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Generic Homeomorphisms Have the POTP

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ABSTRACT

We give our result on the C^0-genericity of the POTP for homeomorphisms. In Section 1 and 2, we study some stabilities in the topological dynamics. In Section 3, we give the main result and an outline of the proof.

§1. Stabilities in the Topological Dynamics

Let M be a compact differentiable manifold with the metric d induced by a Riemannian structure. We denote by Homeo(M) (resp. ClDiff(M)) the set of all homeomorphisms (resp. the C^0-closure of all diffeomorphisms) with the C^0-topology, i.e. the topology induced by the following metric.

$$d(f,g)=\max_{x\in M} d(f(x),g(x))$$

The C^0-topology depend on neither the differentiable structure nor the Riemannian structure.

In the world of the topological dynamics, many concepts of stability as below have been defined and studied by many mathematicians, see the references.

In this note, f and g denote elements of Homeo(M).

<u>(1.1) DEFINITIONS.</u> (1) A sequence $\{x_i\}_{i\in Z}$ is a δ-pseudo-orbit of f if $d(f(x_i), x_{i+1}) < \delta$ for every $i \in Z$.

(2) A sequence $\{x_i\}_{i\in Z}$ is ε-traced by the f-orbit through $x \in M$ if $d(f^i(x), x_i) < \varepsilon$ for every $i \in Z$.

(3) A sequence $\{x_i\}_{i\in Z}$ is ε-set-traced by the f-orbit through $x \in M$ if $\bar{d}(\mathrm{Cl}\{f^i(x)\}, \mathrm{Cl}\{x_i\}) \leqq \varepsilon$.

Here we denote by $\mathrm{Cl}\{x_i\}$ the closure of $\{x_i : i \in Z\}$ and by $\bar{d}$ the Hausdorff metric with respect to d.

(4) f is <u>strong C^0-tolerance stable</u> if for every $\varepsilon > 0$, there exists a $\delta > 0$ such that for every $g \in V_\delta(f)$, every f-orbit is ε-traced by some g-orbit and every g-orbit is ε-traced by some f-orbit.

Here we denote by $V_\delta(f)$ the δ-neighborhood of f in Homeo(M).

(5) f is <u>C^0-tolerance stable</u> if for every $\varepsilon > 0$, there exists a $\delta > 0$ such that for every $g \in V_\delta(f)$, every f-orbit is ε-set-traced by some g-orbit and every g-orbit is ε-set-traced by some f-orbit.

(6) f has the <u>pseudo-orbit tracing property</u> (abbr. POTP) if for every $\varepsilon > 0$, there exists a $\delta > 0$ such that every δ-pseudo-orbit of f is ε-traced by some f-orbit.

(7) f is <u>OE-stable</u> if for every $\varepsilon > 0$, there exists a $\delta > 0$ such that every δ-pseudo-orbit of f is ε-set-traced by some f-orbit.

(8) f is lower semi-conjugate to g under φ if there exists a continuous surjection $\varphi : M \to M$ satisfying $f\varphi = \varphi g$.

(9) f is <u>topologically stable</u> if for every $\varepsilon > 0$, there exists a $\delta > 0$ such that for every $g \in V_\delta(f)$, f is lower semi-conjugate to g under φ satisfying $d(\varphi(x), x) < \varepsilon$ for every $x \in M$.

We rewrite our definitions of the C^0-tolerance stability and the OE-stability after the fashion of Takens [14].

Let $C(M)$ be the metric space of all non-empty closed subsets of M with the Hausdorff metric $\bar{d}$ with respect to d. Then $C(M)$ is again compact. Let $C(C(M))$ be the space of all non-empty closed subsets of $C(M)$ with the Hausdorff metric $\bar{\bar{d}}$ with respect to $\bar{d}$.

A set $A \in C(M)$ is an extended f-orbit if for every $\varepsilon > 0$ and $\delta > 0$, there exists a δ-pseudo-orbit $\{x_i\}_{i \in Z}$ satisfying $\bar{d}(A, \mathrm{Cl}\{x_i\}) \leqq \varepsilon$.

We define $O(f), E(f) \in C(C(M))$ as follows.

$O(f) = \mathrm{Cl}\{\,\mathrm{Cl}\{f^i(x)\} : x \in M\}$

$E(f) = \{\,A : A$ is an extended f-orbit$\}$

It is easy to see that $O(f) \subseteq E(f)$.

(1.2) PROPOSITION. (1) f is C^0-tolerance stable if and only if the map O :Homeo$(M) \rightarrow C(C(M))$ is continuous at f.

(2) f is OE-stable if and only if $O(f) = E(f)$.

PROOF. (1) It is immediate by the fact that every f-orbit is ε-set-traced by some g-orbit and every g-orbit is ε-set-traced by some f-orbit if and only if $\bar{\bar{d}}(O(f), O(g)) \leqq \varepsilon$.

(2) Let us prove "only if" part. Take an $A \in E(f)$ and an $\varepsilon > 0$. Since f is OE-stable, there exists a $\delta > 0$ such that every δ-pseudo-orbit is $(\varepsilon/2)$-set-traced by some f-orbit. Since $A \in E(f)$, there exists a δ-pseudo-orbit $\{x_i\}_{i \in Z}$ satisfying $\bar{d}(A, \mathrm{Cl}\{x_i\}) \leqq \varepsilon/2$. Since $\{x_i\}_{i \in Z}$ is $(\varepsilon/2)$-set-traced by some f-orbit through, say $x \in M$, we obtain $\bar{d}(A, \mathrm{Cl}\{f^i(x)\}) \leqq \varepsilon$. Hence $A \in O(f)$.

Let us prove "if" part. Assume that f is not OE-stable. Namely, there exists an $\varepsilon > 0$ such that for every $\delta > 0$, there exists a δ-pseudo-orbit $\{x_i^{\delta}\}_{i \in Z}$ such that it is not ε-set-traced by any f-orbits. For every $k \geqq 1$, we consider

$A_k = \mathrm{Cl}\{x_i^{1/k}\} \in C(M)$. Since $C(M)$ is compact, there exists a convergent subsequence of $\{A_i\}_{i \in Z}$ and its limit point $A \in C(M)$. It is easy to see that $A \notin O(f)$ and $A \in E(f)$. Hence $O(f) \neq E(f)$.

(1.3) PROPOSITION (Relations between Stabilities).

(1) Topological Stability

$\Downarrow$

(2) Strong C^0-Tolerance Stability $\Rightarrow$ (4) POTP

$\Downarrow$ $\Downarrow$

(3) C^0-Tolerance Stability (5) OE-Stability

PROOF. (1) $\Rightarrow$ (2). It is immediate by the definition.
(2) $\Rightarrow$ (3) and (4) $\Rightarrow$ (5). They are immediate by the fact that the ε-traceability implies the ε-set-traceability.
(2) $\Rightarrow$ (4). The m-dimensional cases ($m \geqq 2$) are obtained by a modification of the proof of Theorem 11 in [16]. The 1-dimensional case is also valid, see Theorem 2.4.

We denote by $P(f)$, $c(f)$, $L(f)$, $\Omega(f)$ and $R(f)$ the set of all periodic points, the closure of all points each of which is both ω- and α-recurrent (it is called the Birkhoff center, see [4]), the closure of all limit points, the nonwandering set and the chain recurrent set respectively. The definitions of the above sets and proofs of the following inclusions are found in [10].

$P(f) \subseteq c(f) \subseteq L(f) \subseteq \Omega(f) \subseteq R(f)$

As an analogue to the topological stability, we define the topological R-stability.

(1.4) DEFINITIONS. (1) f is lower R-semi-conjugate to g under φ if there exists a continuous surjection

$\varphi : R(g) \to R(f)$ satisfying $f\varphi = \varphi g$ on $R(g)$.

(2) f is topologically R-stable if for every $\varepsilon > 0$, there exists a $\delta > 0$ such that for every $g \in V_\delta(f)$, f is lower R-semi-conjugate to g under φ satisfying $d(\varphi(x), x) < \varepsilon$ for every $x \in R(g)$.

(1.5) PROPOSITION. Let f be topologically R-stable. Then:

(1) f has the periodic pseudo-orbit tracing property (abbr. PPOTP), i.e. for every $\varepsilon > 0$, there exists a $\delta > 0$ such that every periodic pseudo-orbit with a period $p \geqq 1$ is ε-traced by some periodic f-orbit with the period p;

(2) $\overline{P}(f) = R(f)$; and

(3) $f|_{R(f)} : R(f) \to R(f)$ has the POTP.

PROOF. (1) The m-dimensional cases $(m \geqq 2)$ is proved by a modification of the proof of Theorem 11 in [16]. The 1-dimensional case is also valid, see Section 2 and Remark 1.6.

(2) It is immediate by (1).

(3) It is proved by a similar argument to the proof of Theorem 1 in [1].

(1.6) REMARK. If $f \in \mathrm{Homeo}(M)$ has the PPOTP, then $f|_{R(f)}$ has the POTP. Conversely, if $f|_{R(f)}$ is expansive with the POTP, then f has the PPOTP.

(1.7) PROPOSIOTION. If $f \in \mathrm{C1Diff}(M)$ is topologically R-stable, then f has a finite number of chain components.

PROOF. It is proved by the same method of the proof in [2].

(1.8) PROPOSITION. If f is topologically stable, then it is topologically R-stable.

PROOF. Assume that f is lower semi-conjugate to some g under some $\varphi : M \to M$. Clearly, we obtain $\varphi(R(g)) \subseteq R(f)$. For every $x \in P(f)$ with a period $p \geqq 1$, $\varphi^{-1}(x)$ is non-empty, closed and f^p-invariant. Since $\varphi^{-1}(x)$ contains a minimal set of f^p, we obtain $\varphi^{-1}(x) \cap c(g) \neq \emptyset$. Therefore $R(f) = \overline{P}(f) \subseteq \varphi(c(g)) \subseteq \varphi(R(g))$. Hence f is lower R-semi-conjugate to g under $\varphi|_{R(g)} : R(g) \to R(f)$.

(1.9) REMARK. We can also define the topological Ω-, L-, c-, and $\overline{P}$-stability. The same conclusions as in Proposition 1.5 and 1.7 are also valid for these stabilities. We can also prove the following implications.

Topological R-Stability $\Rightarrow$ Topological Ω-Stability
$\Rightarrow$ Topological L-Stability $\Rightarrow$ Topological c-Stability

Moreover if we assume expansiveness on the chain recurrent set, then the following implications are valid.

Topological R-Stability $\Leftrightarrow$ Topological Ω-Stability
$\Leftrightarrow$ Topological L-Stability $\Leftrightarrow$ Topological c-Stability

(1.10) REMARK. We can prove that some theorems for the topological stability stated in [2] and [15, Section 4] are also valid for the topological R-, Ω-, L-, c- and $\overline{P}$-stability.

§2. Stabilities of the Topological Dynamics on the Circle

In this section, we consider S^1 as R/Z. There is a very useful tool to observe the orbit structure of orientation preserving $f \in \mathrm{Homeo}(S^1)$. It is the rotation number of f, say $\rho(f) \in R/Z$, see [6]. We introduce the following facts about $f \in \mathrm{Homeo}(S^1)$, see [6]:

① $P(f)=\emptyset$ if and only if $\rho(f^2)$ is irrational;
② if $P(f)=\emptyset$, then for every $x \in M$, we obtain $\omega(x)=\alpha(x)=c(f)=L(f)$;
③ if $P(f)=\emptyset$ and $c(f)=S^1$, then f is topologically conjugate to an irrational rotation; and
④ if $P(f)=\emptyset$ and $c(f)\neq S^1$, then $c(f)$ is perfect, nowhere-dense and minimal.

Let $f \in \mathrm{Homeo}(S^1)$ satisfy $P(f)\neq\emptyset$. If f is orientation preserving, then all of its peridic points have the same period, say $p \geqq 1$. If f is orientation reversing, then two of its periodic points have the same period (say $q \geqq 1$) and the others have the period $p=2q$. Therefore every periodic point of f^p is a fixed point and $P(f)=\overline{P}(f)$. Denote by $\mathrm{pr}: R \to S^1$ the covering map and by $F: R \to R$ the lift of f^p which satisfies $F(x)=x$ for every $x \in \mathrm{pr}^{-1}(P(f))$. Define
$U^+=\mathrm{pr}\{x \in R : F(x) > x\} \subset S^1$ and
$U^-=\mathrm{pr}\{x \in R : F(x) < x\} \subset S^1$.

(2.1) DEFINITIONS. (1) $f \in \mathrm{Homeo}(S^1)$ is of Yano type if $P(f)\neq\emptyset$, $P(f)\neq S^1$, $\mathrm{int}P(f)=\emptyset$, $U^+\neq\emptyset$, $U^-\neq\emptyset$, and every two components in U^+ (resp. U^-) are separated by some component in U^- (resp. U^+).
(2) $f \in \mathrm{Homeo}(S^1)$ is of Morse-Smale type if it is Yano type and $P(f)$ is finite.

Remark that every homeomorphism of Morse-Smale-type is topological conjugate to some Morse-Smale diffeomorphism.

(2.2) THEOREM. If $M=S^1$, then the following are equivalent:
(0) f is of Morse-Smale type;
(1) f is topologically stable; and

(2) f is topologically R-stable.

PROOF. The implications (0)$\Leftrightarrow$(1) were proved by Yano [20], and (0)$\Leftrightarrow$(2) are proved by the same method of his proof.

(2.3) REMARK. On Theorem 2.2, (0) is also equivalent to the following:

(3) f is topological Ω-stable;

(4) f is topological L-stable;

(5) f is topological c-stable; and

(6) f is topological $\overline{P}$-stable.

(2.4) THEOREM. If $M = S^1$, then the following are equivalent:

(0) f is of Yano type;

(1) f is strong C^0-tolerance stable; and

(2) f has the POTP.

(2.5) REMARK. On Theorem 2.4, (0) is also equivalent to the following:

(3) f is persistent (see [3]); and

(4) $\overline{\mathfrak{prm}}(f) = R(f)$ (here $\mathfrak{prm}(f)$ denotes the set of all permanent periodic points, see [9]).

(2.6) LEMMA. (1) Let $f \in \mathrm{Homeo}(S^1)$ have the POTP. Then there exists $\delta > 0$ such that for every $g \in V_\delta(f)$, we obtain $\rho(f^2) = \rho(g^2) = $ rational.

(2) If $f \in \mathrm{Homeo}(S^1)$ is strong C^0-tolerance stable, then the same conclusion as in (1) holds.

PROOF. (1) If $f, g \in \mathrm{Homeo}(S^1)$ satisfy $d(f, g) < \delta < 1/2$, then we can take their lifts $\hat{f}, \hat{g} : R \to R$ satisfying $d(\hat{f}, \hat{g}) < \delta$. If $\delta > 0$ is sufficientry small, then every g-

orbit is (1/2)-traced by some f-orbit. Therefore there exist $x, y \in R$ satisfying $|\hat{f}^{2i}(x) - \hat{g}^{2i}(y)| < 1/2$ for every $i \in Z$. Therefore by the definition of ρ, we obtain $\rho(f^2) = \rho(g^2)$. Since the set of diffeomorphisms having periodic points is dense in Homeo(S^1), $\rho(f^2)$ is rational.

(2) It is proved by the same method of (1).

PROOF OF THEOREM 2.4. The implications (0)$\Leftrightarrow$(2) were proved by Yano [20] and (0)$\Leftrightarrow$(3) are easy. Therefore it is sufficient to prove (0)$\Leftrightarrow$(1).

(0)$\Rightarrow$(1). Assume that f is of Yano type. If f^p is strong C^0-tolerance stable, then so is f. Therefore we can assume without loss of generality that every periodic point of f is a fixed point. For such f and every $\varepsilon > 0$, there exist finite components A_i of U^+ and B_i of U^- $(i = 1, \cdots, n)$ such that A_i and B_i alternately appear and $\varepsilon/5$ is the biggest length of all components of $S^1 - \cup\{A_i \cup B_i\}$. Let $\alpha > 0$ be the smallest length of all A_i and B_i. Let $\beta = \min\{\varepsilon/5, \alpha/5\}$. For $k = 1, 2$, let $A_i^{k\beta} = U_{k\beta}(A_i)$, $B_i^{k\beta} = U_{k\beta}(B_i)$ and $C_j^{k\beta}$ $(j = 1, \cdots, 2n)$ be the components of $S^1 - \cup\{A_i^{k\beta} \cup B_i^{k\beta}\}$.

Take $k \in Z$ sufficiently large so that for every $x \in \cup\{A_i^{\beta} \cup B_i^{\beta}\}$, $f^k(x)$ is in some C_s^{β} and $f^{-k}(x)$ is in some C_t^{β}. Take $\delta > 0$ sufficiently small so that for every $g \in V_\delta(f)$, we obtain $d(f^i, g^i) < \varepsilon$ for $-k \leqq i \leqq k$.

Let $g \in V_\delta(f)$. Then for every $C_j^{2\beta}$, there exists a fixed point of g in it. We verify that every f-orbit is ε-traced by some g-orbit.

Case 1. Given f-orbit is entirely included in some $C_j^{2\beta}$. Since the length of $C_j^{2\beta}$ is less than ε, the f-orbit is ε-traced by some fixed point of g in $C_j^{2\beta}$.

Case 2. Given f-orbit is not entirely included in any $C_j^{2\beta}$. Then the f-orbit passes through some point x contained in

some $A_i^{2\beta}$ or $B_i^{2\beta}$, and there exist $C_s^{2\beta}$ and $C_t^{2\beta}$ such that
$f^i(x), g^i(x) \in C_s^{2\beta}$ for $i \geqq k$ and
$f^i(x), g^i(x) \in C_t^{2\beta}$ for $i \leqq -k$.
Moreover, by the definition of δ, we obtain
$d(f^i(x), g^i(x)) < \varepsilon$ for $-k \leqq i \leqq k$.

Since every g-orbit is a δ-pseudo-orbit of f and f has the POTP, for sufficientry small $\delta > 0$, we can prove that every g-orbit is ε-traced by some f-orbit.

(1)$\Rightarrow$(0). By Lemma 2.6, $P(f) \neq \emptyset$. The rest is easy.

(2.7) THEOREM. If $M = S^1$, then the following are equivalent:

(0) f is either of Yano type or topologically conjugate to some irrational rotation;

(1) f is C^0-tolerance stable; and

(2) f is OE-stable.

PROOF. (0)$\Rightarrow$(1). By Theorem 2.4, Yano type implies the C^0-tolerance stability. Therefore it is sufficient to show that every irrational rotation is C^0-tolerance stable.

Let f be an irrational rotation. For every $\varepsilon > 0$, there exists $n \geqq 1$ such that for every $x \in S^1$, the length of every compornent of $S^1 - \{f^i(x) : -n \leqq i \leqq n\}$ is less than $\varepsilon/3$. For sufficiently small $\delta > 0$ and every $g \in V_\delta(f)$, we obtain $d(f^i, g^i) < \varepsilon/3$ for every $-n \leqq i \leqq n$. Hence for every $x \in S^1$, we obtain $\bar{d}(\mathrm{Cl}\{f^i(x)\}, \mathrm{Cl}\{g^i(x)\}) \leqq \varepsilon$.

(1)$\Rightarrow$(0). If f is C^0-tolerance stable and $P(f) \neq \emptyset$, then it is easy to see that f is of Yano type. Therefore it is sufficient to show that if f is C^0-tolerance stable and $P(f) = \emptyset$, then $c(f) = S^1$.

Assume $P(f) = \emptyset$ and $c(f) \neq S^1$. Then $c(f)$ is nowhere-dense. Take a component (a, b) of $S^1 - c(f)$, $x = (2a + b)/3$, $y = (a + 2b)/3$, and $6\varepsilon = b - a$. Then for every $\delta > 0$, there exist $m, n \geqq 1$ satisfying $d(f^{m}(x), f^{-n}(y)) < \delta$. Therefore we can constract $g \in V_{\delta}(f)$ which has an orbit through x and y. Hence f is not C^{0}-tolerance stable.

(0)⇔(2). They are proved by a similar argument to the proof of (0)⇔(1).

§3. Genericity of the Strong C^{0}-Tolerance Stability

A subset in a topological space is called residual if it includes a countable intersection of open and dense subsets. A topological space is called a Baire space if every residual set is dense in it. In particular, every complete metric space is a Baire space. For example, Homeo(M) is a Baire space. Because the metric $\tilde{d}$ below, which induces the C^{0}-topology, makes Homeo(M) a complete metric space.

$\tilde{d}(f, g) = d(f, g) + d(f^{-1}, g^{-1})$

Moreover, ClDiff(M) is also a Baire space.

Let P be a property for homeomorphisms. We say that generic homeomorphisms in Homeo(M) (resp. ClDiff(M)) satisfy P if the set of homeomorphisms satisfying P is residual in Homeo(M) (resp. ClDiff(M)). There are some result about generic homeomorphisms, for example [9], [19], and our result [8] below.

<u>(3.1) THEOREM.</u> Generic $f \in$ ClDiff(M) satisfies the following:

(0) f is strong C^{0}-tolerance stable;

(1) f has the pseudo-orbit tracing property;

(2) f is C^{0}-tolerance stable;

(3) f is not topologically R-stable; and

(4) f is not topologically stable.

By the above theorem, for every compact differentiable manifold M, we can see the existence of the homeomorphisms which have the POTP but are not topologically stable.

By [5] or [17], if $\dim M \leqq 3$, then $\mathrm{ClDiff}(M) = \mathrm{Homeo}(M)$. Therefore we have the following theorem.

<u>(3.2) THEOREM.</u> If $\dim M \leqq 3$, then generic $f \in \mathrm{Homeo}(M)$ satisfies the properties (0) - (4) in Theorem 3.1.

The above theorem was first proved by K.Yano [20] in the case of $M = S^1$. We prove (0) of Theorem 3.1 by the facts ①, ② below and Proposition 1.3. By Proposition 1.3, the cases (1) and (2) are corollaries of (0). We prove (3) by Proposition 1.7 and the fact ③ below. By Proposition 1.8, the case (4) is a corollary of (0).

① The set of all AS-diffeomorphisms (diffeomorphisms which satisfy Axiom A and the Strong Transversality Condition) is C^0-dense in $\mathrm{ClDiff}(M)$ (see [11] or [12]).

② Every AS-diffeomorphism is topologically stable (see [7]).

③ Generic $f \in \mathrm{ClDiff}(M)$ has infinitely many chain components.

<u>PROOF OF ③.</u> It is easy to see that $\mathrm{ClDiff}(M)$ is a topological group under the composition. Therefore the same conclusions of Theorem 1 in [10] are also proved for $\mathrm{ClDiff}(M)$ by the same argument. The conclusion (h) of Theorem 1 in [9] does not seem same as ③ but the proof of it easily induces ③.

<u>(3.3) REMARK.</u> Palis et al. [9] proved $\overline{\mathrm{prm}}(f) = \Omega(f)$ for generic $f \in \mathrm{Homeo}(M)$ by the C^0-Closing Lemma. In the case of $\dim M \geqq 2$, we can also prove $\overline{\mathrm{prm}}(f) = R(f)$ for generic

$f \in \mathrm{Homeo}(M)$ by Lemma 10 in [16]. In the case of $M = S^1$, the same conclusion is valid, see Remark 2.5. This implies (a)-(e) of Theorem 1 in [9], see Theorem 3.11 in [10].

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