

## Fractal Dimension of Brain Wave

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Fractal natures of brain wave, that is of electroencephalogram (EEG), are investigated, made use of some fractal dimensionalities of it. Especially as for  $\alpha$ -wave, those of frontal and occipital region are compared with respect of the fractal dimension. Significant difference of the dimension gives such a conjecture that those  $\alpha$ -wave are independently created. It is indicated that EEG can be modelled by a deterministic dynamical system with a chaotic nature.

### 1. Introduction

Recently numbers of papers are published on the fractal dimensionality of turbulent time-series of data. One of those time-series data is of brain wave, that is of electroencephalogram (EEG). Although, depending on the method of the fractal analysis on the time-series data and on the experimental condition under which the data are sampled, the obtained fractal dimensions are more or less different from each other, EEG has been an attractive object to be investigated with respect of the fractal nature (Babloyantz, *et al.*, 1985; Mayer-Kress and Layne, 1987; Watt and Hameroff, 1988; Xu-Nan and Xu-Jinghua, 1988; Arle and Simon, 1990).

In this paper, EEG, mainly  $\alpha$ -wave, is investigated with respect of its fractal dimensionality. We have been sampled EEG data through a series of experiments following the international 10-20 system placement (Reilly, 1982). We shall apply three different approach to analyze the data with respect of the fractal dimensionality: a) correlation dimension; b) dimensionality of frequency distribution; c) fractal dimension of graph. The approach of correlation dimension has been applied to estimate the fractal nature of a variety of turbulent time-series data (Kariniemi and Ammala, 1981; Mondanlou and Freeman, 1982; Goldberger, *et al.*, 1984; Cohen and Procaccia, 1985; Pickover and Khorasani, 1986; Liebovitch, *et al.*, 1987; Mpitsos, *et al.*, 1988). Although the dimensionality of frequency distribution

of time-variable is in general difficult to be satisfied for the data obtained from the natural system, we shall investigate if the dimensionality is applicable for EEG or not. The third fractal dimensionality we choose, the fractal dimension of graph will be shown to be useful to estimate the fractal nature of EEG. It corresponds to the dimension analysis on the fractal nature of coastal line (Mandelbrot, 1982; Higuchi, 1988). Making use of these three approaches, we shall consider the fractal nature of EEG and try to discuss some aspects of cerebral system creating EEG.

## 2. Analysis

We analyze time-series of EEG data, making use of three different approaches to pull out the fractal nature of EEG (as for the concrete calculating method, see Appendix:

*Correlation Dimension:* For a time-series of data  $(x_0, x_1, \dots, x_M)$ , where  $x_i$  ( $i = 0, 1, \dots, M$ ) is the  $i$  th data of EEG amplitude and  $M$  is the total number of data obtained with the time interval  $\Delta t$ , we can calculate the following correlation integral:

$$C(r) = \frac{1}{N^2} \sum_i^N \left\{ \sum_j^N H(r - \|\vec{x}_i - \vec{x}_j\|) - 1 \right\},$$

where the function  $H(z)$  is the step function which is 1 for non-negative  $z$  and 0 for negative  $z$ .  $\|\cdot\|$  is a proper norm for the  $d$ -dimensional space.  $N$  is the total number of  $d$ -dimensional vectors  $\vec{x}_i$  ( $i = 1, 2, \dots, M - d + 1$ ) which are constructed from  $(x_0, x_1, \dots, x_M)$  as  $\vec{x}_i = (x_i, x_{i+1}, x_{i+2}, \dots, x_{i+d-1})$  ( $i = 1, 2, \dots, M - d + 1$ ) (Takens, 1980). If the correlation integral  $C(r)$  satisfies

$$C(r) \sim r^D,$$

$\alpha$ -wave	3.33 $\pm$ 0.32 (22)
$\beta$ -wave	5.52 $\pm$ 0.97 (13)
mixed wave	4.5 ~ 5.5

Table 1. Correlation dimension of EEG. The number in the bracket shows that of subjects in the experiment. The mean  $\pm$  standard deviation is shown for the fractal dimension of  $\alpha$ -wave and  $\beta$ -wave. In the case of mixed wave, sufficient data could not be sampled.

then the power  $D$  can be regarded as a fractal dimension, say 'correlation dimension', and it can saturate for a sufficiently large embedded dimension  $d$  (Grassberger and Procaccia, 1983). It implies the dimension of attractor in the  $d$ -dimensional phase space, which is created by a dynamical system governing the time-series. Thus, if we can estimate the correlation dimension  $D$  for EEG, it gives such a possibility that the EEG may be governed by a dynamical system and further may be a low dimensional chaos.

In Table 1, the result for  $\alpha$ -wave,  $\beta$ -wave, and wave mixed both are shown. The mixed wave seems to have an intermediate correlation dimension, compared with those of  $\alpha$ -wave and  $\beta$ -wave, though we could not sample satisfactory data for the case of mixed wave.

*Dimensionality of Frequency Distribution:* If the cumulative frequency distribution  $P(X)$  for an amplitude  $X$  of EEG follows the following geometric law:

$$P(X) \equiv \int_x^{\infty} p(y) dy \approx X^{-\gamma}$$

then the power  $\gamma$  can be regarded as a fractal dimension derived from the frequency distribution (Mandelbrot, 1982). Although it has not yet been given any satisfactory explanation why there are some distributions following such a geometric law for natural phenomena, the dimensionality

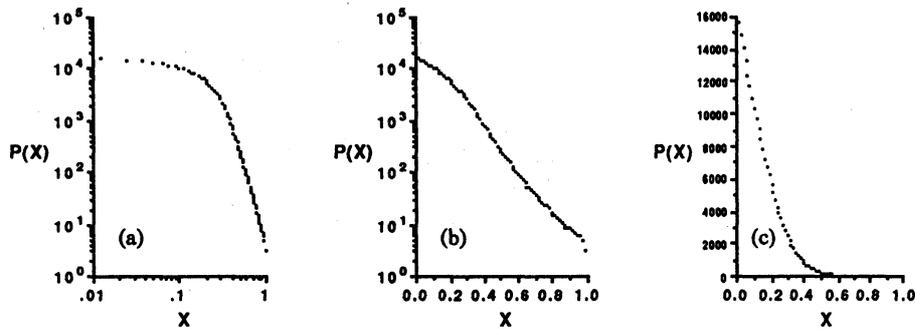


Figure. An example of sampled cumulative frequency distribution of  $\alpha$ -wave amplitude. (a) log-log axis; (b) normal-log axis; (c) normal-normal axis. Maximum amplitude is normalized to 1.

can be regarded as one of strong characteristics of the time-series data, if it can be appropriately estimated.

Although obtained frequency distribution of EEG for some data sets is partially linear with the log-log axis, it generally seem not to follow any geometric law. Instead, as shown in Figure, the distribution for some data seems to be alternatively exponential rather than geometric. Therefore, as our result, it is shown that EEG does not have the fractal nature in terms of the frequency distribution.

*Fractal Dimension of Graph:* Since the temporal variation of EEG is expressed as a 2-dimensional graph, the fractal dimension of the graph can be estimated. We shall apply the estimating method making use of the cumulative length of EEG oscillation. In detail, we deal with the following quantity (Higuchi, 1988):

$$L_{\Delta t} \equiv \sum_{i=1}^{M-1} |x_{i+1} - x_i|$$

	FRONTAL REGION	OCCIPITAL REGION	CORRELATION COEFFICIENT
SPECTRAL FLUCTUATION	0.938 ± 0.144	1.032 ± 0.141	0.832
GRAPH DIMENSION	1.825 ± 0.018	1.862 ± 0.03	0.069

Table 2. Fractal dimension of graph and spectral fluctuation of  $\alpha$ -wave.  $\alpha$ -waves of the frontal and the occipital regions are independently investigated. The mean  $\pm$  standard deviation is shown for the fractal dimension. Correlation coefficient between the data for two regions is calculated, too. 16 sets of data are used.

which corresponds to the cumulative length of EEG oscillation, measured in a time unit  $\Delta t$ . If  $L_{\Delta t} \propto \Delta t^{-\beta}$ , then the power  $\beta$  can be regarded as the fractal dimension of EEG graph. If  $\beta$  can be appropriately determined, it indicates a fractal nature of EEG.

The result for  $\alpha$ -wave is shown in Table 2. The dimension is estimated respectively for  $\alpha$ -waves of frontal region and of occipital one. The dimension is significantly higher in the occipital region than in the frontal. The correlation coefficient between the frontal and the occipital  $\alpha$ -waves is estimated by the data for each subject. The fractal dimensions of graph for those  $\alpha$ -waves have a low correlation coefficient. This result indicates that those  $\alpha$ -waves of the frontal and the occipital regions can be regarded independent each other. In contrast, as shown in Table 2, the spectral fluctuation of  $\alpha$ -wave amplitude shows high correlation between those two regions.

### 3. Conclusion

Fractal nature of EEG is investigated to show such possibility that EEG may be able to be described as a chaotic oscillation driven by a dynamical system.

Our results shows that EEG has a clear fractal nature and thus can be described or at least can be modelled by a deterministic dynamical system with a finite number of variables.

By our fractal analysis on  $\alpha$ -wave, it is concluded that  $\alpha$ -wave of the frontal region is correlated little with that of the occipital one. This implies that there may be two systems driving  $\alpha$ -wave, which have a weak

connectivity between them. Contrarily since the spectral fluctuation results in a high correlation between the frontal and the occipital regions, it is probable that the spectral fluctuation may be originated either to a system different from the system driving  $\alpha$ -wave or to a part common between those two systems driving  $\alpha$ -wave.

Although the data are sampled not sufficient up to now, the fractal analysis on  $\beta$ -wave and on  $(\alpha, \beta)$ -mixed wave indicates such possibility that  $\beta$ -wave may be driven by the system same with that driving  $\alpha$ -wave.

The quantitative results by our analysis will be possibly different more or less from those in past and future. This may be because the fractal dimension strongly depends on the method to calculate it by data and is influenced by the quality of data very much, and because EEG is very sensitive to the condition in and out of the subject in the experiment (Xu-Nan and Xu-Jinghua, 1988). However, the accumulation of researches on EEG will lastly clarify the fractal nature of it. We hope that our work will contribute at least to the promotion of such research.

## Appendix

In this appendix, it is shown how the calculation is carried out for each of three methods applied to EEG data in order to estimated the fractal dimensionality.

*Correlation Dimension:* For  $d$ -dimensional embedded vectors constructed from EEG data as mentioned in the main text, we calculate the number  $\#_r(\vec{x}_{k_j})$  of vectors in a distance  $r$  from a randomly selected vector  $\vec{x}_{k_j}$  determined by a randomly selected integer  $k_j$  ( $j = 1, 2, \dots, L$ ) less than  $N + 1$ .  $L$  is the number of randomly selected vectors which are used to calculate this value.  $L$  should be sufficiently large. Then the correlation integral  $C(r)$  corresponds to

$$\frac{\sum_{j=1}^L \#_r(\vec{x}_{k_j})}{L} / N^2,$$

Therefore if  $C(r) \sim r^D$ , we can estimate the correlation dimension  $D$  from the

linear relation, that is the slope of graph, between  $\log r$  and  $\log \sum_{j=1}^L \#_r(\vec{x}_{k_j})$ .

*Dimensionality of Frequency Distribution:* It is necessary only to count the number of data of EEG amplitude more than  $X$  in order to obtain the cumulative frequency distribution  $P(X)$  for an amplitude  $X$  from the data.

Actually,

$$P(X) \approx \frac{\sum_{i=1}^M H(X - x_i)}{M},$$

where the function  $H(z)$  is 1 for non-negative  $z$  and 0 otherwise.  $M$  is the total number of data. If the relation  $P(X) \propto X^{-\gamma}$  is satisfied, the power  $\gamma$  is

determined from the slope of graph between  $\log X$  and  $\log \sum_{i=1}^M H(X - x_i)$ .

*Fractal Dimension of Graph:* The value  $L_{\Delta t}$  can be easily calculated from the data for a time-interval  $\Delta t$ . Then, with the same data set for  $\Delta t$ , we can calculate  $L_{2\Delta t}$ ,  $L_{3\Delta t}$ , and so on, making use of the following correspondence:

$$\begin{array}{l}
 \Delta t \longleftrightarrow \{x_1, x_2, x_3, \dots, x_M\} \\
 \\
 2\Delta t \longleftrightarrow \{x_1, x_3, x_5, \dots\} \\
 \qquad \qquad \qquad \{x_2, x_4, x_6, \dots\} \\
 \\
 3\Delta t \longleftrightarrow \{x_1, x_3, x_7, \dots\} \\
 \qquad \cdot \qquad \{x_2, x_5, x_8, \dots\} \\
 \qquad \cdot \qquad \{x_3, x_6, x_9, \dots\} \\
 \qquad \cdot
 \end{array}$$

Therefore, if  $L_{\Delta t} \propto \Delta t^{-\beta}$ , the power  $\beta$  can be estimated from the slope of graph between  $\log n$  and  $\log \Sigma L_{n\Delta t}$ , where the sum  $\Sigma$  is carried over  $n$  corresponding sets for an given  $n$  as shown above.

## References

- J. E. Arle and H. Simon. 1990. "An application of fractal dimension to the detection of transients in the electroencephalogram." *Electroencephalography and clinical Neurophysiology*. 75, 296-305.
- A. Babloyantz, J. M. Salazar and C. Nicolis. 1985. "Evidence of chaotic dynamics of brain activity during the sleep cycle." *Phys. Lett.* 111A, 152-156.
- A. Cohen and I. Procaccia. 1985. "Computing the Kolmogorov entropy from time series signals of dissipative and conservative dynamic systems." *Phys. Rev. A*. 31A, 1872-1882.
- A. L. Goldberger, L. J. Findley, M. R. Blackburn and A. J. Mandell. 1984. "Nonlinear dynamics in heart failure: implications of long-wavelength cardiopulmonary oscillations." *Am. Heart J.* 107, 612-615.
- P. Grassberger and I. Procaccia. 1983. "Measuring the strangeness of strange attractors." *Physica*. 9D, 189-208.
- T. Higuchi. 1988. "Approach to an irregular time series on the basis of the fractal theory." *Physica D*. 31, 277-283.

- V. Kariniemi and P. Ammala. 1981. "Short-term variability of fetal heart rate during pregnancies with normal and insufficient placental function." *Am. J. Obstet. Gynecol.* **139**, 33.
- L. S. Liebovitch, J. Fischberg, J. P. Koniarek, I. Todorova and M. Wang. 1987. "Fractal model of ion-channel kinetics." *Biochim. Biophys. Acta.* **896**, 173-180.
- B. B. Mandelbrot. 1982. *The Fractal Geometry of Nature* San Francisco: Freeman.
- G. Mayer-Kress and S. P. Layne. 1987. "Dimensionality of human electroencephalogram." *Perspectives in Biological Dynamics and Theoretical Medicine.*, S. H. Koslow, A. J. Mandell and M. F. Shlesinger (ed.), Austria: Urban and Schwarzenberg.
- H. D. Mondanlou and R. K. Freeman. 1982. "Sinusoidal fetal heart rate pattern: its definition and clinical significance." *Am. J. Obstet. Gynecol.* **142**, 1033-1038.
- G. J. Mpitsos, R. M. Burton, H. C. Creech and S. O. Soinila. 1988. "Evidence for chaos in spike trains of neurons that generate rhythmic motor patterns." *Brain Res. Bull.* **21**, 529-538.
- C. A. Pickover and A. L. Khorasani. 1986. "Fractal characterization of speech waveform graphs." *Comput. Graphics.* **10**, 51-61.
- E. L. Reilly. 1982. "EEG recording and operation of the apparatus." *Electroencephalography.*, E. Niedermeyer and F. L. da Silva (ed.), Austria: Urban and Schwarzenberg.
- F. Takens. 1980. "Detecting strange attractors in turbulence." *Dynamical Systems and Turbulence.*, D. A. Rand and L. S. Young (ed.), 365-381. New York: Springer.
- R. C. Watt and S. R. Hameroff. 1988. "Phase space electroencephalograms (EEG): a new mode of intraoperative EEG analysis." *Int. J. Clin. Monit. Comput.* **5**, 3-13.
- Xu-Nan and Xu-Jinghua. 1988. "The fractal dimension of EEG as a physical measure of conscious human brain activities." *Bull. Math. Biol.* **50**, 559-565.