数理解析研究所講究録
第 750 巻 1991 年 1-2

Radon transform an real hyperbolic spaces

Let H^n be the real hyperbolic space of dimension n, $1 \le k \le n-1$, and $\Sigma = \Sigma_k$ be the family of all totally geodesic submanifolds of H^n of dimension k. Σ carries a natural manifold structure. Moreover, each $\xi \in \Sigma$ carries the induced surface measure $d_k x$ from the volume element $d_n x$ in H^n. We can therefore define the k-dimensional Radon transform $R = R_k$ by

$$Rf(\xi) = \int_\xi f(x)\, d_k x \quad ,$$

at least for functions $f \in C_0^\infty(H^n)$. We have $Rf \in C_0^\infty(\Sigma)$.

Let $x \in H^n$, Σ_x the family of all $\xi \in \Sigma$ such that $x \in \xi$. It also carries a natural measure $d\xi$, which is "independent of x" in an obvious sense. We define the adjoint operator R^* as follows:

$$R^*\varphi(x) = \int_{\Sigma_x} \varphi(\xi)\, d\xi \quad .$$

Given a function κ of a single real variable t we can define a "radial convolution operator" K on H^n by

$$Kf(x) = \int_{H^n} f(y)\, \kappa(d(x,y))\, d_n y \quad ,$$

where $d(x,y)$ denotes the hyperbolic distance between x and y.

The operator R^*R is a radial convolution operator (as shown by Helgason) and for k even it has an inversion formula of the form

$$P(\Delta)\ R^*R = id\ ,$$

where P is an explicit polynomial in the Laplace-Beltrami operator (Helgason) .

In joint work with E. Casadio Tarabusi, I have shown for k odd that if $\kappa(t) = \cosh t(\sinh t)^{k-n}$ and K is the corresponding convolution operator, then there is an explicit polynomial Q such that

$$Q(\Delta)\ K\ R^*R = id\ .$$

The original case of interest was the x-ray transform in the hyperbolic disk, because of its applications to Applied Potential Tomography. In this case $k=1$, $n=2$, and if we take

$$\kappa(t) = \frac{\cosh t}{\sinh t} - 1$$

the corresponding operator K has an integrable kernel and the inversion formula for R^*R is

$$-\frac{1}{4\pi}\ \Delta\ K\ R^*R = id\ .$$

Helgason has a different inversion formula of the operator R involving Abel integral equations.

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