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On Quasi-Hadamard Product of Certain p-Valent Functions with Negative Coefficients

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1. INTRODUCTION

In [1], Kumar showed some results for the quasi-Hadamard product of certain univalent functions with negative coefficients.

In the present note, we show that Kumar's results [1] are generalized to the case of certain p-valent functions with negative coefficients.

Let $A(p)$ be the class of analytic and p-valent function $f(z)$ of the form

$$f(z) = z^p - \sum_{n=1}^{\infty} a_{p+n} z^{p+n} \qquad (a_{p+n} \geqq 0,\ p \in N)$$

in the unit disk $U = \{z : |z| < 1\}$.

Let $T^*(p,\alpha)$ and $C(p,\alpha)$ denote the subclasses of $A(p)$ which satisfy $\mathrm{Re}\left\{ \frac{zf'(z)}{f(z)} \right\} > \alpha$ and $\mathrm{Re}\left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > \alpha$, for $0 \leqq \alpha < p$, respectively.
Clearly the function in $T^*(p,\alpha)$ and $C(p,\alpha)$ are p-valent starlike function and p-valent convex function of order α, respectively.

For these classes, Owa has obtained the following results in [2].

LEMMA 1. A function f(z) is in the class $T^*(p,\alpha)$ if and only if

$$\sum_{n=1}^{\infty}(p + n - \alpha) \leqq p - \alpha.$$

The result is sharp.

LEMMA 2. A function f(z) is in the class $C(p,\alpha)$ if and only if

$$\sum_{n=1}^{\infty}(p + n)(p + n - \alpha)a_{p+n} \leqq p(p - \alpha).$$

The result is sharp.

Let $A_0(p)$ denote the class of analytic and p-valent function f(z) of the form

$$f(z) = a_p z^p - \sum_{n=1}^{\infty} a_{p+n} z^{p+n} \quad (a_p > 0,\ a_{p+n} \geqq 0,\ p \in \mathbf{N})$$

in the unit disk **U**.

Furthermore, let $T_0^*(p,\alpha)$ and $C_0(p,\alpha)$ be the subclasses of $A_0(p)$ as follows:

$$T_0^*(p,\alpha) = \left\{ f(z) \in A_0(p) : \mathrm{Re}\left\{ \frac{zf'(z)}{f(z)} \right\} > \alpha \quad (0 \leqq \alpha < p) \right\}$$

and

$$C_0(p,\alpha) = \left\{ f(z) \in A_0(p) : \mathrm{Re}\left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > \alpha \quad (0 \leqq \alpha < p) \right\}.$$

For these classes, by Lemma 1 and Lemma 2, we easily obtain the following theorems, respectively.

<u>THEOREM 1.</u> A function f(z) in the class $T_0^*(p,\alpha)$ if and only if

$$\sum_{n=1}^{\infty} (p + n - \alpha)a_{p+n} \leqq (p - \alpha)a_p.$$

<u>THEOREM 2.</u> A function f(z) is in the class $C_0(p,\alpha)$ if and only if

$$\sum_{n=1}^{\infty} (p + n)(p + n - \alpha)a_{p+n} \leqq p(p - \alpha)a_p.$$

We now introduce the subclass $S_0(k,p,\alpha)$ of the class $A_0(p)$ as follows.
A function f(z) belongs to the class $S_0(k,p,\alpha)$ if and only if

$$\sum_{n=1}^{\infty} \left(\frac{p + n}{p}\right)^k (p + n - \alpha)a_{p+n} \leqq (p - \alpha)a_p,$$

where k is any real number.
Evidently, $S_0(0,p,\alpha) \equiv T_0^*(p,\alpha)$ and $S_0(1,p,\alpha) \equiv C_0(p,\alpha)$.

From now on, let the functions of the class $A_0(p)$ be the following forms:

$$f_i(z) = a_{p,i}z^p - \sum_{n=1}^{\infty} a_{p+n,i}z^{p+n} \quad (a_{p,i} > 0,\ a_{p+n,i} \geqq 0)$$

and

$$g_j(z) = b_{p,j}z^p - \sum_{n=1}^{\infty} b_{p+n,j}z^{p+n} \quad (b_{p,j} > 0,\ b_{p+n,j} \geqq 0),$$

respectively.

Let us define the quasi-Hadamard product $f_i*g_j(z)$ of the functions $f_i(z)$ and $g_j(z)$ by

$$f_i(z)*g_j(z) = a_{p,i}b_{p,j}z^p - \sum_{n=1}^{\infty} a_{p+n,i}b_{p+n,j}z^{p+n}.$$

2. RESULTS

Consequently, we have the following theorems. we can prove these theorems by using the same way as Kumar [1].

THEOREM 3. Let the functions $f_i(z)$ belong to the classes $T_0^*(p,\alpha_i)$ for each $i = 1,2,3,\cdots,m$, respectively. Then the quasi-Hadamard product $f_1*f_2*f_3*\cdots*f_m(z)$ belongs to the class $S_0(m-1,p,\beta)$, where $\beta = \max\{\alpha_1,\alpha_2,\alpha_3,\cdots,\alpha_m\}$.

THEOREM 4. Let the functions $f_i(z)$ belong to the classes $C_0(p,\alpha_i)$ for each $i = 1,2,3,\cdots,m$, respectively. Then the quasi-Hadamard product $f_1*f_2*f_3*\cdots*f_m(z)$ belongs to the class $S_0(2m-1,p,\beta)$, where $\beta = \max\{\alpha_1,\alpha_2,\alpha_3,\cdots,\alpha_m\}$.

THEOREM 5. Let the functions $f_i(z)$ belong to the classes $T_0^*(p,\alpha_i)$ for each $i = 1,2,3,\cdots,m$ and for each $j = 1,2,\cdots,q$, let the functions $g_j(z)$ belong to the classes $C_0(p,\beta_j)$, respectively. Then the quasi-Hadamard product $f_1*f_2*f_3*\cdots*f_m*g_1*g_2*g_3*\cdots*g_q(z)$ belongs to the class $S_0(m+2q-1,p,\gamma)$, where $\gamma = \max\{\alpha_1,\alpha_2,\alpha_3,\cdots,\alpha_m,\beta_1,\beta_2,\beta_3,\cdots,\beta_q\}$.

THEOREM 6. Let the functions $f_i(z)$ belong to the class $C_0(p,\alpha)$ for each $i = 1,2,3,\cdots,m$ and let $0 \leqq \alpha \leqq r_0$, where r_0 is a root of the equation $(p + 1)^m(p - mr) - p(p - r)^m = 0$ in the interval $(0,\frac{p}{m})$. Then the quasi-Hadamard product $f_1*f_2*f_3*\cdots*f_m(z)$ belongs to the class $S_0(m-1,p,m\alpha)$.

REMARK. If we put $p = 1$ in these theorems, we have the Kumar's results [1].

REFERENCES

[1] V. Kumar, Quasi-Hadamard product of certain univalent functions, J. Math. Anal. Appl. 126, 70-77(1987).

[2] S. Owa, On certain classes of p-valent functions with negative coefficients, SIMON STEVIN, A Quart. J. Pure Appl. Math. 59, No4 385-402(1985).