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A numerical study of a Lagrangian renormalized closure for two-dimensional anisotropic turbulence

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Two-points/spectral closures have contributed much to the understanding of the physics of turbulence. However, because of the complexity of the closure equations, the application of the closures has been rather limitted to homogeneous and isotropic turbulence, and the application to more realistic cases, i.e., inhomogeneous and/or anisotropic cases has been regarded to be difficult in practice. The principal difficulty is associated with the convolution sums like $\int Q(\boldsymbol{p})Q(\boldsymbol{q})G(\boldsymbol{k})\delta^3(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})$ (in a symbolic notation), which arise from the nonlinearity of the Navier Stokes equations. The computation of such sums is in general quite difficult if the assumption of isotropy can not be used. In order to make closures applicable to realistc cases, it is therefore necessary to develope an efficient method to compute such convolution sums.

Only few studies seem to have been done in this direction. Among them are the pioneering works by Leslie(1973), Herring(1974) and Cambon et al.(1981). In the study of homogeneous anisotropic turbulence, they expanded spectral quantities such as the energy spectrum $Q(\boldsymbol{k})$ and the response function $G(\boldsymbol{k})$ around the isotropic state. By retaining only few terms in such expansions, they could reduce the full complicated closure equations to simpler solvable forms. Such a method may be efficient for nearly isotropic turbulence. It is, however, not clear that it is also efficent when the anisotropy is not weak. It is therefore interesting to develope a method which is efficient not only for weakly but also for strongly anisotropic cases.

Recently we have developed a method of computing the convolution sums by using Fast-Fourier-Transforms. The advantage of this method is that it enables us to directly compute spectral quantities on each discretized wavenumber vector $\boldsymbol{k}$, so that we need not assume the turbulence to be nearly isotropic. Moreover, no preliminary analytical reduction, which would be sometimes quite heavy task, is required.

We applied this method to a closure for two-dimensional anisotropic turbulence obeying the Navier-Stokes equations. Among various closures, we used here a Lagrangian renormalized apporximation (say, LRA) proposed ealier by one of the present authors (Kaneda,1981). The reason of our using the LRA is triple-folded;

(1) The LRA is a deductive and self-consistent approximation which contains no ad-hoc adjusting parameter or quantity.

(2) The equations of the LRA are much simpler than those of the other Lagrangian closures such as the ALHDIA and the SBALHDIA. The generalization of the isotropic LRA equations to anisotropic and/or inhomogeneous cases is much easier than that of the TFM equations.

(3) The LRA applied to isoptropic turbulence has been found in good agreement with experiments and numerical simulations over a wide range of Reynolds number in both two and three dimensions. The LRA is competitive in performance to the TFM.

We have made a computer code (say, A-Code) implementing the method noted above. In order to test the code, we made several runs for isotropic cases and compared the results with those obtained by another code (say, I-code) which is efficient but only applicable to isotropic cases. The agreement was found to be satisfactory. We have also made a direct numerical simulation code, and compared the results of a few runs with those of the numerical simulation by Herring et al.(1974). All of the results by the A-code, I-code and our simulation code were in good agreement with those of Herring et al.

Regarding the discretized mesh width Δk in the wavenumber space and the time interval Δt to solve closure equations, it was conjectured that they may be taken quite larger than those to simulate random turbulent field, since closures deal with only averaged quantities whose dependence on $\boldsymbol{k}$ and the time t is milder than that of the turbulent field. In order to test this conjecture, we made runs with various Δk and Δt. It was found that the results are quite insensitive to Δk, and the admissible Δk is larger than expected. Moreover, the comparison of these runs with and without aliasing errors suggests that the aliasing is not very crucial. These facts are helpful for saving machine time and memory.

We have then made simulation- and closure- runs of anisotropic decaying turbulence without mean flow. An interesting phenomenon found by the simulations is an over-relaxation of return to isotropy. It was found that initially non zero(i.e. anisotropic) $\Delta E \equiv \langle u_x^2 \rangle - \langle u_y^2 \rangle$ tends to zero at early stage, but it then over-relaxes, i.e. changes the sign and the deviation from zero increases with time at the later stage; ΔE approaches to zero not monotonically but oscillatingly. Such an over-relaxation is well reproduced also by the closure computation (cf.Fig.1). By the way, such an over-relaxation is also observed in the simulations of inviscid case. We therefore think that the non-linearity of the Navier-Stokes equations plays the key role in this phenomenon.

Figure 2 shows an example of the decay of vorticity components. The closure result is seen to be in good agreement with the simulation. The comparison of Figs.1 and 2 shows that the return to isotroy is faster at high wavenumbers. Since various gaphic facilities are now available, we can get informations from various graphics. Figure 3 is an example of such graph ics for the response function in the wavenumber space, from which we can see immediately how anisotropic it is.

Finally we would like to mention two points;

(1) It is possible to extend the present method of computing convolution sums to the case of non-uniform discretized mesh in the wavenumber space, so as to acceralate the computation at high Reynolds number.

(2) The present method and our code can be easily generalized for the other two-dimensional turbulent phenomena including the Navier-Stokes turbulence with linear mean flow and also β-plane turbulence in geophysical fluid mechanics.

References

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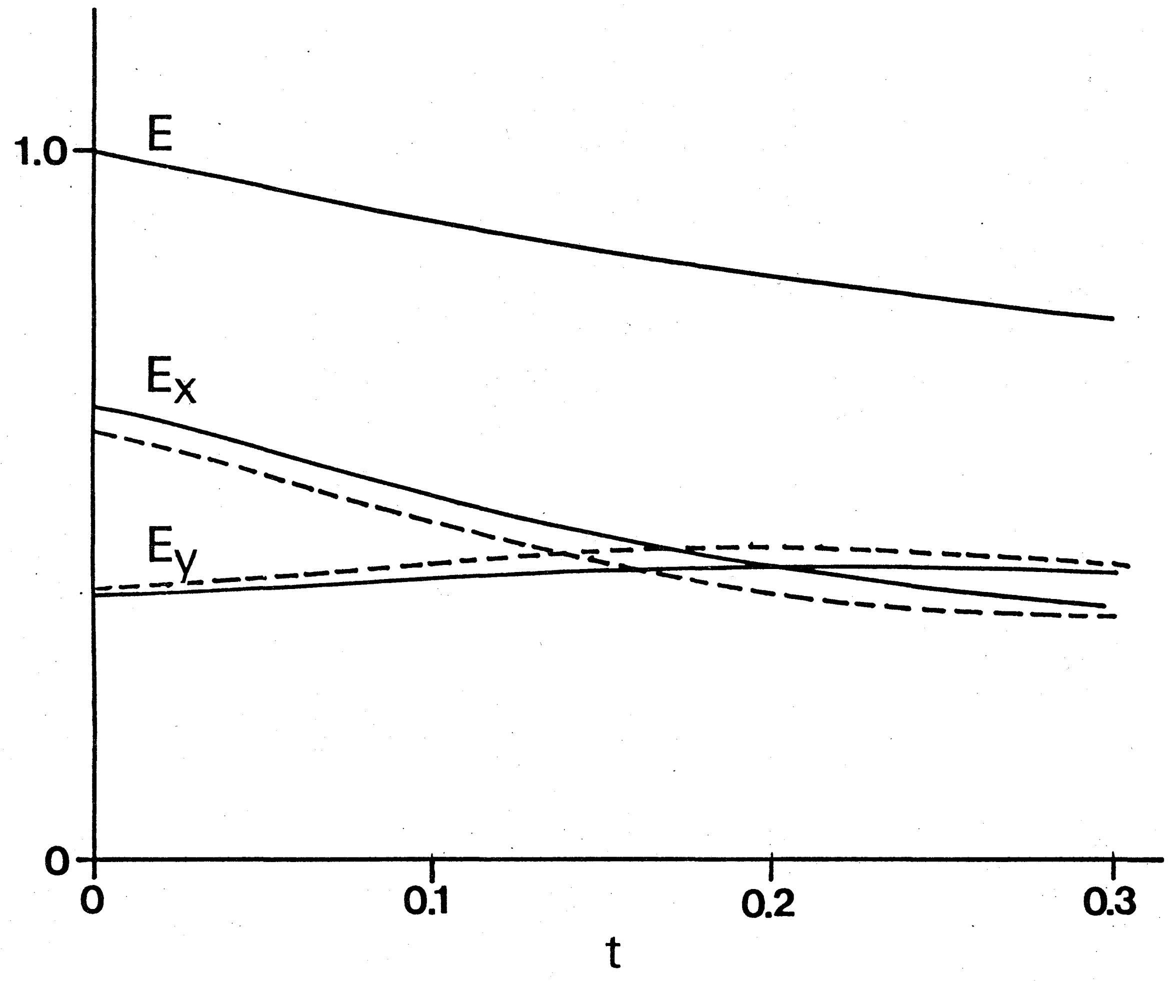

Fig.1

Evolution of energy components $E_x = \langle \sum u_x(\mathbf{k}) u_x(-\mathbf{k}) \rangle$, $E_y = \langle \sum u_y(\mathbf{k}) u_y(-\mathbf{k}) \rangle$ and the total energy $E = E_x + E_y$ (normalized by $E(t=0)$). Solid lines are closure values($\Delta t=0.0075$, $\Delta k=3$, $R_L=39$) and dashed lines are simulation values(one realization, $\Delta t=0.001$, $\Delta k=1$, $R_L=38$). The initial spectrum is $E(k,0)=16\sqrt{2/\pi}\, v_0 (k/k_0)^4 k_0^{-1} \exp[-(3k_x^2 + 0.3k_y^2)/k_0^2]$ with $v_0=1.0$ and $k_0=8.0$, $\nu=0.005$.

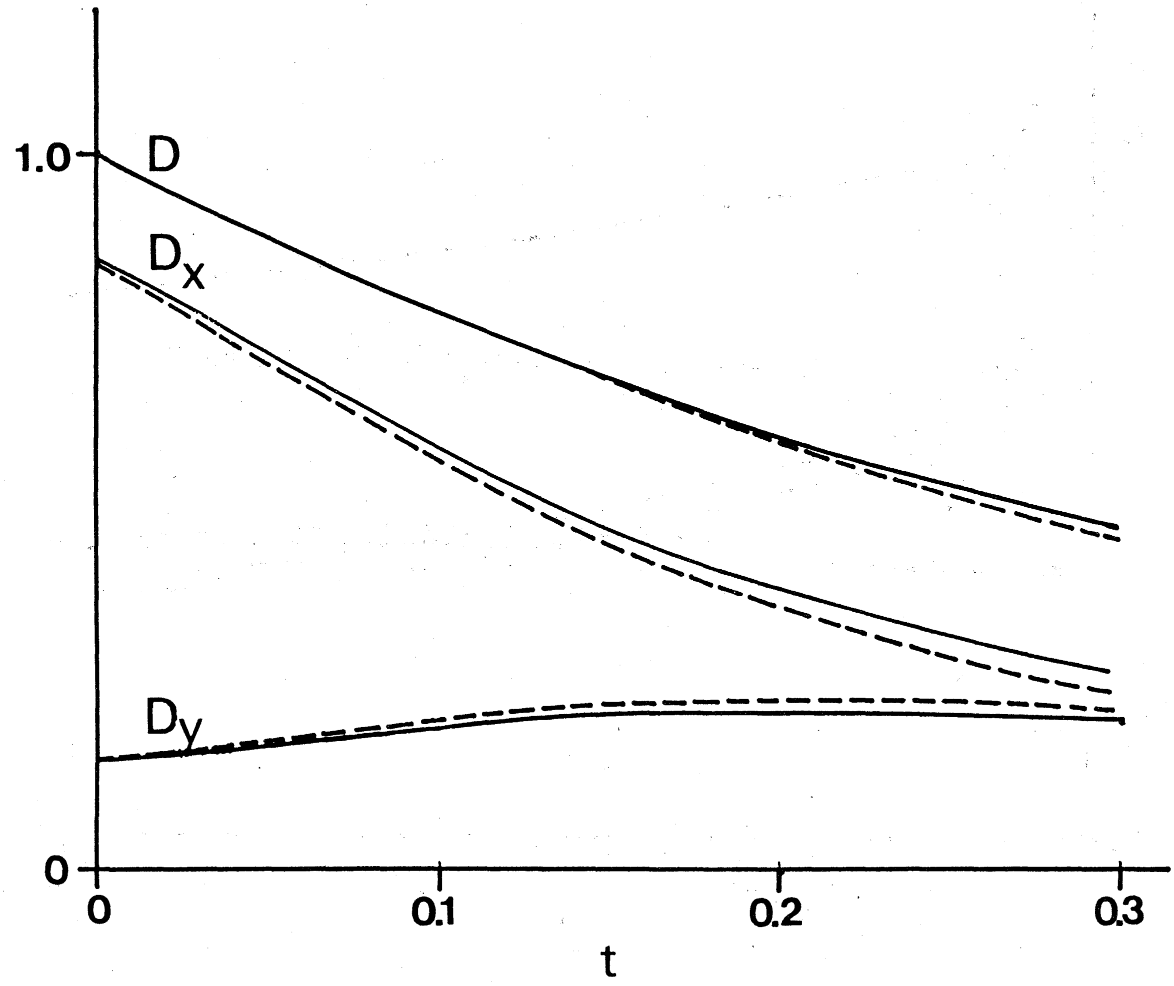

Fig.2

Evolution of vorticity components $D_x = \langle \sum k^2 u_x(\mathbf{k}) u_x(-\mathbf{k}) \rangle$, $D_y = \langle \sum k^2 u_y(\mathbf{k}) u_y(-\mathbf{k}) \rangle$ and the total vorticity $D = D_x + D_y$ (normalized by $D(t=0)$) of the run of Fig.1

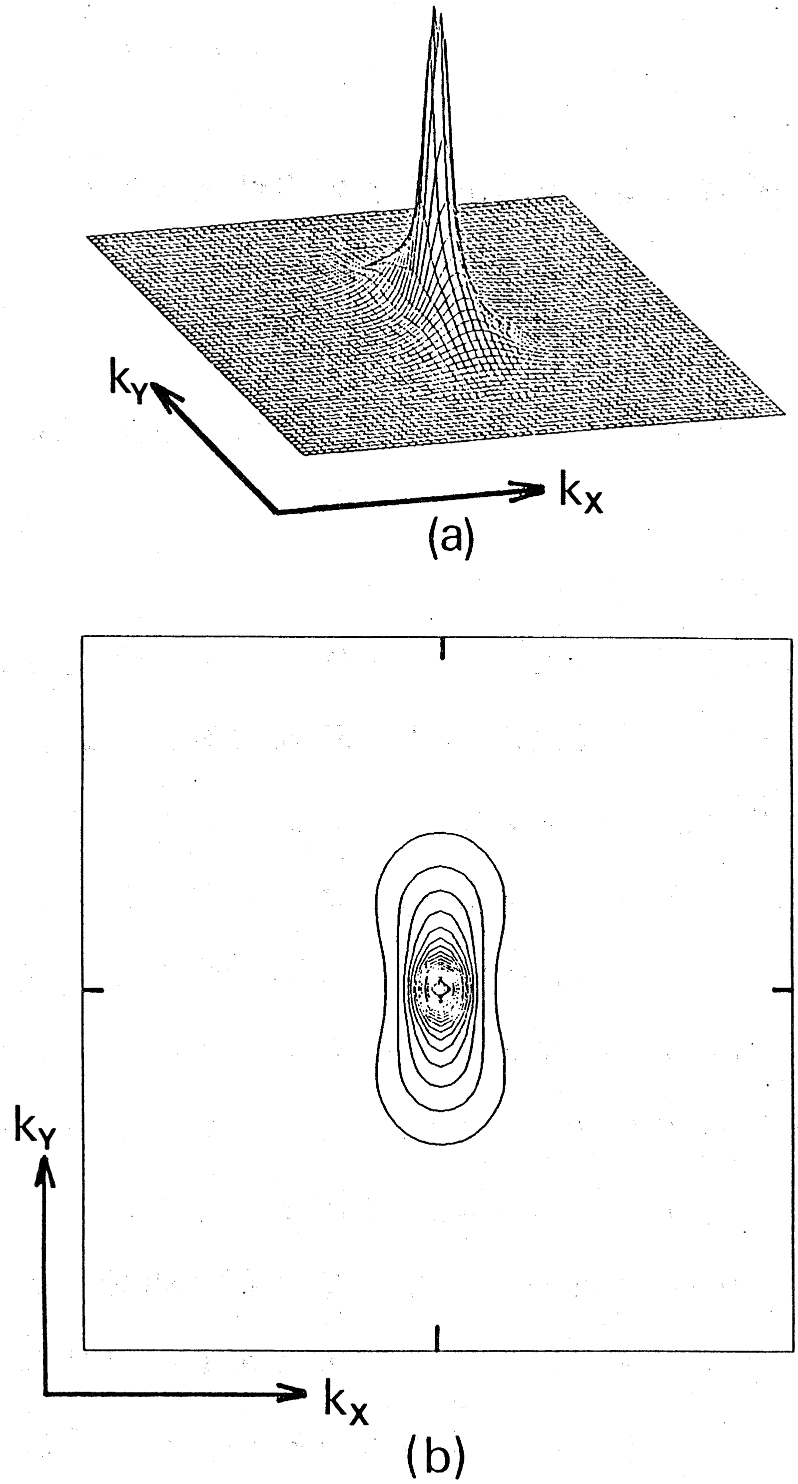

Fig.3

(a) Perspective view of the response function G(**k**,t,s) with t=0.225,s=0.
(b) Contour for G(**k**,t,s) with t=0.225,s=0.