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Algebraic Riemann manifolds

Kazuo Yamato 大和一夫

College of General Education, Nagoya University

We give a criterion by which we decide whether two given Riemann manifolds $M,\overline{M}$ are isometric or not. We recall the following classical theorem.

THEOREM (C^ω ISOMETRY THEOREM). *Let $M,\overline{M}$ be real analytic Riemann manifolds of dimension n. Let $p \in M$, $\bar{p} \in \overline{M}$. Suppose that there exists a linear isometry $I : T_p(M) \to T_{\bar{p}}(\overline{M})$ which preserves the curvature tensors $R,\overline{R}$, and their covariant differentials $\nabla^k R$, $\nabla^k \overline{R}$ of any order k. Then the mapping I can be extended to an isometry h between neighborhoods of $p,\bar{p}$. Hence in particular if $M,\overline{M}$ are complete, connected, and simply connected, then M, $\overline{M}$ are isometric.*

By replacing C^ω with the Nash category C^Ω, and introducing the notion "minimal differential polynomial" ϕ_M of a C^Ω Riemann manifold M, we observe that the proof of this theorem implies the following criterion.

THEOREM 1. *Let $M,\overline{M}$ be C^Ω Riemann manifolds of dimension n. Let $p \in M$, $\bar{p} \in \overline{M}$. Suppose that*

(1) *the minimal differential polynomials ϕ_M, $\phi_{\overline{M}}$ coincide,*

(2) *the two point $p,\bar{p}$ are "nonsingular" with respect to ϕ_M, $\phi_{\overline{M}}$, respectively, and*

(3) *there exists a linear isometry $I : T_p(M) \to T_{\bar{p}}(\overline{M})$ which preserves the curvature tensors R, $\overline{R}$, and their first $4n-5$ covariant differentials $\nabla^k R$, $\nabla^k \overline{R}$.*

Then the mapping I can be extended to an isometry h between neighborhoods of p,$\bar{p}$.

As an application we obtain

THEOREM 2. *Let M be a compact C^Ω Riemann manifold of dimension n. Suppose that M is nowhere homogeneous, i.e. for any distinct points p,q of M, there exists no isometry $h, h(p) = q$, between neighborhoods of p, q. Then M is C^Ω embeddable, and the embedding is given by means of general scalar curvatures. If any point of M is nonsingular with respect to ϕ_M, then some finite number of general scalar curvatures of order at most 4n-5 give a one to one mapping of M into a vector space.*

REFERENCE

K.Yamato, *Algebraic Riemann manifolds*, Nagoya Math. J. **115** (1989) (to appear).