数理解析研究所講究録
第 652 巻 1988 年 207-209

Statistical closure using bulk properties for turbulent shear flows

Akira Yoshizawa
吉 澤 徴

Of a variety of turbulence models, the k-ε model is widely used for engineering purposes. Its importance is diminishing, however, from the viewpoint of scientific turbulence research. This fact is attributed to the deficiency that the k-ε model cannot predict anisotropy of turbulent intensities accurately.

Previously, the author studied the Reynolds stress with the aid of a two-scale direct-interaction approximation (TSDIA).[1] This formalism is founded on an asymptotic expansion based on a scale parameter distinguishing the slow variation of mean flows from the fast variation of turbulent fluctuations. Consequently, TSDIA may be regarded as a derivative expansion formalism incorporated with DIA[2,3] by Kraichnan (for details about the structure of scale expansion in TSDIA, see Ref.4). For the Reynolds stress $R^{\alpha\beta}$, TSDIA gives

$$R^{\alpha\beta} \equiv - \langle u'^{\alpha} u'^{\beta} \rangle$$

$$= - \frac{2}{3} k\delta^{\alpha\beta} + \nu_e \left(\frac{\partial U^{\alpha}}{\partial x^{\beta}} + \frac{\partial U^{\beta}}{\partial x^{\alpha}}\right) + \tau^{\alpha\beta} \qquad (1)$$

(< > denotes the ensemble mean). Here, $\vec{U}$ and $\vec{u}'$ are the ensemble mean and fluctuation of velocity, k is the turbulent kinetic energy ($k \equiv \langle u'^a u'^a \rangle/2$; the summation convention is applied to repeated superscripts), and $\delta^{\alpha\beta}$ is the Kronecker delta symbol. The first two terms on the right-hand side give the familiar eddy-viscosity representation. The appearance of isotropic eddy viscosity ν_e is due to an assumption about isotropy of the basic field or the lowest-order term in the two-scale expansion of the fluctuating field. On the other hand, $\tau^{\alpha\beta}$ expresses the deviation from the isotropic eddy-viscosity representation (the details are given in Ref.1). Equation (1) was incorporated into the k-ε model to be applied successfully to the study of anisotropic turbulent intensities in channel flows.[5] This improvement of the k-ε closure for turbulent shear flows indicates a possibility that models of k-ε type as well as the second-order models can remain a useful method for simulating complicated flows such as recirculating flows.

At this time, it is a major concern about turbulence modeling (including the second-order models) that no statistical foundation has yet been given to a model equation for ε (the dissipation rate of turbulent kinetic energy), unlike a model equation for k. In reality, the structure of the transport equation for k is simple, and its modeling is not difficult. Some theoretical support can be given to the modeling by using turbulence theories such as DIA.[6]

On the other hand, the structure of the transport equation for ε is extremely complicated. As a result, the ε equation in the k-ε model has not been constructed statistically so far on the basis of the exact transport equation for ε. This situation will become a big obstacle when we desire to incorporate anisotropic effects such as and buoyancy force into the ε equation. This point is a major motivation of present work.

In the present paper, we can make use of an analytical expression for k from TSDIA to show that a model ε equation is derived with the aid of the concept of transferability of model. The model equation is very similar to the counterpart in the k-ε model.

The details of this work are given in Ref.7.

1. A. Yoshizawa, Phys. Fluids 27, 1377 (1984).
2. R.H. Kraichnan, J. Fluid Mech. 5, 497 (1959).
3. R.H. Kraichnan, Phys. Fluids 7, 1048 (1964).
4. A. Yoshizawa, Phys. Fluids 28, 3226 (1985).
5. S. Nisijima and A. Yoshizawa, AIAA J. 25, 414 (1987).
6. A. Yoshizawa, Phys. Fluids 28, 59 (1985).
7. A. Yoshizawa, Phys. Fluids 30, 628 (1987).