数理解析研究所講究録
第758巻 1991年 1-6

Strongly countably complete spaces と fragment について

神戸大院生 長水寿寛 (Toshihiro Nagamizu)

Dfinition 1. Let X be a topological space and ρ be a metric on X. X is said to be fragmeted by ρ (or ρ-fragmented) if for each $\varepsilon>0$ and each nonempty subset A of X there is an open subset U of X such that $U\cap A\neq\phi$ and $\rho\text{-diam}(U\cap A)\leqq\varepsilon$.

The topological space X is said to be fragmentable if there exists a metric on X which fragments X.

Definition 2. A well ordered family $\mathcal{U}=\{U_\xi \mid 0\leqq\xi<\xi_0\}$ of subsets of the topological space X is said to be a relatively open partitioning of X ,if

(1) $U_0=\phi$;

(2) U_ξ is contained in $X\setminus(\cup_{\eta<\xi} U_\eta)$ and is relatively open in it for every ξ , $0<\xi<\xi_0$;

(3) $X=\cup_{\xi<\xi_0} U_\xi$.

A family $\mathcal{U}$ of subsets of X is said to be a <u>σ-relatively open partitioning of X</u>, if $\mathcal{U}=\cup_{n=1}^{\infty}\mathcal{U}^n$, where $\mathcal{U}^n$, n=1,2,... are relatively open partitionings of X.

$\mathcal{U}$ is said to <u>separete the points of X</u>, if whenever x and y are two different elements of X there exists n such that x and y belong to different elements of the partitioning $\mathcal{U}^n$.

In this case we say that X admits a <u>separating σ-relatively open partitioning</u>.

In [9], N.K.Ribarska proved the following two theorems.

Theorem (N.K.Ribarska). The topological space X admits σ-relaitve open partitioning if and only if there exists a metric which fragments X.

Theorem A (N.K.Ribarska). Let X be a compact Hausdorff space. If X is a fragmentable then there exists a complete metric ρ on X such that X is ρ-fragmented and the topology generated by ρ is stronger than the original topology on X.

A.V.Arhangel'skii proved that a functionally complete compact Hausdorff space is an Eberlein compact space ([1]). And each Eberlein compact space is Radon-Nikodým compact space which is homeomorphic to a norm-fragmented w -compact subset of a dual Banach spce ([7]). Hence we obtain the following theorem.

Theorem B (I.Namioka [7]). Let X be an Eberlein compact space. Then X is fragmented by a lower semi-continuous metric.

In this note we extend these results (Thorem A and Theorem B) to the class of the strongly countably complete spaces.

Definition 3. A topological space X is said to be <u>strongly countably complete</u> (<u>s.c.c.</u>) if there exists a strongly countably complete sequence of open coverings of X.

Theorem 1. Let X be a completely regular s.c.c. space. If X is a fragmentable then there exists a complete metric on X such that X is ρ-fragmented and the topology generated by ρ is stronger than the original topology on X.

Theorem 2. Every completely regular s.c.c. functionally complete space is fragmented by a lower semi-continuous metric.

Definition 4. Let X be a topological space. X is said to be a Namioka space if the following condition is satisfied for any compact space Y;

(*) for any separately continuous function $f: X\times Y\to R$ there exists a dence G_δ subset A of X such that f is jointly continuous at each point of $A\times Y$ (where R is a real line).

Remark. Completely regular s.c.c. space is a Namioka space. Each closed subspace of a s.c.c. space is also s.c.c., hence Namioka space.

Theorem 3. Let X be a completely regular functionally complete space. If each closed subspace of X is a Namioka space then X is fragmented by a lower semi-continuous metric.

From Theorem 3 we get Theorem 2.

References

[1] A.V.Arhangel'skii, On some topological spces that occur in functional analysis, Russian Math. Math. Surveys, 31 (1976), 14-30.

[2] Z.Frolik, Baire spaces and some generalizations of complete metric spaces, Czech. Math. J., 11 (1961), 359-379.

[3] J.E.Jayne and C.A.Rogers, Borel selectors for upper semi-continuous set-valued maps, Acta Math., 155 (1985), 41-79.

[4] J.L.Kelly and I.Nanioka, et al. Linear topological spaces (Springer, New York, 1976).

[5] J.P.Lee and Z.Piotrowski, A note on spaces related to Namioka spaces, Bull. Austral. Math., 31 (1985) 285-292.

[6] I.Namioka, Separately continuity and jointly continuity, Pacific J. Math., vol 51, No2 (1974), 515-531.

[7] I.Namioka, Eberlein and Radon-Nikodým compact spaces, Lecture note of a course given at University College London, 1985/86. See also Mathematika, 34 (1987).

[8] Z.Piotrowski, Seprate and joint conyinuity,(preprint).

[9] N.K.Ribarska, Internal characterizationof fragmentable spaces, Mathmatika, 34 (1987), 243-257.

[10] M.Talagrand, Espaces de Baire et espaces de Namioka, Math. Ann., 270 (1985), 159-164.

Depertment of Mathematica and
System Fundamentals
The Graduate School of
Science and Technology
Kobe University
Nada, Kobe
Japan