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GENERALIZED MODULAR SYMBOLS AND COHOMOLOGY OF ARITHMETIC GROUPS

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This talk was to a large extent a report on some joint work with Avner Ash [1].

Let G be the group of real points of a semi-simple $\mathbb{Q}$-group, K a maximal compact subgroup of G, $X = G/K$ and Γ an arithmetic subgroup of $G(\mathbb{Q})$. The real cohomology of Γ may be identified with $H^{\cdot}(\Gamma\backslash X; \mathbb{R})$. In this paper, we give two related geometric constructions of infinite, locally finite, cycles whose dual cohomology classes are non-zero, in fact restrict non-trivially to the cohomology of certain faces of the Borel-Serre boundary. One family of such cycles consists of the fundamental classes of the so-called "generalized modular symbols", namely the quotients $(\Gamma\cap M)\backslash X_M$, where M is the group of real points of a Levi $\mathbb{Q}$-subgroup of a parabolic $\mathbb{Q}$-subgroup P of G, X_M the quotient of M by a maximal compact subgroup. For suitable choices of Γ, they admit natural embeddings in $\Gamma\backslash X$ are orientable and are shown to have a strictly positive intersection with compact cycles associated to the unipotent radical of P. The cohomology classes thus obtained are all not square integrable.

[1] A. Ash and A. Borel, "*Generalized modular symbols*", Proceedings of a Workshop on cohomology of arithmetic groups, Luminy 1989, to appear in Springer Lecture Notes in Mathemaitcs.