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Negativity and Vanishing of Microfunction Solution Sheaves at the Boundary

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Introduction. Let M be a real analytic manifold with a complexification X. Let V be a $\mathbb{C}^\times$-conic involutive submanifold of $\overset{\circ}{T}{}^*X (= T^*X \setminus X)$, and let $\mathfrak{M}$ be a coherent $\mathcal{E}_X$-module with constant multiplicity along V. Moreover let Ω be an open subset of M with real analytic boundary $N = \partial\Omega$. The aim of this note is to give vanishing theorems for the cohomology groups of the complex $\mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_{\Omega|X})$ where $\mathcal{C}_{\Omega|X}$ is the complex of microfunctions at the boundary introduced by P. Schapira [S].

The vanishing of the complex $\mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_M)$ has been studied by M. Sato *et al.* [SKK], M. Kashiwara [K] and Kashiwara-Schapira [K-S2], and we study in this talk an analogous problem at the boundary.

Main Theorems. Let M, X, Ω and N be as in §1.1. Then we give

THEOREM 1. *Let $p \in \overset{\circ}{T}{}^*_M X$ with $\pi_X(p) \in N$, and let $V = \{q \in \overset{\circ}{T}{}^*X;\ f(q) = 0\}$ be given by a homogeneous holomorphic function f satisfying the condition*

$$\{f, f^c\}(p) < 0. \tag{1}$$

Assume that there exists a homogeneous holomorphic function ψ for which the following conditions (2), (3), (4), (5) are satisfied.

$$d\psi \wedge \omega_X \neq 0 \text{ at } p. \quad (\omega_X \text{ is the canonical 1-form of } T^*X.) \tag{2}$$

$$V \cap \overline{V} \subset \{\psi = 0\}. \tag{3}$$

$$\mathrm{Im}\ \psi\,|_{T^*_M X} = 0. \tag{4}$$

$$\pi_X^{-1}(\Omega) \cap T^*_M X \subset \{\psi > 0\} \text{ in a neighborhood of } p. \tag{5}$$

Let $\mathfrak{M}$ be a coherent $\mathcal{E}_X$-module with constant multiplicity along V defined in a neighborhood of p. Then we have

$$\mathrm{H}^0\, \mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_{\Omega|X})_p = 0.$$

THEOREM 2. *Let p, V, Ω be as in Theorem 1. Let W ($\subset V$) be a $\mathbf{C}^{\times}$-conic involutive variety in $\overset{\circ}{T}{}^*X$ through p with q (≥ 1) negative eigenvalues of $\mathcal{L}_\Lambda(W)(p)$. Let $\mathfrak{M}$ be a coherent $\mathcal{E}_X$-module with constant multiplicity along W defined in a neighborhood of p. Then we have*

$$\mathrm{H}^j\,\mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_{\Omega|X})_p = 0 \qquad (j < q).$$

Moreover if $\mathcal{L}_\Lambda(W)(p)$ is non-degenerate, then we have the vanishing of the left-hand side for $j \neq q$.

Next we give a generalization of Theorem 1.

THEOREM 3. *Let $p \in \overset{\circ}{T}{}^*_M X$ with $\pi_X(p) \in N$, and let W be a $\mathbf{C}^{\times}$-conic involutive variety of codimension d with $p \in W$ and $\mathcal{L}_\Lambda(W)(p) < 0$. Assume that there exists a homogeneous holomorphic function ψ with the properties;*

$$d\psi \wedge \omega_X \neq 0 \quad \text{at } p, \tag{6}$$

$$\mathrm{Im}\ \psi\ |_{T^*_M X} = 0, \tag{7}$$

$$W \cap \overline{W} \subset \{\psi = 0\}, \tag{8}$$

$$\pi_X^{-1}(\Omega) \cap T^*_M X \subset \{\psi > 0\} \text{ in a neighborhood of } p. \tag{9}$$

Let $\mathfrak{M}$ be a coherent $\mathcal{E}_X$-module with constant multiplicity along W. Then we have

$$\mathrm{H}^j\,\mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_{\Omega|X})_p = 0 \qquad (j \neq d).$$

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