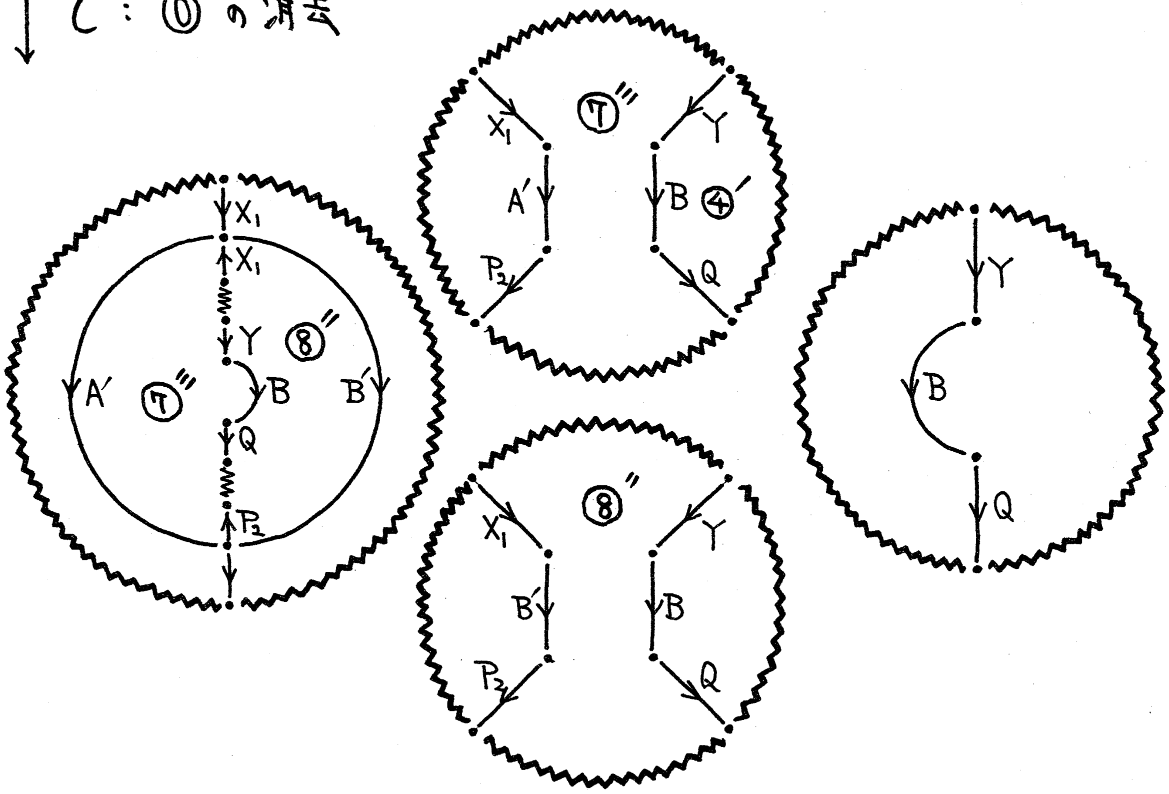
ℓ : ⑥ の消去



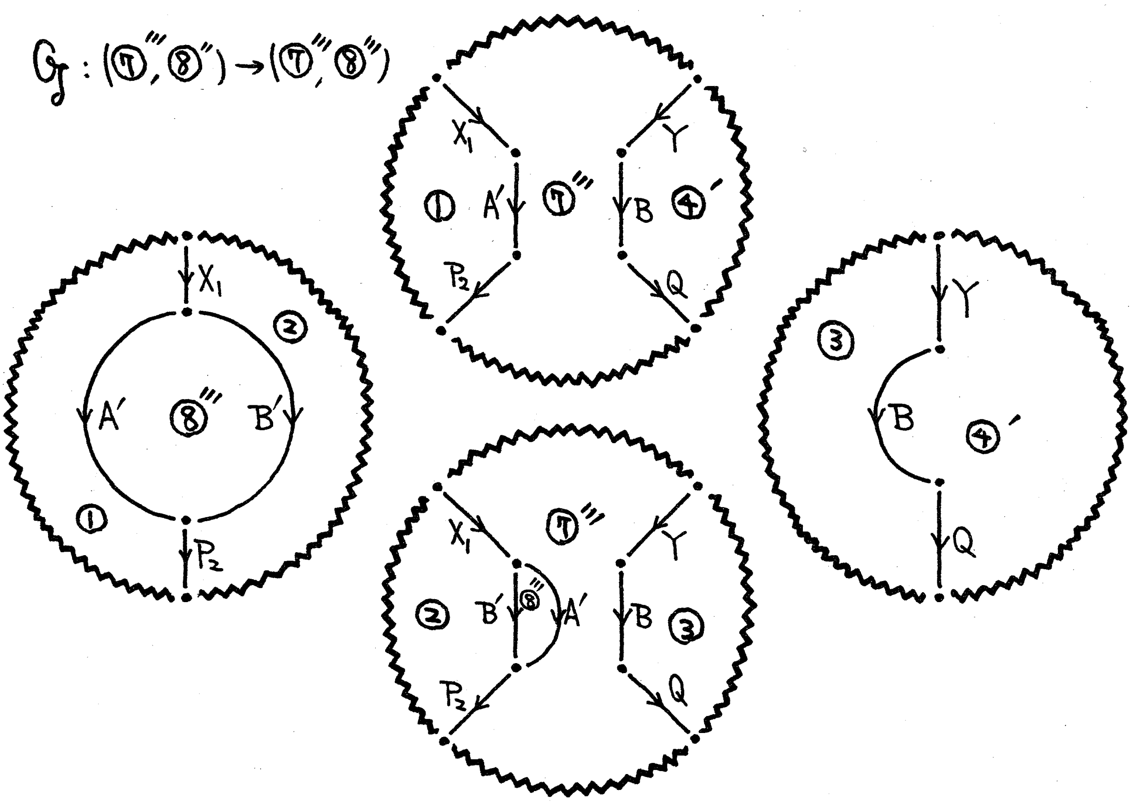
g: (⓪, ⑦″) → (⓪, ⑦‴)
(⓪, ④) → (⓪, ④′)



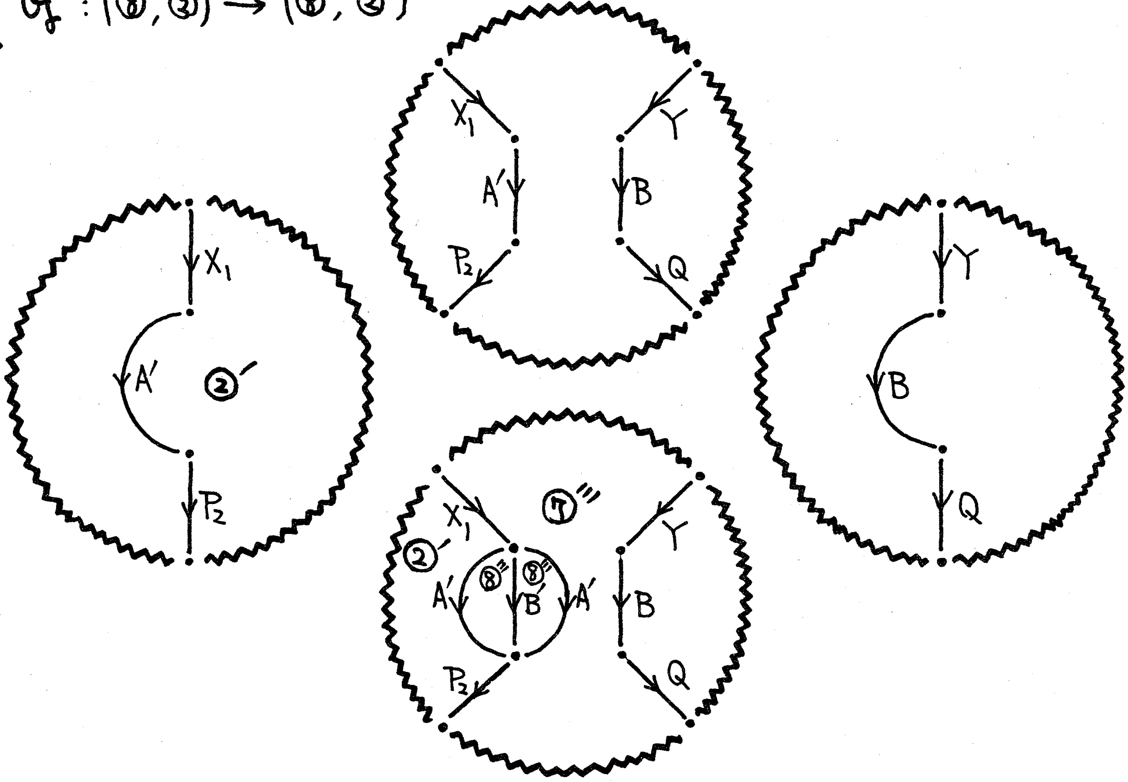
C : 0 の消去



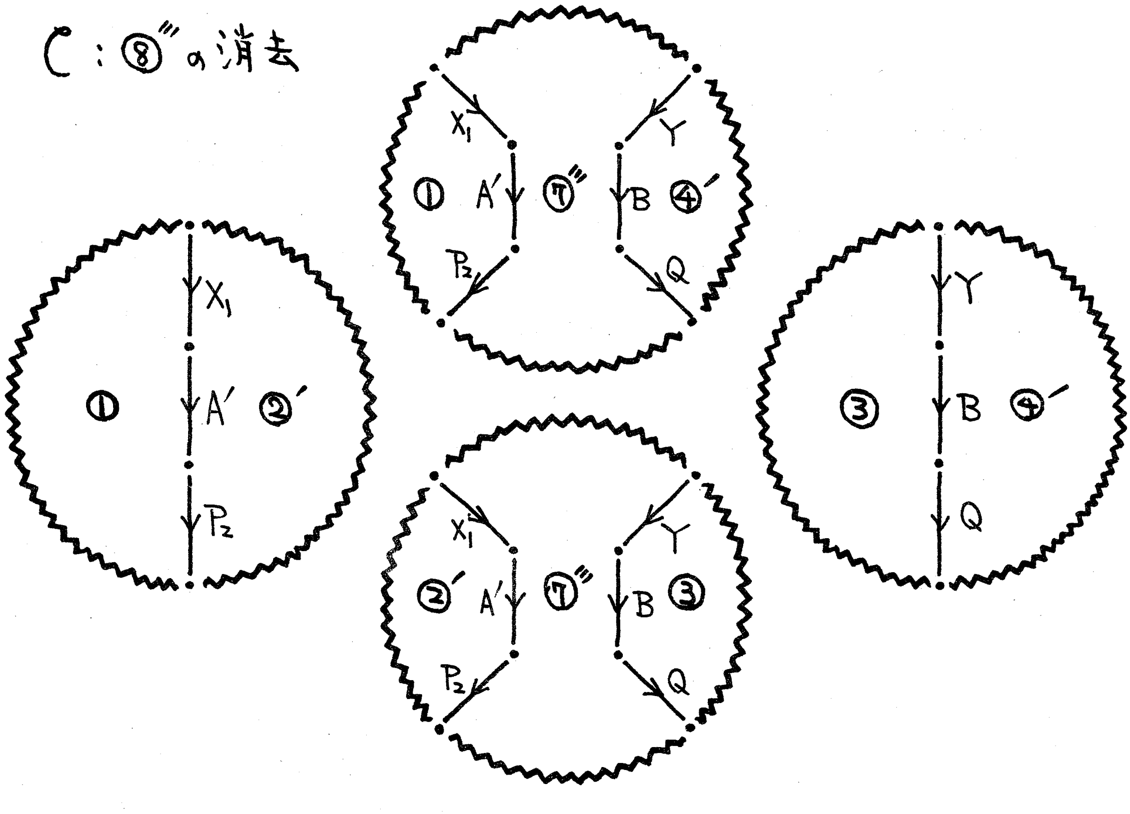
$G : (7''', 8'') \to (7''', 8''')$



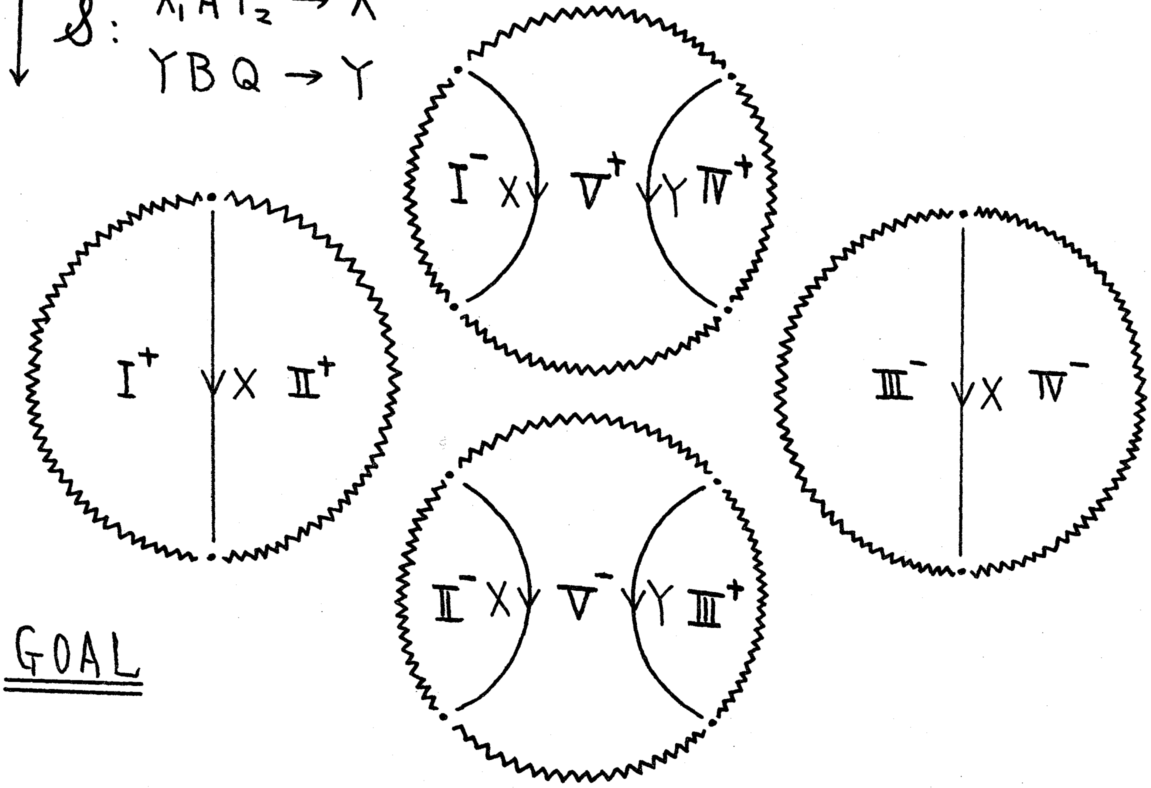
Gj : (⑧''', ②) → (⑧''', ②')



C : ⑧''の消去



$$\Big\downarrow\ \delta:\ \begin{array}{l} X_1 A' P_2 \to X \\ YBQ \to Y \end{array}$$



GOAL

したがって

Prop D_2-変形は regular polygram を regular polygram に移す δ-変形である。

(以上)