数理解析研究所講究録
第 567 巻 1985 年 104-122

On the Unit Groups of Burnside Rings.

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1. Introduction.

Let G be a finite group and let $\underline{\underline{Set}}_f{}^G$ be the category of finite (right) G-sets and G-maps. The Grothendieck ring of this category (with respect to coproduct + and product ×) is called the Burnside ring of G and is denoted by A(G).

A super class function is a map of the set of subgroups of G to $\underline{\underline{Z}}$ which is constant on each conjugate class of subgroups. Let $\widetilde{A}(G) = \underline{\underline{Z}}^{Cl(G)}$ be the ring of super class functions. For any subgroup S of G, the map $[X] \longrightarrow |X^S|$ extends to a ring homomorphism $\varphi_S : A(G) \longrightarrow \underline{\underline{Z}}$, and so we have a ring homomorphism

$$\varphi = \prod_{(S)} \varphi_S : A(G) \longrightarrow \widetilde{A}(G) = \underline{\underline{Z}}^{Cl(G)}.$$

This map is injective. Thus we can identify any element x of A(G) with the super class function $\varphi(x)$, and so we simply write $x(S) := \varphi(x)(S) = \varphi_S(x)$ for a subgroup S. See tom Dieck [Di79] Chapter 1.

Now, by geometric methods, tom Dieck proved that for an $\underline{\underline{R}}G$-module V, the function

$$u(V) : S \longmapsto \operatorname{sgn} \dim V^S$$

belongs to the Burnside ring A(G), where $\operatorname{sgn} n := (-1)^n$ ([Di79] Proposition 5.5.9). The first purpose of this paper is to prove this fact by purely algebraic methods. In fact we shall prove the following theorem in Section 2.

<u>Theorem A</u>. <u>Let</u> G <u>be a finite group and let</u> V <u>be a</u> $\underline{\underline{C}}G$-<u>module with real valued character</u>. <u>Then the function</u>

$$u(V) : S \longmapsto \operatorname{sgn} \dim_{\underline{\underline{C}}} V^S$$

<u>is a member of the Burnside ring</u> A(G).

Since clearly $u(V)^2 = 1$ and $u(V \oplus W) = u(V)\, u(W)$, we have a group homomorphism into the unit group :

$$u = u_G : \overline{R}(G) \longrightarrow A(G)^*,$$

where $\overline{R}(G)$ is the ring of real valued virtual characters of G. We call this map u_G a <u>tom Dieck homomorphism</u>.

There are various maps between Burnside rings (and unit groups), and the assignment $A^* : H \longmapsto A(H)^*$ together with restrictions and multiplicative inductions forms a so called G-functor (= a Mackey functor from the category of finite G-sets) and further that the tom Dieck homomorphism gives a morphism between G-functors. Since A^* is a G-functor, we have that $A(G)^*$ is a module over $A(G)$ (and also over $A(G)_{(2)}$, the localization at 2). In fact, the action $A(G)^* \times A(G) \longrightarrow A(G)^*$ is induced by the exponentaial map $(Y, X) \longrightarrow Y^X$ (the set of all maps of X to Y). From the theory of Burnside rings and G-functors, we can show some transfer theorems about $A(G)^*$. See Section 3. The proof will appear in another paper.

Notation and terminology. We always denote by G a finite group. The set of G-conjugate classes (H) of subgroups H of G is denoted by Cl(G). For subsets A, B of G, we mean by $A =_G B$ (resp. $A \leq_G B$) that A and B are conjugate in G (resp. A is G-conjugate to a subgroup of B). We put $A^g := g^{-1}Ag$. When a group G acts on a set X, we denote by X^G the set of elements fixed by G. The ordinary character ring of G is denoted by R(G). For a ring R, the unit group of R is denoted by R^*. The inner product of characters χ and θ is denoted by $\langle \chi, \theta \rangle$. Other notation and terminology are standard. See [Go68], [Di79].

2. Proof of Theorem A.

In this section, we prove Theorem A. As in the introduction, let G be a finite group and let $Cl(G)$ denote the set of conjugate classes of subgroups of G. We mean by (S) the class of a subgroup S. We set $WS := N_G(S)/S$ for a subgroup S of G.

Lemma 2.1. *There is an exact sequence of abelian groups*:

$$0 \longrightarrow A(G) \xrightarrow{\varphi} \mathbb{Z}^{Cl(G)} \xrightarrow{\psi} \prod_{(S)} (\mathbb{Z}/|WS|\mathbb{Z}) \longrightarrow 0,$$

where φ *is the injective ring homomorphism given in the introduction, and for a super class function* x, *the* S-*component of* $\psi(x)$ *is defined by*

$$(x)_S := \sum_{gS \in WS} x(\langle g \rangle S) \quad \bmod |WS| .$$

This lemma is well-known and its proof is found in, for example, tom Dieck [Di79] 1.3. We can now prove Theorem A.

Proof of Theorem A. Let χ be the real valued character afforded by the $\mathbb{C}G$-module V, and let $u(V)$ be the present super class function:

$$u(V) : (S) \longmapsto \operatorname{sgn} \dim V^S.$$

By Lemma 2.1, we must show that for each subgroup S of G,

(1) $$\sum_{gS \in WS} u(V)(\langle g \rangle S) \equiv 0 \mod |WS|.$$

Let χ' be the character afforded by the $\mathbb{C}WS$-module V^H, so that by an easy representation theory, we have that

$$\chi'(gS) = \frac{1}{|S|} \sum_{h \in S} \chi(gh), \qquad gS \in WS,$$

and so χ' is also a real valued character. Thus in order to prove (1), we may assume that $S = 1$. Set $u_\chi(g) := u(V)(\langle g \rangle)$, then it has the value

$$u_\chi(g) = \operatorname{sgn} \langle \chi_{|\langle g \rangle}, 1_{\langle g \rangle} \rangle,$$

where $\langle \ , \ \rangle$ stands for the inner product of characters. Now when $S = 1$, (1) becomes

(2) $$\sum_{g \in G} u_\chi(g) \equiv 0 \mod |G|.$$

In order to prove (2), it will suffice to show that u is a virtual character of G. In fact, we can show that

(3) $$u_\chi = (-1)^{\chi(1)} \det \chi,$$

where $\det \chi$ is the linear character of G defined by the composition

$$\det \chi : G \longrightarrow GL(V) \xrightarrow{\det} \mathbb{C}^*.$$

See Yoshida [Yo78]. In order to prove (3), we may assume that G is cyclic. Since $u_{\chi+\theta} = u_\chi u_\theta$ and $\det(\chi+\theta) = \det\chi\cdot\det\theta$, we may further assume that either χ is a real valued linear character or $\chi = \lambda + \bar{\lambda}$ for some nonreal linear character λ. In the first case, we have that $\langle \chi_{|\langle g\rangle}, 1_{\langle g\rangle}\rangle = 1$ if g is in the kernel of and $= 0$ otherwise, and so $u = -1$ or $+1$, respectively. Since $\det\chi = \chi$, (3) holds in this case. Next assume that $\chi = \lambda + \bar{\lambda}$, so that $\langle \chi_{|\langle g\rangle}, 1_{\langle g\rangle}\rangle = 0$ or 2 and $\det\chi = 1_G$. Thus (3) holds also in this case. The theorem is proved.

3. Some transfer theorems for the unit groups.

Let p be a prime. We put

$$\mathbb{Z}_{(p)} := \{ a/b \in \mathbb{Q} \mid a \in \mathbb{Z},\ b \in \mathbb{Z} - p\mathbb{Z} \},$$
$$A(G)_{(p)} := \mathbb{Z}_{(p)} \otimes_{\mathbb{Z}} A(G).$$

For a finite group H, the subgroup generated by all p'-elements of H is denoted by $O^p(H)$. When $O^p(Q) = Q$ (that is, Q has no normal subgroup of index p), the group

Q is called to be p-<u>perfect</u>. Let $Cl_p(G) \subseteq Cl(G)$ denote the classes of p-perfect subgroups.

There is a one-to-one correspondence between primitive idempotents of $A(G)_{(p)}$ and $Cl_p(G)$ (cf. [Di79] 1.4). An explicit formula of primitive idempotents was obtained by Gluck [Gl81] and Yoshida [Yo83]. Let μ be the Mobius function of the subgroup lattice of G and δ_G the function defined by

$$\delta_G(H, K) := \begin{cases} 1 & \text{if } H =_G K \\ 0 & \text{otherwise.} \end{cases}$$

Each primitive idempotent of $A(G)_{(p)}$ is then written in the form

$$e^p_{G,Q} = \sum_{(D)\in Cl(G)} \lambda_{G,Q}(D)\ [D \backslash G],$$

where $(Q) \in Cl_p(G)$ and

$$\lambda_{G,Q}(D) := \frac{1}{|N_G(D)|} \sum_{K \leq G} \mu(D, K)\ \delta_G(O^p(K), Q).$$

As a super class function, $e^p_{G,Q}$ has the value

$$e^p_{G,Q}(S) = \begin{cases} 1 & \text{if } O^p(S) =_G Q \\ 0 & \text{otherwise.} \end{cases}$$

For the finite group G, let $\underline{\underline{Set}}_f{}^G$ denote the category of finite (right) G-sets and G-maps. For two G-sets X and Y, let Y^X be the G-set consisting of all mappings of X to Y with G-action defined by $\alpha^g(x) := \alpha(xg^{-1})\cdot g$. For an element $a = [A] - [B]$ of $A(G)$, we furthermore define the exponential map

$$(-)^a : A(G)^* \longrightarrow A(G)^* ; \; u \longmapsto u^{A+B}.$$

We often write $u \uparrow a$ for u^a. By this action, $A(G)^*$ is an $A(G)_{(2)}$-module whose annihilator contains $2A(G)_{(2)}$.

<u>Theorem B</u>. (i) <u>There is a decomposition</u>

$$A(G)^* = \prod_{(Q)} A(G)^* \uparrow e^2_{G,Q},$$

<u>where</u> (Q) <u>runs over</u> $Cl_2(G)$, <u>classes of</u> 2-<u>perfect subgroups</u>.

(ii) <u>Let</u> Q <u>be a</u> 2-<u>perfect subgroup of</u> G <u>and let</u> P <u>be a subgroup of</u> $N := N_G(Q)$ <u>such that</u> P/Q <u>is a Sylow</u> 2-<u>subgroup of</u> N/Q. <u>Then there are group isomorphisms</u>:

$$A(G)^* \uparrow e^2_{G,Q} \cong A(N)^* \uparrow e^2_{N,Q} \cong (A(P)^*)^N \uparrow e^2_{P,Q}.$$

<u>where the last group is the subgroup of</u> $A(P)^* \uparrow e^P_{P,Q}$ <u>consisting of all elements</u> x <u>such that</u>

$$res^P_{P^n \cap P}\, con^n_P(x) = res^P_{P^n \cap P}(x)$$

for any element n of N.

Theorem C. Let N be a finite group with 2-perfect normal subgroup Q. Put $W := N/Q$. Let $\bar{P} = P/Q$ be a Sylow 2-subgroup of W. Then the following groups are isomorphic:

(a) $A(N)^* \uparrow e^2_{N,Q}$,

(b) $\{\bar{u} \in A(W)^* \uparrow e^2_{W,1} \mid \bar{u}(S/Q) = 1 \text{ if } O^{2'}(S) \not\geq Q\}$,

(c) $\{\bar{v} \in A(\bar{P})^{*W} \mid \bar{v}(S/Q) = 1 \text{ if } O^{2'}(S) \not\geq Q\}$,

where $O^{2'}(S)$ is the subgroup of S generated by all 2-elements and $A(\bar{P})^{*W}$ is the set of elements v of $A(\bar{P})^*$ such that

$$\mathrm{res}_{\bar{P}^w \cap \bar{P}} \, \mathrm{con}^g(x) = \mathrm{res}_{\bar{P}^w \cap \bar{P}}(x) \quad \text{for all } w \text{ in } W.$$

Theorem D. Assume that the finite group G has an abelian Sylow 2-subgroup. Let Q be a 2-perfect subgroup of G. Put $N := N_G(Q)$, $W := N/Q$, and let P/Q be an (abelian) Sylow 2-subgroup of W. Put $L := N_G(P)$ $(\leq N)$, $\bar{L} := L/Q'$, $\bar{Q} := Q/Q'$, and $\bar{P} := P/Q'$, where Q' is the intersection of subgroups of Q of odd prime index. Then the following hold :

(i) $A(G)^* \uparrow e^2_{GQ} \cong A(\bar{L})^* \uparrow e^2_{\bar{L}\bar{Q}}$.

(ii) <u>If</u> Q <u>is perfect, then</u>

$$A(\bar{L})^* \uparrow e^2_{\bar{LQ}} \cong A(P/Q)^{*L/Q} \cong C_{(P/QP^2)}(L/Q),$$

<u>where</u> C <u>stands for the centralizer group</u>.

(iii) <u>Assume that</u> Q <u>is not perfect</u>. <u>If</u> $\bar{P}$ <u>is generated by elements</u> t <u>with</u> $C_{\bar{Q}}(t) \neq 1$, <u>then</u> $A(\bar{L})^* \uparrow e^2_{\bar{LQ}} = 1$, <u>and otherwise it is of order</u> 2.

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Appendix

(References on Burnside rings)

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