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Knowledge Network and Conceptual Semantics

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1. Introduction

In this paper we introduce a knowledge representation as the set of locally formulated logical systems which are connected each other by the symbolic relations. For this purpose we regard our knowledge as the logical systems characterized by axioms and introduce the symbolic correspondence between them. We also introduce modal operators to discuss the semantics for natural language. For the purpose we construct Kripke model from the knowledge networks. The interpretation of the meaning of modal operators such as $\Diamond$ (may) and $\Box$ (must) are given on the model. We also discuss maintenances of knowledge network and problems of default reasoning by using the modal operators. Furthermore we construct three valued model which become equivalent to data semantics introduced by F. Veltman. The model gives another aspect of the interpretation of knowledge network.

As a result we have introduced non transcendental foundation for semantics. These approach aims opposite direction of realist theory of meanings. Meaning should be given not by transcendental real world but by our own restricted knowledge.

2. Knowledge Unit and Network

In this section we give basic formulations and some propositions. We use the usual first order predicate calculus. As a rule of inference we use the Modus-Ponens and the Generalization [1,7].

Def.2.1 Vocabulary and Type

The set of constants of a language L is denoted by V_c, the set of m-variable functional symbols by Vf^m and the set of n-variable predicate symbols by V_r^n. We call m,n the type of functional symbols and the type of predicate symbols respectively.

Let $V_f=\cup\{V_f^i \mid i\in N\}, V_r=\{V_r^j \mid j\in N\}$, $V=V_c\cup V_f\cup V_r$, where V is called the vocabulary of language L.

The language L is defined by using vocabulary. The theory on the language is defined by determining the set of axioms which represent some local knowledge. Then the theory is called a knowledge unit and denoted by an ordered pair <V, Γ> or simply Γ, where Γ denote the set of axioms. We recognize our local knowledge as a knowledge unit. Now we assume the consistency of the axioms. Next we introduce relations between units.

Def.2.2 Simple Interpretation

We introduce the one to one functions between vocabularies of knowledge units $<V_1, \Gamma_1>$, $<V_2, \Gamma_2>$ such as (1) $I_c: V_{1c} \rightarrow V_{2c}$ (2) $I_f^i: V_{1f}^i \rightarrow V_{2f}^i$ (3) $I_r^j: V_{1r}^j \rightarrow V_{2r}^j$, $i, j \in N$.

Def.2.3 Interpretation

The interpretation between formulas is defined as follows, where J denote the interpretation.

(A) Interpretation of Term

(1) Let x be a variable then $J(x)=x$

(2) Let term be a constant c then $J(c)=I_c(c)$.

(3) Let f be a m-variable function symbol and $t_1,\ldots, t_m$ be terms, then $J(f(t_1,\ldots,t_m))=I_f^m(f)(J(t_1),\ldots,J(t_m))$

(B) Interpretation of Atomic Formula

(1) Let t_1, t_2 be terms and $t_1=t_2$, then $J(t_1=t_2)=$"$J(t_1)=J(t_2)$"

(2) Let P be a n-variable predicate symbol and $t_1,\ldots, t_n$ be terms, then $J(P(t_1,\ldots,t_n))=I_r^n(P)(J(t_1),\ldots,J(t_n))$

(C) Interpretation of Formula

(1) Let Φ,Ψ be formulas, then $J(\Phi\wedge\Psi)=J(\Phi)\wedge J(\Psi)$, $J(\neg\Phi)=\neg J(\Phi)$

(2) Let x be a variable and F be a formula, then $J(\forall xF)=\forall xJ(F)$

Symbol "I" is commonly used for denoting any types of interpretation.

Def.2.4 Extension of Vocabulary

The simple extension of vocabulary V is defined as follows, where V' denote the extension.

(1) $V'=V_c' \cup V_f' \cup V_r'$

(2) $V_c' \supseteq V_c, V_f' \supseteq V_f, V_r' \supseteq V_r$

(3) The constant, function and predicate symbols, which belong to V_c', V_f' and V_r' and which do not belong to V_c, V_f and V_r respectively, must be introduced as follows.

(3a) Let Ψ be the formula of a theory Γ which include only one free variable y. If $\Gamma \vdash \exists! y\, \Psi(y)$ holds then we can add the new constant symbol c which satisfy $\Psi(c)$.

(3b) Let Ψ be the formula of a theory Γ which include n+1 variables $x_1,\ldots,x_n,y$. If $\Gamma \vdash \forall x_1,\ldots,x_n \exists! y\, \Psi(x_1,\ldots,x_n,y)$ holds then we can add the new function symbol f which satisfy such condition as "$y=f(x_1,\ldots,x_n) \equiv \Psi(x_1,\ldots,x_n,y)$".

(3c) Let Ψ be the formula of a theory Γ which include n free variables. Then we can add the new predicate symbol P which satisfy such condition as "$r(x_1,\ldots,x_n) \equiv \Psi(x_1,\ldots,x_n)$".

We can get further extension by using these new symbols. Thus we define extension by definition as a result of finite simple extension and denote it by V_{ex}.

Def.2.5 Extension of Theory

Let $<V,\Gamma>$ be a theory and V_{ex} be a extension by definition of V. Then the extension by definition of the theory is defined by introducing following axioms according to the new symbols ,where $<V_{ex},\Gamma_{ex}>$ denote the extended theory.

(A) $\Psi(c)$

(B) $\forall x_1,\ldots,x_n \exists! y\ \ y=f(x_1,\ldots,x_n)\equiv\Psi(x_1,\ldots,x_n,y)$

(C) $\forall x_1,\ldots,x_n \exists! y\ \ r(x_1,\ldots,x_n)\equiv\Psi(x_1,\ldots,x_n)$

We often omit suffix "ex" without confusion.

Def.2.6 Comprehension

Let "I" be an interpretation between $<V_1,\Gamma_1>$and$<V_2,\Gamma_2>$. The interpretation "I" is called comprehension if $I(\Phi)$ become a theorem of Γ_2 for any axiom $\Phi\in\Gamma_1$. Then we call that the theory Γ_2 is comprehended by Γ_1 or Γ_1 comprehend Γ_2 and it is denoted by $\Gamma_1\Rightarrow\Gamma_2$. If a comprehension is an identity map then it is called trivial.

The notion of comprehension include the notion of satisfy as a special case if we treat the theory with the complete axioms as the structure.

Prop.2.1

Let "I" be a comprehension from theory Γ_1 to Γ_2 or Γ_{2ex}.

(1) If Φ is a theory of Γ_1 then $I(\Phi)$ become a theory of Γ_2.

(2) If $I(\Phi)$ is shown to be a theory of Γ_2 for a sentence Φ of Γ_1 then $\neg\Phi$ can not be shown to be a theorem of Γ_1.

Proof 2.1 (1) (A) If Φ is an axiom of Γ_1 then $\Gamma_2 \vdash I(\Phi)$ hold because of the definition of the comprehension. (B) If Φ is not an axiom then there exist the sequence of the proof on Γ_1. Corresponding to the sequence we can construct the sequence of proof on Γ_2. For this purpose the following commutative diagram of proof must be shown for the schema of Modus-Ponens and Generalization.

$$\begin{array}{ccccccc} I(\Phi),\ I(\Phi\to\Psi) & \Rightarrow & I(\Psi) & & I(\Phi) & \Rightarrow & \forall x I(\Phi) \\ \Uparrow & & \Uparrow & & \Uparrow & & \Uparrow \\ \Phi,\ \Phi\to\Psi & \Rightarrow & \Psi & & \Phi & \Rightarrow & \forall x\Phi \end{array}$$

It is shown from such properties of property of comprehension as

(a) $I(\Phi\to\Psi)$="$I(\Phi)\to I(\Psi)$",

(b) $I(\forall x\Phi)$="$\forall x I(\Phi)$".

Prop.2.2

(1) Composition of interpretations also become interpretation.

(2) Composition of comprehensions also become comprehension.

In the previous definition knowledge unit is defined by first order Language. Next we introduce set theoretical language for extending its ability of description.

Def.2.7 Set Theoretical Language

The language of set theory with urelement is a special type of the first order language with equality and defined as follows[2,6]. (1) the set of predicate symbols R={∈}. (2) the set of functional symbols F={pr,ap}. (3) the set of constant symbols C.

The symbol ∈, pr and ap are commonly used in any set theoretical languages. Thus the vocabulary is characterized only by the set of constant symbols. The function pr and ap denote an ordered pair and a value of function respectively.

Set theoretical knowledge unit are also defined as an ordered pair of the set of vocabulary and Axioms, where every set theoretical theory have common axioms for describing set theoretical operation. We adopt the axioms of the set theory with urelement [5]. The knowledge unit can be characterized by the set of its constant symbols.

Def.2.8 Set Theoretical Structure

Set theoretical structure is also introduced as a model of set theoretical theory as follows.

$U=\langle V_s,\{\in\},\{pr,ap\},C\rangle$ where V_s is the superstructure on the set of urelements S [1]. The structure can also be characterized by C.

Def.2.9 Interpretation

Simple interpretation between set theoretical language is also defined as a function between vocabularies of the knowledge units $\langle V_1, \Gamma_1\rangle$ and $\langle V_2, \Gamma_2\rangle$, where $V_1=R_1\cup F_1\cup C_1$, $V_2=R_2\cup F_2\cup C_2$ and $R_1=R_2$, $F_1=F_2$. Thus the symbol $\in$, ap and pr are always interpreted to the same symbols as follows. (1) $I(\in)=\in$ (2) $I(ap)=ap$ (3) $I(pr)=pr$. Therefore the interpretation between constant symbols is essential in the interpretation.

Def.2.10 Concept

Let $\Psi(x_1,\ldots,x_n)$ be a formula which include n free variables then we call Ψ a n-variable concept or simply a concept. Where a sentence is regarded as 0-variable concept.

Def.2.11 Equivalence of concepts

If $\Psi(x_1,\ldots,x_n)$ and $\Phi(x_1,\ldots,x_n)$ are concepts on a knowledge unit Γ. Then Ψ and Φ are called equivalent on Γ if and only if $\forall x_1,\ldots,x_n\ \Psi(x_1,\ldots,x_n) \equiv \Phi(x_1,\ldots,x_n)$ holds in Γ.

Concepts which are equivalent with each other on some unit might be not equivalent on the other unit.

Def.2.12 Concrete and Abstract

If "I" is a comprehension from $\langle V_1,\Gamma_1\rangle$ to $\langle V_2,\Gamma_2\rangle$, then $\langle V_1,\Gamma_1\rangle$ is called an abstract knowledge and $\langle V_2,\Gamma_2\rangle$ is called a concrete knowledge. The notions are relative. If "I_1" and "I_2" are comprehensions from $\langle V_1,\Gamma_1\rangle$ to $\langle V_2,\Gamma_2\rangle$ and from $\langle V_2,\Gamma_2\rangle$ to $\langle V_3,\Gamma_3\rangle$ respectively. Then $\langle V_2,\Gamma_2\rangle$ is more abstract than $\langle V_3,\Gamma_3\rangle$ and more concrete than $\langle V_1,\Gamma_1\rangle$

Def.2.13 Retraction

Let "I" be a comprehension from $\langle V_1,\Gamma_1\rangle$ to $\langle V_2,\Gamma_2\rangle$. If there exist the concept Ψ and the term s of $\langle V_1,\Gamma_1\rangle$ according to a concept Φ and a term t of $\langle V_2,\Gamma_2\rangle$ and $I(\Psi)=\Phi$ and $I(s)=t$ hold, then the concept Ψ and the term s are called the retracted concept and term respectively. There do not always exist retracted concepts and terms. Then Ψ and s are called abstract concept and term respectively and Φ and t are called concrete ones.

Def.2.14 Strategy for Inference

If "I_1" and "I_2" are comprehensions from $<V_1,\Gamma_1>$ to $<V_2,\Gamma_2>$ and from $<V_2,\Gamma_2>$ to $<V_3,\Gamma_3>$ respectively. Then the strategy to prove a sentence Ψ of $<V_2,\Gamma_2>$ is classified into next three types.

(1) If Ψ is directly proved in $<V_2,\Gamma_2>$ then the inference is called the standard one or inner theory inference.

(2) If Ψ can be retracted into the sentence $I_1^{-1}(\Psi)$ of $<V_1,\Gamma_1>$ and $I_1^{-1}(\Psi)$ is proved in $<V_1,\Gamma_1>$ then the inference is called abstract one. If the retraction of Ψ exist then the abstract inference is always proper because of the proposition 2.1

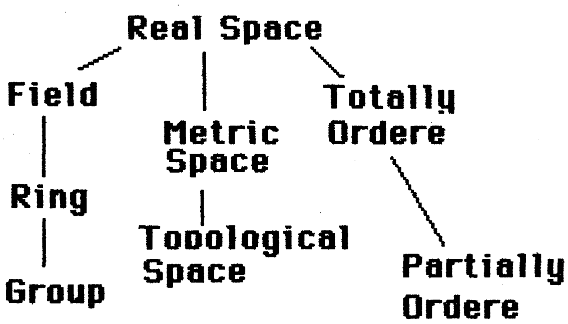
(3) If $I(\Psi)$ is proved in Γ_3 then the inference is called concrete one. Concrete inference is always applicable but not always proper because it only assure that $\neg\Psi$ does not hold in Γ_2.

Def.2.15 knowledge Network

Let TS be a set of theories and IS be a set of comprehension among the theories. Then KN=<TS,IS> is called knowledge network if IS include identity interpretation and IS include any composition of comprehension.

Example. 2.1 Mathematical Knowledge

The following diagram shows the knowledge network among mathematical theories.



Where the upper theories of the lines in the diagram is comprehend by the lower theories of the lines. For example Field is comprehended by Ring naturally. But Metric space is required to be extended by definition for being comprehended by Topological space in a strict sense. Let $<\Omega_m,\rho,R>$ be a metric space where $\rho:\Omega\rightarrow R$ is a metric function and $<\Omega_T,\Theta_T>$ be a topological space. Then it is required to extend such symbol Θ_m that $I(\Theta_T)=\Theta_m$ hold to the metric space by definition Where Θ_m is defined as the set of union of elements of the open base which is introduced by using metric function. Then $<\Omega_T,\Theta_T>$ comprehend $<\Omega_m,\Theta_m,\rho,R>$ under the such interpretation that $I(\Theta_T)=\Theta_m$ and $I(\Omega_T)=\Omega_m$. For a working mathematician this symbolic interpretation is trivial and intuitive process and that $<\Omega_m,\rho,R>$ and $<\Omega_m,\Theta_m,\rho,R>$ are regarded as the same system. We distinguish them in our framework.

3.Application

3.1 Application for meta systems model

We formulate some Meta systemic properties by using our notation.

The system which have common properties among systems is defined as follows.

Def. 3.1 Common System

Let Σ, $\Gamma_1,\ldots,\Gamma_n$ be knowledge units. Then Σ is called the common system among $\{\Gamma_1,\ldots,\Gamma_n\}$ if there exist comprehensions $I_1,\ldots,I_n$ from Σ to Γ_i respectively.

In the following cases we assume that n=2.

Def.3.2 Direct Analogy

Let Σ, Γ_1 and Γ_2 be knowledge units. If Σ is a common system between Γ_1 and Γ_2, Σ is a subset of axioms of Γ_1 and $I_1:\Sigma\rightarrow\Gamma_1$ is identity symbol map, then Σ is called a direct analogy from Γ_1 to Γ_2.

Def.3.3 Abductive Analogy

Let Σ, Γ_1 and Γ_2 be knowledge units. If Σ is a common system between Γ_1 and Γ_2 and that $I_1:\Sigma\rightarrow\Gamma_1$ and $I_2:\Sigma\rightarrow\Gamma_2$ are not identity symbol maps, then Σ is called an abductive analogy.

Def. 3.4 COMMON[Γ_1, Γ_2]

COMMON[Γ_1, Γ_2] denote the set of all common systems between Γ_1 and Γ_2 in which $I^1_i=I_{ij}\bullet I^1_j$ and $I^2_i=I_{ij}\bullet I^2_j$ hold, where I^1_i and I^2_i are comprehension from common systems Σ_i to Γ_1 and Γ_2 respectively and I_{ij} denote a comprehension from common system Σ_i to Σ_j if the comprehension exist between them.

Prop.3.1 Maximal Common System

COMMON[Γ_1, Γ_2] has a maximal common system.

Proof 3.1 (**sketch**) (1) We introduce symbol $\_$ which denote the mutual comprehension between systems. (2) COMMON[Γ_1, Γ_2]/$\_$ is introduced and shown to be a partially ordered set. Let CH be the index set of a chain in COMMON[Γ_1, Γ_2]/$\_$. (3) Direct limit of language L_i of system $\Sigma_i\in$COMMON[Γ_1, Γ_2]/$\_$, $i\in$CH, is defined as follows. Let $C=\cup\{C_i|\ i\in CH\}$, $V_p=\cup\{V_{pi}|\ i\in CH\}$, $V_r=\cup\{V_{ri}|\ i\in CH\}$. Let $c_1\in C_j$, $c_2\in C_k$. Then $c_1\approx c_2$ is defined as $\exists I$: comprehension $I:\Sigma_i\Rightarrow\Sigma_j$ or $I:\Sigma_j\Rightarrow\Sigma_i$ and $I(c_1)=c_2$ or $I(c_2)=c_1$. $V_p/\approx$ and $V_r/\approx$ are defined as the same way. Let $L=<C,V_p,V_r>$ and $L^*=<C/\approx,V_p/\approx,V_r/\approx>$. Then L^* become a direct limit. Ψ^* is defined as a formula of L^*. (4) Direct limit of theories is defined by the similar way and denoted by Σ^*. It is also shown that Σ^* belong to COMMON[Γ_1, Γ_2]. Then Σ^* become the maximal system.

Def. 3.5 Reduction and Emergency

Let Ω and Γ be systems and Γ' be the system which has the same vocabulary of the one of Γ and has more axioms which express specialized boundary conditions. Γ'_{ex} denote its extension and Γ_{ex}' denote an specialization of Γ_{ex}.

If Ω comprehend Γ'_{ex} or Γ_{ex}' then it is called that the system Ω is reduced to Γ'_{ex} or Γ_{ex}' respectively as an extension and specialization of Γ. It is shown in the following diagram.

$$
\begin{array}{llll}
\text{extension} & & \text{specialization} & \\
\Gamma'_{ex} \Leftarrow & \Gamma' & \Gamma_{ex}' \Leftarrow & \Gamma_{ex} \\
\Uparrow & \Uparrow \text{ specialization} & \Uparrow & \Uparrow \text{ extension} \\
\Omega & \Gamma & \Omega & \Gamma
\end{array}
$$

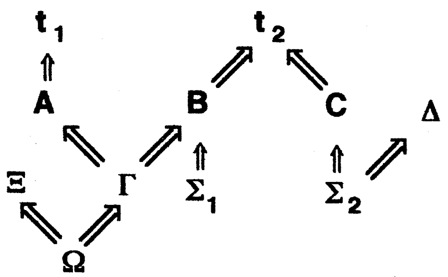
In the diagram we call that the extension has information emergency and the specialization has boundary emergency. Γ'_{ex}or Γ_{ex}' is called a realization of Ω based on Γ.

Ex. 3.1

Let Ω be a digital system and Γ be the electronic circuit system. Then the digital system is realized through specializing the electronic circuit by giving the boundary condition and through extending the system by defining new symbols. For example the symbols "1"and "0" of digital system are defined as "1" = 5 volt and "0" = 0 volt and any digital system is realized by specialized electronic circuit. In other word the digital system is reduced to the electronic circuit. It is also possible to reduce the digital system to another concrete system such as hydrodynamic system.

Ex. 3.2 Systems Recognition

An example of metamodel of systems recognition is shown in the following diagram by using our formulation.



In the diagram t_1 and t_2 denote real objects, A, B and C denote the structured constructed by the experiment and observation of real objects, which are called observational structure and Δ denotes the structure constructed by imaginative experiment, which is called imaginative structure. The system Γ comprehend different objects through two observational structure A and B. Γ and Σ_1 comprehend the same observational structure from different point of view. Σ_2 comprehend both the observational structure and the imaginative one. Ω is a common system between Ξ and Γ and it offers an analogy for two different systems.

3.2 Application for natural language

Example 3.1

(A) Names of knowledge units

We introduce a knowledge network consist of next five knowledge units.

(1) Bird (2) Swallow (3) Penguin (4) Living creature (5) Insect

(B) Contents of knowledge units

(1) Bird

V_r={Have(x,y), Numbers-of-leg(x,y), Bird(x), Can_fly(x)}, V_c={Wing,2}

Bird={∀x Bird(x) → Have(x,Wing), ∀x Bird(x) → Can_fly(x), ∀x Bird(x) → Numbers-of-leg(x,2)}

(2) Swallow

V_r={Have(x,y), Numbers-of-leg(x,y), Swallow(x), Migratory_bird(x), Can_fly(x)}, V_c={Wing,2}

Swallow={∀x Swallow(x) → Have(x,Wing), ∀x Swallow(x) → Can_fly(x), ∀x Swallow(x) → Migratory_bird(x) ,∀x Swallow(x) → Numbers-of-leg(x,2)}

(3) Penguin

V_r={Have(x,y), Numbers-of-leg(x,y), Penguin(x), Fly(x)}, V_c={Wing,2}

Penguin={∀x Penguin(x) → Have(x,Wing), ∀x Penguin(x) → ¬Can_fly(x), ∀x Penguin(x) → Numbers-of-leg(x,2)}

(4) Living creature

V_r={Breathe(x,y), Living creature(x) }, V_c={ Air}

Living creature={∀x Living creature(x) → Breathe(x,Air)}

(5) Insect

V_r={Breathe(x,y), Numbers-of-leg(x,y), Insect(x)}, V_c={Air, 6}

Insect={∀x Insect(x)(x) → Breathe(x,Air), ∀x Insect(x) → Numbers-of-leg(x,6)}

(C) Figure of comprehension relations

Penguin
  I_1 ⇑ I_2
  Bird ⇒ Swallow
  I_3 ⇑ I_4
Living creature ⇒ Insect

(D) Interpretations

Interpretation of predicate and constant symbols between the knowledge units are given as follows.

(1) I_1:Bird ⇒ Penguin I_1(Bird(x))=Penguin(x)

(2) I_2:Bird ⇒ Swallow I_2(Bird(x))=Swallow(x)

(3) I_3:Living creature ⇒ BirdI_3(Living creature(x))=Bird(x)

(4) I_4:Living creature ⇒ Insect I_4(Living creature(x))=Insect(x)

The other symbols are interpreted to the same name of predicate and constant symbols respectively in this example.

It is more natural for describing natural language to construct knowledge unit by using urelement set theory. This is now under construction.

In this example the interpretation I_2 and I_4 are comprehension. But I_1 and I_3 are not comprehension. Nevertheless I_3 should be recognized as comprehension. In the knowledge representation of natural language in our framework interpretations might be declared to be comprehension without checking the necessary conditions for comprehension. On the contrary the pre declaration allow to convert the knowledge from abstract units to more concrete units. In the previous example we do not want to describe such knowledge as " A Bird breathes air" in the unit of bird explicitly. If we have to describe these kinds of knowledge in every more concrete knowledge units than living creature then knowledge of concrete units become too big. It is more natural to consider that there is a knowledge inheritance between the units after the pre declaration of comprehension . In other word the pre declaration is a kind of methods of knowledge representations in the knowledge network. But these pre declaration might cause inconsistency and we need maintenance of knowledge. On the other hand I_1 is not a comprehension. Because Penguin can not fly. The maintenance of knowledge is required to recognize I_1 as a comprehension. In the next section we introduce the concept of modality for giving a formal base of the pre declaration and the maintenance of knowledge.

4. Knowledge network and possible world

In this section we construct possible world from a knowledge network to give a interpretation of modal operators such as ◊ and Ó. The construction shows that we do not use possible world from transcendental point of view but construct it depending on our restricted knowledge which are expressed by a knowledge network.

Def.4.1 Σ_{HEX}, CompHE(Σ), BA(Σ), CanoBA(Σ),CanoST(Σ)

Let KN=<TS, IS> be a knowledge network and $\Sigma \in TS$ be a theory. Then Σ_{HEX} denotes the Henkin extension of Σ. Σ_{HEX} include the special axioms such as "$\exists xP(x) \rightarrow P(c[\exists xP(x)])$". $L_{HEX}[\Sigma]$ denotes the language of Σ_{HEX}. which includes such constant symbols as "$c[\exists xP(x)]$". The set of complete extensions of Σ_{HEX} is expressed by CompHE(Σ) and Σ_{CHEX} denotes its element. CanoST(Σ) denotes canonical structure which is constructed by Σ.

B(Σ) denote the set of closed terms of Σ and CanoBA(Σ) denote the base set of canonical structure of Σ which is constructed from the equivalence classes on closed terms. We assume that every theory include at least one constant symbol to construct canonical structure.

Def.4.2 $W_{CHE}[\Gamma,KN]$, $W_{CST}[\Sigma,KN]$, $W_{CST}[\Sigma]$,KN_{ext}, TH[ω], HTH[ω]

$W_{CHE}[\Sigma,KN]=\cup\{CompHE(\Gamma) \mid I:\Gamma \Leftarrow \Sigma,\ \Sigma,\Gamma \in TS,\ I \in IS\}$

$W_{CST}[\Sigma,KN]=\{\omega \mid \omega=CanoST(\lambda),\ \lambda \in W_{CHE}[\Sigma,KN]\}$

$W_{CST}[\Sigma]=\{\omega \mid \omega=CanoST(\lambda),\ \lambda \in CompHE(\Sigma)\ \}$

We identify $W_{CHE}[\Sigma,KN]$ with $W_{CST}[\Sigma,KN]$ in the case of no confusion.

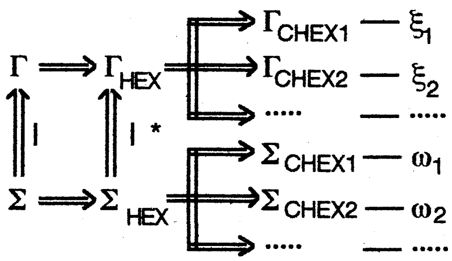
KN_{ext}=<TS_{ext}, IS_{ext}> where KN_{ext} is a knowledge network and $TS_{ext} \supseteq TS$, $IS_{ext} \supseteq IS$ and $\forall \Sigma \in KN_{ext}\ \exists\ \Gamma \in KN\ \exists\ I \in IS_{ext}\ I:\Sigma \Leftarrow \Gamma$ hold.

Let $\omega \in W_{CHE}[\Sigma,KN]$ or $\omega \in W_{CST}[\Sigma,KN]$ then TH[ω] denotes the theory from which ω is constructed and HTH[ω] denotes its Henkin extension.

Cor.4.1

(1) If $\lambda_1,\lambda_2 \in$ CompHE(Σ) then CanoBA(HTH[λ_1])=CanoBA(HTH[λ_2])

Cor.4.2
There is a suitable embedding mapping by which $B(\Sigma) \supset CanoBA(\Sigma) \supset CanoBA(\Sigma_{HE})$ hold.
Def.4.3 I^*
Let $I:\Gamma \Leftarrow \Sigma$. We define the function I^* from $CanoBA(\Sigma_{HE})$ to $CanoBA(\Gamma_{HE})$ as follows.
(1) Let c be a constant of $L(\Sigma)$ then $I^*(c)=I(c)$
(2) For a new constant $c[\exists xP(x)]$ which is introduced by Henkin extension, $I^*(c[\exists xP(x)])=I(c[\exists xP(x)])=c[\exists xI(P)(x)]$.
(3) For functional symbols and predicate symbols I^* is the same to I.
I^* is also extended naturally on the mapping from $CanoBA(\Sigma_{HEX})$ to $CanoBA(\Gamma_{HEX})$ We use the same symbol "I^*" without confusion.
Cor.4.3 I^* is a comprehension from Σ_{HEX} to Γ_{HEX}.
Proof The interpretation of the special axioms for constants also become the special axioms for constants. For other axioms I is a comprehension. Thus I^* become comprehension.
Cor.4.4 Let Σ be a consistent theory, Σ_{HEX} be its Henkin extension and Σ_{CHEX} be a its complete Henkin extension then canonical structure of Σ_{CHEX} is a model of Σ and Σ_{HEX}.
Proof. See reference [7].
Our constructions are shown as follows.



Def. 4.4 ML[Σ], SML[Σ]
ML[Σ] denotes the language which is the extension of L[Σ] by adding modality symbols such as ◊ and □. SML[Σ] denote the simple modal extension of L[Σ] which consist of $\Diamond\phi$ or $\Box\phi$ where ϕ belong to L[Σ].
Next we construct Kripke model from our knowledge network.
Def. 4.5 KNMM[Σ,KN]
KNMM[Σ,KN]=<W,R,D,Q,V>
Let KN=<TS,IS> be a knowledge network then the model KNMM[Σ,KN]=<W,R,D,Q,V> is constructed as follows.
(1) $W=W[\Sigma,KN]=W_{CST}[\Sigma,KN]$ (2) $\forall \omega,\xi \in W \ \ \omega R \xi \leftrightarrow_{def} \exists I \in IS \ \ I:TH[\xi] \Leftarrow TH[\omega]$
(3) $D=\cup\{CanoBA(\Gamma_{HEX}) \mid I:\Gamma \Leftarrow \Sigma,\ \Gamma \in TS,\ I \in IS\}$
(4) $\forall \omega \in W\ D[\omega]=Q(\omega)=CanoBA(HTH[\omega])$
(5) Valuation function $V_\omega(\phi)$ which assign 1 or 0 to a sentence
[1] Let $\phi \in ML[\Sigma]$ be a sentence
$V_\omega(\phi)=1$ if and only if $\omega \models I(\phi)$ where $I:\Sigma$
$V_\omega(\phi)=0$ if and only if $\omega \models I(\phi)$

$V_\omega(\phi(a))=1$ if and only if $\omega \models I(\phi)(^\wedge a)$ $V_\omega(\phi(a))=0$ if and only if $\omega \models I(\neg\phi)(^\wedge a)$

Where $a \in D[\omega]$ and $^\wedge a$ is its a natural mapping to dom(ω).

[2]

$V_\omega(\neg\phi)=1$ if and only if $V_\omega(\phi)=0$, $V_\omega(\neg\phi)=0$ if and only if $V_\omega(\phi)=1$

$V_\omega(\phi \vee \psi)=1$ if and only if $V_\omega(\phi)=1$ or $V_\omega(\phi)=1$.

$V_\omega(\phi \vee \psi)=0$ if and only if $V_\omega(\phi)=0$ and $V_\omega(\phi)=0$.

$V_\omega(\phi \wedge \psi)=1$ if and only if $V_\omega(\phi)=1$ and $V_\omega(\phi)=1$.

$V_\omega(\phi \wedge \psi)=0$ if and only if $V_\omega(\phi)=0$ or $V_\omega(\phi)=0$.

$V_\omega(\forall x\phi(x))=1$ if and only if $V_\omega(\phi(a))=1$ for any $a \in D[\omega]$

$V_\omega(\forall x\phi(x))=0$ if and only there exist $a \in D[\omega]$ $V_\omega(\phi(a))=0$.

$V_\omega(\Box\phi)=1$ if and only if $\forall\xi\in W$ $\omega R\xi \rightarrow V_\omega(\phi)=1$.

$V_\omega(\Diamond\phi)=1$ if and only if $\exists\xi\in W$ $\omega R\xi$ and $V_\omega(\phi)=1$.

Where $\Diamond\phi$ is defined as $\neg \Box \neg\phi$.

Theorem 4.1 KNMM[Γ,KN] is a Kripke S4 model.

Proof We have to show following four conditions for <W,R,D,Q,V>
(1) R is a transitive and reflective relation on W., (2) $D=\cup\{D[\omega] | \omega\in W\}$, (3) $D[\omega]=Q(\omega)$ and if $\omega_i R\omega_j$ then $Q(\omega_j)\supseteq I^*(Q(\omega_i))$, (4) valuation function satisfy the condition for Kripke model. Condition (1) is clear from the definition of I. (2), (3),(4) are also clear from the definition.

Cor.4.5 Let KN=<TS, IS> be a knowledge network. Then there exist accompanied knowledge network denoted by HeEx[KN] or KN_{HEX} which consist of Henkin extension of knowledge units of KN.

Proof We construct the network. Let HeEx[KN]=$\langle TS^*, IS^*\rangle$, where $TS^*=\{\Sigma_{HEX} | \Sigma\in TS\}$, $IS^*=\{I^* | I\in IS\}$. Then HeEx[KN] also become a knowledge network.

We use HeEx[KN] instead of KN if necessary and there is no confusion. Then we can use such sentence as Bird(c[∃xBird(x)]) without referring the extension in the previous example.

5. Default reasoning and maintenance of knowledge network

In this section we study how maintenances of knowledge are described by using the concept of modality which was introduced in the previous section. For this purpose we at first show the relation between Kripke model on a knowledge network and proof of a sentence ϕ on a knowledge unit.

Prop. 5.1 Let KN=<TS, IS> be a knowledge network ,$\Sigma\in TS$ and $\omega\in W_{CST}[\Sigma]$. If $\phi \in L[\Sigma]$ is a closed sentence and modality symbols are not used in it.Then

(1) $V_\omega(\Box\phi)=1 \leftrightarrow \Sigma\vdash \phi$ (2) $V_\omega(\Diamond\phi)=1 \leftrightarrow \neg\Sigma\vdash \neg\phi$

Proof. (1) $V_\omega(\Box\phi)=1 \leftrightarrow_{def} \forall\xi\in W_{CST}[\Sigma,KN]\ \omega R\xi \rightarrow V_\xi(\phi)=1$

$\leftrightarrow \forall\xi\in W_{CST}[\Sigma,KN]\ \exists I\in IS\ I{:}Th[\omega] \Rightarrow Th[\xi] \rightarrow \xi \models I(\phi)$

$\leftrightarrow \forall\Gamma\in TS\ \exists I\in IS\ I{:}\Gamma\Leftarrow\Sigma\ \forall\xi\in W_{CST}[\Gamma] \rightarrow \xi \models I(\phi)$

$\leftrightarrow \forall\Gamma\in TS\ \exists I\in IS\ I{:}\Gamma\Leftarrow\Sigma\ \Gamma\vdash\phi$

$\leftrightarrow \Sigma\vdash \phi$

Def.5.1 Modal extension of knowledge unit $<\Sigma, \Sigma_{MOD}>$

Let Σ be a knowledge unit. Then $<\Sigma, \Sigma_{MOD}>$ is called the modal extension of knowledge unit Σ where Σ_{MOD} denotes the finite set of axioms which consist of sentences of SML[Σ]. Then $\phi \in \Sigma$ is called main axiom and $\psi \in \Sigma_{MOD}$ is called sub axiom of the unit.

Def.5.2 Meta rules for inference

(1) Let $\Diamond\phi \in \Sigma_{MOD}$. If $\Gamma \in ST$ and $\exists I \in S\ I:\Gamma \Leftarrow \Sigma$ then we can add $I(\phi)$ to Γ for inference by default depending on prop.5.1 (2). If there happen inconsistency on Γ then $I(\phi)$ must be eliminated.

(2) Let $\Box\phi \in \Sigma_{MOD}$. $\Box\phi$ means the declaration that $I:\Gamma \Leftarrow \Sigma \cup \{\phi\}$ is a comprehension without checking the necessary conditions for any Γ which is comprehended by Σ,i.e., $\exists I \in S\ I:\Gamma \Leftarrow \Sigma$. Then we must add $I(\phi)$ and necessary vocabularies of ϕ to Γ for inference by default depending on prop.5.1 (1). If there happen inconsistency on Γ then maintenance is not simple. In that case we must reconsider the contents of unit Γ or $<\Sigma, \Sigma_{MOD}>$

In the example 3.1 we can introduce new knowledge units as follows by using our new modal notation and meta rules.

(1) $<Bird, Bird_{MOD}> = <\{\forall x\ Bird(x) \rightarrow Have(x,Wing),\ \forall x\ Bird(x) \rightarrow Numbers\text{-}of\text{-}leg(x,2)\},\ \{\Box\ (\forall x\ Bird(x) \rightarrow Can\_fly(x))\}>$

(2) $<Living\ creature,\ Living\ creature_{MOD}> = <\{\},\{\Box\ \forall x\ Living\ creature(x) \rightarrow Breathe(x,Air)\}>$, where $\{\}$ is the empty set.

Then $I(\forall x\ Living\ creature(x) \rightarrow Breathe(x,Air))$ must be added to Insect, Bird, Penguin and Swallow for inference by default. $I(\forall x\ Bird(x) \rightarrow Can\_fly(x))$ must be also added to Penguin and Swallow for inference by default. But this cause inconsistency in the unit of Penguin. One way for maintenance is to use "$\Diamond$" and to change $<Bird, Bird_{MOD}>$ to $<\{\forall x\ Bird(x) \rightarrow Have(x,Wing),\ \forall x\ Bird(x) \rightarrow Numbers\text{-}of\text{-}leg(x,2)\},\ \{\Diamond\ (\forall x\ Bird(x) \rightarrow Can\_fly(x))\}>$. Then Penguin can be comprehended by Bird. If inconsistency happened by adding the sentence to Penguin then it must be eliminated by the meta rule. The other method for the maintenance is to divide the unit into two different units such as Bird andCan_fly_Bird which are shown as follows.

Penguin
$I_1 \Uparrow\ \ I_2 \qquad\qquad I_2$
Bird $\Rightarrow$ Can_fly_Bird $\Rightarrow$ Swallow
$I_3 \Uparrow \qquad\qquad I_4$
Living creature $\Rightarrow$ Insect

Where $<Bird, Bird_{MOD}> = <\{\forall x\ Bird(x) \rightarrow Have(x,Wing),\ \forall x\ Bird(x) \rightarrow Numbers\text{-}of\text{-}leg(x,2)\},\ \{\}>$ and $<Can\_fly\_Bird,\ Can\_fly\_Bird_{MOD}> = <\{\forall x\ Bird(x) \rightarrow Have(x,Wing),\ \forall x\ Bird(x) \rightarrow Numbers\text{-}of\text{-}leg(x,2)\},\ \{\Box\ (\forall x\ Bird(x) \rightarrow Can\_fly(x)\ )\}>$

We can also apply our framework to other types of default reasoning.

6. Data Semantics and Knowledge Network

In this section we will give a relation between data semantics and our framework. As a result we construct a information model of data semantics from a knowledge network without changing classical inference rules. Data semantics was introduced by F. Veltman[8,9,10,12]. The similarity between data semantics and knowledge network is pointed out by Nagao[11].

Next we give a information model from knowledge network. In the following definition the formulas are limited in the set of closed sentences.

Def. 6.1 Comparison between data semantics and knowledge network

Let $M=<S, \geq, V_s>$ be an information model and $KN=<TS, IS>$ be a knowledge network. Then M_{d_o} denotes the information model constructed from knowledge network and is defined as $M_{d_o}=<TS, \Leftarrow, V_\Sigma>$ where $\Leftarrow$ denotes the comprehension relation and V_Σ is defined as follows compared with the definitions of Veltman, where [V i] shows the definitions of Veltman and [DO i] shows ones of mine.

(1) Atomic formula

[V1] $_s\models \phi \leftrightarrow_{def} V_s(\phi)=1$ $_s=| \phi \leftrightarrow_{def} V_s(\phi)=0$

[DO1] $V_\Sigma(\phi)=1 \leftrightarrow_{def} \forall\omega\in W_{CST}[\Sigma]\ \ \omega\models\phi$

$V_\Sigma(\phi)=0 \leftrightarrow_{def} \forall\omega\in W_{CST}[\Sigma]\ \ \omega\models\neg\phi$

$V_\Sigma(\phi)=U \leftrightarrow_{def}$ otherwise

Notice: $V_s(\phi)$ and $V_\Sigma(\phi)$ are three valued function.

(2)

[V2] $_s\models \neg\phi \leftrightarrow_{def}\, _s=| \phi$ $\qquad _s=| \neg\phi \leftrightarrow_{def}\, _s\models \phi$

[DO2] $V_\Sigma(\neg\phi)=1 \leftrightarrow_{def} V_\Sigma(\phi)=0$ $\ V_\Sigma(\neg\phi)=0 \leftrightarrow_{def} V_\Sigma(\phi)=1$

(3)

[V3] $_s\models \phi\wedge\psi \leftrightarrow_{def}\, _s\models \phi$ and $_s\models \psi$ $\qquad _s=| \phi\wedge\psi \leftrightarrow_{def}\, _s=| \phi$ or $_s=| \psi$

[DO3] $V_\Sigma(\phi\wedge\psi)=1 \leftrightarrow_{def} V_\Sigma(\phi)=1$ and $V_\Sigma(\psi)=1$

$V_\Sigma(\phi\wedge\psi)=0 \leftrightarrow_{def} V_\Sigma(\phi)=0$ or $V_\Sigma(\psi)=0$

Cor.6.1 $V_\Sigma(\phi\wedge\psi)=0 \leftrightarrow \forall\omega\in W_{CST}[\Sigma]\ \ \omega\models\neg(\phi\wedge\psi)$

Proof. $V_\Sigma(\phi\wedge\psi)=0 \leftrightarrow V_\Sigma(\phi)=0$ or $V_\Sigma(\psi)=0 \leftrightarrow$

$\forall\omega\in W_{CST}[\Sigma]\ \ \omega\models\neg\phi$ or $\forall\omega\in W_{CST}[\Sigma]\ \ \omega\models\neg\psi$

$\forall\omega\in W_{CST}[\Sigma]\ \ \omega\models\neg\phi \vee \omega\models\neg\psi$

$\forall\omega\in W_{CST}[\Sigma]\ \ \omega\models\neg(\phi\wedge\psi)$

(4)

[V4] $_s\models \phi\vee\psi \leftrightarrow_{def}\, _s\models \phi$ or $_s\models \psi$ $\qquad _s=| \phi\vee\psi \leftrightarrow_{def}\, _s=| \phi$ and $_s=| \psi$

[DO4] $V_\Sigma(\phi\vee\psi)=1 \leftrightarrow_{def} V_\Sigma(\phi)=1$ or $V_\Sigma(\psi)=1$

$V_\Sigma(\phi\vee\psi)=0 \leftrightarrow_{def} V_\Sigma(\phi)=0$ and $V_\Sigma(\psi)=0$

Cor.6.2 $V_\Sigma(\phi\vee\psi)=0 \leftrightarrow \forall\omega\in W_{CST}[\Sigma]\ \ \omega\models\neg(\phi\vee\psi)$

$V_\Sigma(\phi\vee\psi)=1 \leftrightarrow \forall\omega\in W_{CST}[\Sigma]\ \ \omega\models\phi\vee\psi$

(5)

[V5] $_s\models \phi\rightarrow\rightarrow\psi \leftrightarrow_{def} \neg\{\exists s'\geq s\ \ _{s'}\models\phi$ and $_{s'}=| \psi\}$

$\leftrightarrow \forall s'\geq s\ \ _{s'}\models\phi \rightarrow \neg\, _{s'}=| \psi$

$_s=| \phi\rightarrow\rightarrow\psi \leftrightarrow_{def} \exists s'\geq s\ \ _{s'}\models\phi$ and $_{s'}=| \psi$

Where "$\rightarrow\rightarrow$" denotes the special inference symbol by Veltman.

[DO5] $V_\Sigma(\phi\rightarrow\rightarrow\psi)=1$

$\leftrightarrow_{def}$ $\neg\{\exists\xi\in W_{CST}[\Sigma,KN]\exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1$ and $V_{Th[\xi]}(\psi)=0\}$

$\leftrightarrow \forall\ \xi\in W_{CST}[\Sigma,KN]\forall I\in S\,(I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1)\rightarrow V_{Th[\xi]}(\psi)\neq 0$

$V_\Sigma(\phi\rightarrow\rightarrow\psi)=0$

$\leftrightarrow_{def}$ $\exists\xi\in W_{CST}[\Sigma,KN]\exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1$ and $V_{Th[\xi]}(\psi)=0$

(6)

[V6] $_s|\!=\Diamond_v\,\phi\ \leftrightarrow_{def}\exists s'\geq s\ _{s'}|\!=\phi\quad _s\!=\!|\ \Diamond_v\,\phi\ \leftrightarrow_{def}\neg\{\exists s'\geq s\ _{s'}|\!=\phi\}$

[DO6] $V_\Sigma(\Diamond_{do}\,\phi)=1 \leftrightarrow_{def} \exists\xi\in W_{CST}[\Sigma,KN]\ \exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1$

$V_\Sigma(\Diamond_{do}\,\phi)=0 \leftrightarrow_{def} \neg\{\exists\xi\in W_{CST}[\Sigma,KN]\ \exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1\}$

$\leftrightarrow\ \forall\xi\in W_{CST}[\Sigma,KN]\ \forall I\in S\ \ I:Th[\xi]\Leftarrow\Sigma\rightarrow V_{Th[\xi]}(\phi)\neq 1$

(7)

[V7] $_s|\!=\Box_v\,\phi\ \leftrightarrow_{def}\neg\{\exists s'\geq s\ _{s'}\!=\!|\phi\}\qquad _s\!=\!|\ \Box_v\,\phi\ \leftrightarrow_{def}\neg\{\exists s'\geq s\ _{s'}\!=\!|\phi\}$

[DO7] $V_\Sigma(\Box_{do}\,\phi)=1 \leftrightarrow_{def} \neg\{\exists\xi\in W_{CST}[\Sigma,KN]\ \exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=0)$

$\leftrightarrow \forall\xi\in W_{CST}[\Sigma,KN]\ \forall I\in S\ \ I:Th[\xi]\Leftarrow\Sigma\rightarrow V_{Th[\xi]}(\phi)\neq 0$

$V_\Sigma(\Box_{do}\,\phi)=0 \leftrightarrow_{def} \exists\xi\in W_{CST}[\Sigma,KN]\ \exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=0$

Prop.6.1 $\neg\Box_{do}\neg\phi\leftrightarrow\Diamond_{do}\,\phi$

Proof (1) $\neg V_\Sigma(\Box_{do}\neg\phi)=1\leftrightarrow\exists\xi\in W_{CST}[\Sigma,KN]\ V_{Th[\xi]}(\neg\phi)=0$

$\leftrightarrow\exists\xi\in W_{CST}[\Sigma,KN]\ V_{Th[\xi]}(\phi)=1\leftrightarrow V_\Sigma(\Diamond_{do}\,\phi)=1$

(2) $\neg V_\Sigma(\Box_{do}\neg\phi)=0\leftrightarrow\neg\{\exists\xi\in W_{CST}[\Sigma,KN]\ V_{Th[\xi]}(\neg\phi)=0\}$

$\leftrightarrow\ \forall\xi\in W_{CST}[\Sigma,KN]\ V_{Th[\xi]}(\phi)\neq 1\ \leftrightarrow V_\Sigma(\Diamond_{do}\,\phi)=0$

Prop.6.2

(1) $V_\Sigma(\phi\rightarrow\rightarrow\psi)=1\ \leftrightarrow V_\Sigma(\Box_{do}\{\neg\phi\vee\psi\})=1$

(2) $V_\Sigma(\neg\{\phi\rightarrow\rightarrow\psi\})=1\ \leftrightarrow V_\Sigma(\Diamond_{do}\{\phi\wedge\neg\psi\}$

Proof.

(1) $V_\Sigma(\phi\rightarrow\rightarrow\psi)=1$

$\leftrightarrow\neg\{\exists\xi\in W_{CST}[\Sigma,KN]\exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1$ and $V_{Th[\xi]}(\psi)=0\}$

$V_\Sigma(\Box_{do}\{\neg\phi\vee\psi\})=1\ \leftrightarrow\neg\{\exists\xi\in W_{CST}[\Sigma,KN]\ \exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\neg\phi\vee\psi)=0)$

$\leftrightarrow\neg\{\exists\xi\in W_{CST}[\Sigma,KN]\ \exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\neg\phi)=0$ and $V_{Th[\xi]}(\psi)=0\}$

$\leftrightarrow\neg\{\exists\xi\in W_{CST}[\Sigma,KN]\ \exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1$ and $V_{Th[\xi]}(\psi)=0$

(2) $V_\Sigma(\neg\{\phi\rightarrow\rightarrow\psi\})=1\ \leftrightarrow V_\Sigma(\phi\rightarrow\rightarrow\psi)=0$

$\leftrightarrow\exists\xi\in W_{CST}[\Sigma,KN]\exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1$ and $V_{Th[\xi]}(\psi)=0$

$V_\Sigma(\Diamond_{do}\{\phi\wedge\neg\psi\})=1\ \leftrightarrow\exists\xi\in W_{CST}[\Sigma,KN]\ \exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi\wedge\neg\psi)=1$

$\leftrightarrow\exists\xi\in W_{CST}[\Sigma,KN]\ \exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1$ and $V_{Th[\xi]}(\neg\psi)=1$

$\leftrightarrow\exists\xi\in W_{CST}[\Sigma,KN]\ \exists I\in S\ I:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1$ and $V_{Th[\xi]}(\psi)=0$

Cor.6.3 If modality symbols.are not used in ϕ. Then

(1) $\Sigma|\!-\phi\leftrightarrow\forall\omega\in W_{CST}[\Sigma]\ \ \omega|\!=\phi$

(2) $\neg\,\Sigma|\!-\ \neg\phi\leftrightarrow\exists\omega\in W_{CST}[\Sigma]\ \omega|\!=\phi$

(3a) $\{\forall\Gamma\in TS_{ext}\ \forall I\in S_{ext}\ I:\Gamma\Leftarrow\Sigma\ \rightarrow\neg\,\Gamma|\!-\neg I(\phi)$ for any $KN_{ext}\}\ \leftrightarrow\Sigma|\!-\ \phi$,

(3b) $\{\forall\Gamma\in TS_{ext}\ \forall I\in S_{ext}\ I:\Gamma\Leftarrow\Sigma\ \rightarrow\Gamma|\!-I(\phi)$ for any $KN_{ext}\}\ \leftrightarrow\Sigma|\!-\ \phi$

(4) $\{\exists\Gamma\in TS_{ext}\exists I\in S_{ext}\ I:\Gamma\Leftarrow\Sigma$ and $\Gamma|\!-I(\phi)$ for some $KN_{ext}\}\leftrightarrow\ \neg\,\Sigma|\!-\ \neg\phi$

Proof. Omitted

Prop.6.3 If modality symbols.are not used in ϕ and KN Then

(1) $V_\Sigma(\Box_{do}\,\phi)=1$ for any $KN_{ext}\leftrightarrow\Sigma|\!-\phi$

(2) $V_\Sigma(\Box_{do}\,\phi)=0$ for some KN_{ext}

$\leftrightarrow \exists\Gamma\in TS_{ext}\,\exists\in l\,S_{ext}$ $l:\Gamma\Leftarrow\Sigma$ and $\Gamma\vdash\ \neg l(\phi)$ for some KN_{ext}

$\leftrightarrow \neg\Sigma\vdash\ \phi$

(3) $V_\Sigma(\Diamond_{do}\phi)=1$ for some KN_{ext}

$\leftrightarrow \exists\Gamma\in TS_{ext}\,\exists\in l\,S_{ext}$ $l:\Gamma\Leftarrow\Sigma$ and $\Gamma\vdash l(\phi)$ for some KN_{ext}

$\leftrightarrow\ \neg\ \Sigma\vdash\ \neg\phi$

(4) $V_\Sigma(\Diamond_{do}\phi)=0$ for any KN_{ext}

$\leftrightarrow \forall\Gamma\in TS_{ext}\,\forall l\in S_{ext}$ $l:\Gamma\Leftarrow\Sigma\ \rightarrow\neg\ \Gamma\vdash l(\phi)$ for any KN_{ext}

$\leftrightarrow \Sigma\vdash\ \neg\phi$

Proof.

(1) $V_\Sigma(\Box_{do}\phi)=1$ for any KN_{ext}

$\leftrightarrow$ for any KN_{ext} $\forall\xi\in W_{CST}[\Sigma,KN_{ext}]$ $\forall l\in S$ $l:Th[\xi]\Leftarrow\Sigma\rightarrow V_{Th[\xi]}(\phi)\neq 0$

$\leftrightarrow$ for any KN_{ext} $\forall\xi\in W_{CST}[\Sigma,KN_{ext}]$ $\forall l\in S$ $l:Th[\xi]\Leftarrow\Sigma\rightarrow\neg(\forall\omega\in W_{CST}[Th[\xi]]$ $\omega\models\neg l(\phi))$

$\leftrightarrow$ for any KN_{ext} $\forall\xi\in W_{CST}[\Sigma,KN_{ext}]$ $\forall l\in S$ $l:Th[\xi]\Leftarrow\Sigma\rightarrow\neg(Th[\xi]\ \vdash\ \neg l(\phi)\ )$

$\leftrightarrow$ for any KN_{ext} $\forall\Gamma\in TS_{ext}$ $\forall l\in S_{ext}$ $l:\Gamma\Leftarrow\Sigma\ \rightarrow\neg\ \Gamma\vdash\neg l(\phi)$

$\leftrightarrow \Sigma\vdash\ \phi$

(2) $V_\Sigma(\Box_{do}\phi)=0$

$\leftrightarrow$ for some KN_{ext} $\exists\xi\in W_{CST}[\Sigma,KN_{ext}]$ $\exists l\in S_{ext}$ $l:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=0$

$\leftrightarrow$ for some KN_{ext} $\exists\xi\in W_{CST}[\Sigma,KN_{ext}]$ $\exists l\in S_{ext}$ $l:Th[\xi]\Leftarrow\Sigma$ and $\forall\omega\in W_{CST}[Th[\xi]]$ $\omega\models\neg l(\phi)$

$\leftrightarrow$ for some KN_{ext} $\exists\xi\in W_{CST}[\Sigma,KN_{ext}]$ $\exists l\in S_{ext}$ $l:Th[\xi]\Leftarrow\Sigma$ and $Th[\xi]\ \vdash\ \neg l(\phi)$

$\leftrightarrow$ for some KN_{ext} $\exists\Gamma\in TS_{ext}\,\exists\in l\,S_{ext}$ $l:\Gamma\Leftarrow\Sigma$ and $\Gamma\vdash\neg l(\phi)$

$\leftrightarrow\ \neg\ \Sigma\vdash\ \phi$

(3) $V_\Sigma(\Diamond_{do}\phi)=1$

$\leftrightarrow$ for some KN_{ext} $\exists\xi\in W_{CST}[\Sigma,KN_{ext}]$ $\exists l\in S_{ext}$ $l:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1$

$\leftrightarrow$ for some KN_{ext} $\exists\xi\in W_{CST}[\Sigma,KN_{ext}]$ $\exists l\in S_{ext}$ $l:Th[\xi]\Leftarrow\Sigma$ and $\forall\omega\in W_{CST}[Th[\xi]]$ $\omega\models l(\phi)$

$\leftrightarrow$ for some KN_{ext} $\exists\xi\in W_{CST}[\Sigma,KN_{ext}]$ $\exists l\in S_{ext}$ $l:Th[\xi]\Leftarrow\Sigma$ and $Th[\xi]\ \vdash\ l(\phi)$

$\leftrightarrow$ for some KN_{ext} $\exists\Gamma\in TS_{ext}\,\exists l\in S_{ext}$ $l:\Gamma\Leftarrow\Sigma$ and $\Gamma\vdash l(\phi)$

$\leftrightarrow\ \neg\ \Sigma\vdash\ \neg\phi$

(4) $V_\Sigma(\Diamond_{do}\phi)=0$

$\leftrightarrow$ for any KN_{ext} $\neg\{\exists\xi\in W_{CST}[\Sigma,KN_{ext}]$ $\exists l\in S_{ext}$ $l:Th[\xi]\Leftarrow\Sigma$ and $V_{Th[\xi]}(\phi)=1\}$

$\leftrightarrow$ for any KN_{ext} $\neg\{\exists\Gamma\in TS_{ext}\,\exists l\in S_{ext}$ $l:\Gamma\Leftarrow\Sigma$ and $\Gamma\vdash l(\phi)\}$

$\leftrightarrow$ for any KN_{ext} $\forall\Gamma\in TS_{ext}$ $\forall l\in S_{ext}$ $l:\Gamma\Leftarrow\Sigma\ \rightarrow\neg\Gamma\vdash l(\phi)\}$

$\leftrightarrow \Sigma\vdash\ \neg\phi$

Thus the modal operators $\Box_v$ and $\Diamond_v$ on data model become equivalent to the modal operators $\Box_{do}$ and $\Diamond_{do}$ defined on the data model which is constructed from knowledge networks. It is also easy to compare our model with Veltman's model about other properties such as T-stable and F-stable. As a result we have constructed a data model without changing classical logic of inference by using the knowledge network. We have already introduced the other modal operators $\Box$ and $\Diamond$ on KNMM[Σ,KN] which is also constructed from knowledge networks. We have discussed about the maintenance of knowledge on the model. We can also discuss the problem depending on our new modal operators on the data model.

7. Conclusion

In this short paper we have presented only the part of our framework[3]. It is possible to apply our theory more concrete problems and more philosophical problems as well.

Our semantics is based on conceptualism view of the world. Meaning should be given by our own knowledge itself. This is our thesis. Thus we call these types of semantics conceptual semantics. Of course our knowledge depend on the real world. However meaning should be primarily concerned with not a reference to the real world but a concept of the real world . Thus we insist that possible world should be treated depending on not a transcendental real worlds but a constructed ones from our own knowledge. We have introduced the concept of knowledge network for these purpose. The knowledge network express not only the hierarchical structure of knowledge itself but also the extension of the information about the world, which is treated mainly in the data semantics. Our philosophical stand point is not so popular in these days[13]. Nevertheless we need conceptual semantics for a base of our epistemology as well as of computer science.

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