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Formulation of complex dynamical systems with decision makers

— Foundation for simulation model of complex systems —

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Preface

Opinions differ on whether disciplines so called social sciences, such as economics, sociology, management science and so forth, are truely sciences or not. In fact, following questions indicate weak points of those areas as scieces. Is it possible to design repeatable examinations on subjects in those areas? Do each of them has a certain formal theoretical language peculior to each investigation? However, it may be sure that eccential activities of scientists are to find and confirm some kind of law about their subjects which are complex and so are not easy to understand as a whole. In this sence, every social science undoubtedly aims at science. Further, problems of social sciences, which may include decision-makers, should be more complex and more difficult to understand rather than those of natulal sciences.

We call an object complex when we face some structural difficulty in describing or forecasting the behavior of it. Such objects may have part or all of following properties.

(1) Large scale. Number of elements is large.

(2) To some extent each element behaves autonomously under the influence of behaviors of other elements. When we explain behaviors of such elements, we should describe some complex relationship or behavior rules among elements. Therefore, it should be inevitably more complex than usual state transition explanation of single dynamical system.

(3) Element systems are complexly related to each other. Various kinds of relations exist in various hierarchical levels. Further, it may be possible that relations themselves are changing dynamically.

(4) A language suitable to description of a certain level system may not appropriate for another level. We should choose most suitable language to describe emergent property peculiar to each level.

In mathematical general systems theory(MGST), any object whose behavior can be seen as a binary relation $S \subset X \times Y$ between an input set X and an output set Y is called a general system. Furthermore, a n-ary relation $S \subset \Pi(S_i \mid i=1,2,\cdots,n)$ which includes element systems $S_i \subset X_i \times Y_i$ $(i=1,2,\cdots,n)$ is called a complex system. Because $S \subset \Pi S_i \subset \Pi(X_i \times Y_i)$ $\Pi X_i \times \Pi Y_i$, S can be identified as a general system with an input set ΠX_i and an output set ΠY_i. Complex system S is seen as a super system accompanied with lower systems S_i, and essential property of complex system is recognized as hierarchical property in the sence of such a formulation.

This formulation gives us a general point of view about complex objects, and so it should give us some kind of guideline for making operational models of real complex objects. However, it is doubtful that this viewpoint can be directly applicable to real problems, especially in the area of social sciences. To begin with, social scientists does not always use the scientific procedure, i.e. modeling and examination, in their problem solving. It seems that their analyses or critisisms are ofen formed from their subjective judgement based on their experience. Why they do not use scientific methodology? Main reason of it should be unefficiency of using scientific procedure. Shortage of guideline of making operational model may be one of the reason.

In this short paper we will try to give a set theoretical formulation of complex dynamical system which is useful for making simulation models of real complex systems.

§ 1 Problem situation

In this paper, we are concerned about complex time systems with following properties.

(1) It contains pulral autonomous subjects called controller or decision maker and pulral objects controlled by them.

(2) Decisions of subjects are related to each other. It is a complex decision making problem.

(3) There are some mechanisms to coordinate decision makers.

(But, real existence of some coordinater is not necessarily required.)

Objects treated in social sciences must have these properties in common. It seems very difficult or unefficient to describe global behaviors of such objects scientifically. In fact, social scientists often avoid it. However, system theoretically speaking, those systems are nothing else but complex time systems. And so, conceptually, description of behavior of the whole system shuould be possible by enumerating time series of each behavior of element systems, and of coordinator if it exists. Of course, we know that it is difficult or impossible to put into practice.

(a) Behavior of $E_i = [Q_i + S_i]$

(b) Behavior of coordinator C, if it exists.

If we could enumerate every possible consequences about (a) and (b), we get the denotational representation of systems behavior. There is no doubt that such the representation gives us a complete description of the time system. But it should be impossible in most of cases. If partial enumeration of past real behaviors could be possible, it may not so useful at forecasting future behavior of it. Because a time series of past behavior is at most one consequence from infinitely large number of possibilities, we can't use such description to forecast future behaviors.

However, we can precisely recognize the existence of some problem only from such a description about past behaviors. When we recognize existence of some problem about a system, we can start problem solving process as follows.

Process of problem solving

1 2 3 4

problem → recognition of a object → modeling → examination on the model → making altanatives → execution

↑ ↑↓ ↑↓ ↓

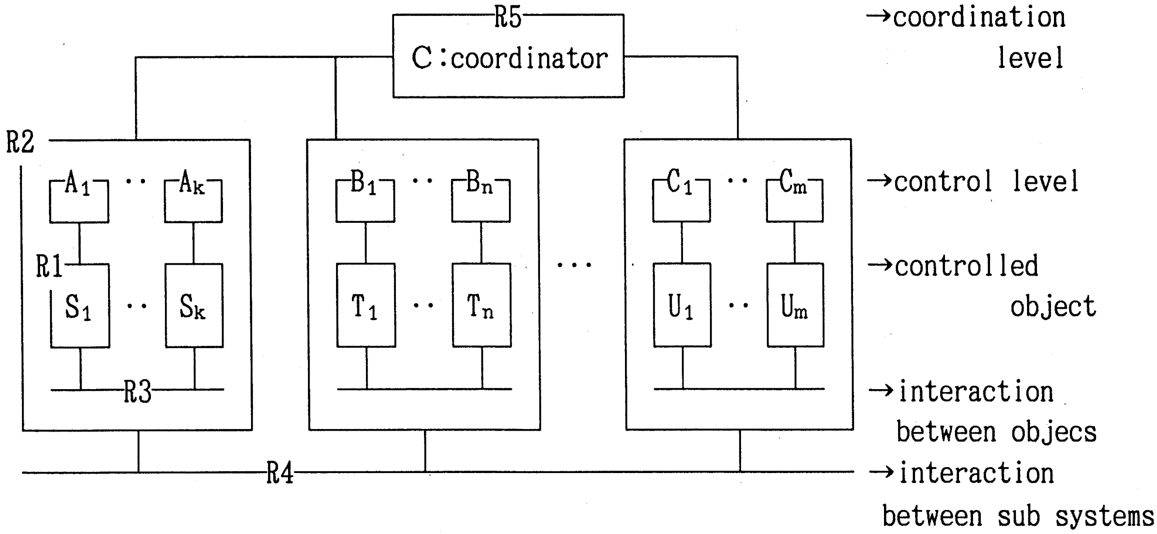
We may be successful in scientific approach to complex system problems only when we could create a powerful model for forecasting, i.e. a connotative expression of complex systems behavior.

§2. Hierarchical recognition of structure and rule

Complex systems, which we put in question here, are objects of which structures and behaviors are difficult to understand as a whole. To begin with, most of us in the present day may get used to analytical thinking rather than synthetical one. Therefore, it is relatively easy for us to answer the following analytical questions. Which unit do you think as an element of it? Which group of elements do you think as a subsystem of it? What do you understand about principles or rules about behaviors of them? What do you understand about relationship between them? Therefore, it is natural that we try to simplify complex system problem by dividing it into subsystem problems and classify them hierarchically.

From this point of view, we get a following scheme of two level system.

Fig. 1 Two level system model



After we recognize the structure of the object hierarchically, we will aim at rules or constraints about behavior of each level. To build a connotative model which is powerful in forecasting, we should discern what kinds of constraints or rules are put on decisions of each subjects. Those constraints or rules can be recognized along the hierarchical levels of the physical structure, as follows.

(1) R1. Underlying constraints: Controlled objects placed in lowest level in Fig. 1 should be subject to some fundamental constraints or conditions,

so called performance, capacity, efficiency, power … e.t.c.

(2) R2. Rules of field: Controllers are going to make decisions and operations according to some rules imposed by the field in which they exist.

(3) R3. Interference rules among element systems: Rules about relationships among states of element systems.

(4) R4. Interference rules among subsystems: Rules about relationships among states of subsystems.

(5) R5. Coordination rules: If some kind of coordination mechanisims exist, elements and subsystems are going to behave under the influence of them.

(6) R6. Interaction between the whole system and the environment: Conditions about input and output relations between the whole system and the environment.

Based on above observations, we will try to give a formulation of dynamical complex systems.

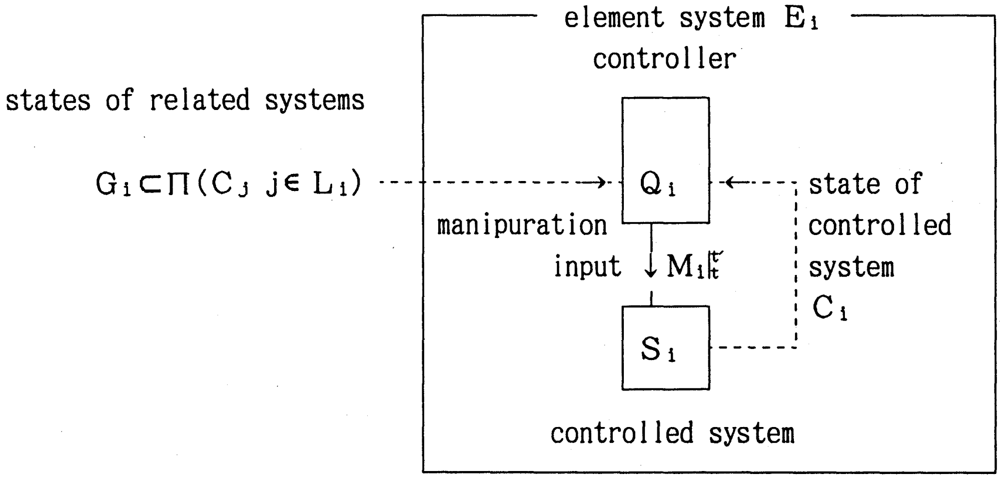
§ 3 Formulation of dynamical complex system

In solving complex systems problems, various modeling methods have been adopted. Limiting to operational ones, they are either of mathematical models or computer simulation model, and or mixture of the both. We think that simulation models have a merit of flexibility in description of systems behaviors and this merit seems remarkable at treatment of complex systems problem. However, of cource, whether a simulation model works effectively as a tool in poblem solving largely depends on a sence or a experience of the model builder.

One of the reason of it is a shortage of general theory which provide us with a reliable guideline of simulation model building. In this section we will try to give a general fomulation of dynamical complex system model which should be useful as a guideline in constructing simulation models.

First of all, we will formulate elementary units in a complex system. It may be natural that a certain combination of controlled object and its controller can be seen as a unit or an element system in a complex system. For example, a car and its driver, a machine and its operator, and a conputer and an office worker using it can be seen as a unit or an element system in a whole traffic system, factory system and office system, respectively. Such an element system can be described generally as a diagram below.

Fig.2 conceptual diagram of element system



Let notate a dynamical complex system $(E \mid i \in I)$, a family of element systems, and an element system $E_i = (Q_i, S_i)$, a couple of controller Q_i and controlled system S_i. Here, S_i is a state transition system which takes manipulation input from Q_i. Each Q_i is a system which is altering manipulation input to S_i, according to information about states of certain systems, not only S_i but also other element systems of which states infulence behavior of E_i.

Here, we make a list of fundamental sets for our formulation of complex dynamical system.

(1) T: Time set with total order $K \subset T \times T$.

(2) I: Index set of elements.

(3) $C \subset \Pi(C_i \mid i \in I)$: State set of global system.

(4) $C_i \subset \Pi(B_{ij} \mid j \in E(i))$ for $i \in I$: State set of ith element system.

(5) $M \subset \Pi(M_i \mid i \in I)$: Manipulation variable of global system.

(6) $M_i \subset P_i^T$ for $i \in I$: Manipulation variable of ith element system.

Each $m \in M_i$ is a function from time set T into a set P_i.

(7) $D \subset C$: States of manipulation alteration.

Based on these sets, we give our first formulation of complex system.

[Formulation 1]

(1) State transition function of S_i : $\varphi_i: C_i \times M_i \times K \to C_i$

(2) Manipulation alteration function of Q_i : $\mu_i: M_i \times C \times T \to M_i$

(3) Rules of manipulation alteration: $c \in D_i \to \mu_i(m, c, t) = m|^t \cdot \eta_i(m, c, t)$

$\eta_i : M_i \times C \times T \rightarrow M_i|_t$

C_i : set of states of S_i

M_i : set of time functions which

represent manipulation trajectory of Q_i

$K = \{(t, t') \mid t, t' \in T \ \& \ t \leqq t'\}$

Here, we remark that μ_i does not determine manipulation input in the far distance future. μ_i determines only temporary manipulation plan which may be revised in some future.

Simulation model based on [Formulation 1] will be constructed as follows.

(1) Setting up initial conditions : starting time $t_0 \in T$、ending time t_{max}、initial state $c \in C$ and initial manipulation plan $m \in M$ are set up.

(2) Repeat the followings until $t = t_{max}$.

[1] For each $i \in I$, under the assumption that manipulation plan $m(i) \in M_i$ will be maintained, the earliest $t_i \in T$, when it will become

$\varphi(c, m, t, t_i) \in D_i$ ··· ① , is estimated.

[2] The earliest time $t' := t_k$ s.t. $\varphi(c, m, t, t_k) \in D_k$ and $k \in I$ is searched.

[3] m(k), the manipulation plan of the element k, is revised by μ_k.

[4] Global state transition is made from t to t', that is $c := \varphi(c, m, t, t')$.

The time goes ahead, that is $t := t'$.

By the way, judgement whether $c \in D_i$ or not generally depends on combination of states of all element systems. However, in most of realistic cases, the judgement can be done by observation of states of a few elements which are directly related to ith element. This means that manipulation alteration states D_i of element i is separable from that of the global system D. The following formulation is on the assumption of separability of D.

[Formulation 2] : Case that each D_i can be separable from D.

(1) State transition function S_i : $\varphi_i : C_i \times M_i \times K \rightarrow C$

(2) Manipulation alteration function of Q_i : $\hat{\mu}_i : M_i \times G_i \times T \rightarrow M_i$

(3) Rules of manipulation alteration :

(a) Alteration states : $G_i \subset \Pi(C_j \mid j \in L_i)$

$L_i \subset I$ indicates indeces of element systems which are directly related to ith element.

(b) Alteration function : $c \in G_i \rightarrow \hat{\mu}_i(m, c, t) = m|^t \cdot \eta_i(m, c, t)$

$\eta_i : M_i \times G_i \times T \rightarrow M_i|_t$

In this case, simulation model will be constructed as follows.

(1) Setting up initial conditions : starting time $t_0 \in T$, ending time t_{max} initial state $c \in C$, initial manipulation plan $m \in M$, manipulation alteration time t_i of each element $i \in I$ and earliest manipulation element $k \in I$ s.t. $t_k = \min\{t_i \mid i \in I\}$ are set up. And let $t' := t_k$.

(2) Repeat followings until $t = t_{max}$.

[1] $c := \varphi(c, m, t, t')$: Global state transition is made from t to t'.

The time goes ahed to t', that is $t := t'$.

[2] $m(k) := \hat{\mu}_k(m(k), (c(j) \mid j \in L_k), t)$: Recalculation of element k.

[3] For each $i \in I$ s.t. $k \in L_i$, earliest t_i such that

$$(\varphi_i(c(j), m(j), t, t_i) \mid j \in L_i) \in G_i \cdots ②$$ is calculated.

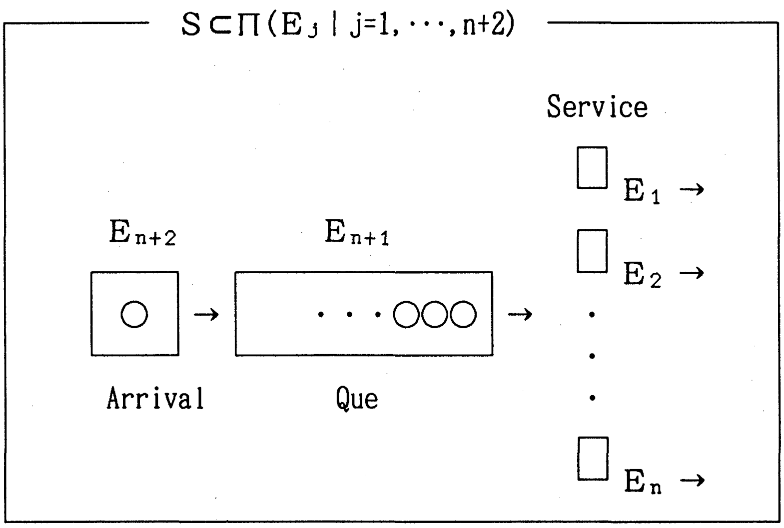
If there is no t_i which satisfies ②, then let $t_i := t_{max}$.

[4] $k \in I$, $t' := t_k$ s.t. $t_k = \min \{t_i \mid i \in I\}$ are searched.

§ 4 Simple example of the formulation

In this section, we will give an simple example of queing system to show the relation between our formulation and discrete type simulation model.

Queing system with pararrel service



Global system : $\mathscr{S} = \{E_1, \cdots, E_n, E_{n+1}, E_{n+2}\}$

<u>for $1 \leqq i \leqq n$</u>

(1) $C_i = \{0, 1\}$

(2) $M_i = \{m : T \to \{-1, 0, 1\} \mid m^{-1}(1)$ and $m^{-1}(-1)$ are finite$\}$

(3) $L_i = \{1, \cdots, n, n+1\}$,　$H_i = \Pi(C_j \mid j \in L_i)$

(4) $\varphi_i : C_i \times M_i|_t^{t'} \times K \to C_i$; $(c, m, (t, t'))$　$c + m^{-1}(1) - m^{-1}(-1)$

(5) $\mu_i : M_i \times H_i \times T \to M_i$; $(m, c, t) \mapsto \begin{cases} m|^t \circ \eta_i(m, c, t) & \text{if } c \in G_i \\ m & \text{otherwize} \end{cases}$

(6) (G_i, η_i)

(a) $G_i = \{c \in H_i \mid c(i)=0 \ \& \ (\forall j<i)(c(j)=1) \ \& \ c(n+1)>0\}$

(b) $\eta_i : M_i \times T \to M_i|_t$; $(m, t) \mapsto \eta_i(m, t) : T|_t \to \{-1, 0, 1\}$

$$\eta_i(m,t)(\tau) = \begin{cases} 1 & \text{if } \tau = t \\ -1 & \text{if } \tau = t - \ln(\text{rand})/\text{survice ratio} \\ m(\tau) & \text{otherwize} \end{cases}$$

<u>i=n+1</u>

(1) $C_{n+1} = \{0, 1, 2, \cdots\}$

(2) $M_{n+1} = \{m : T \to Z \mid (\forall z)(z \in Z \ \& \ z \neq 0 \to m^{-1}(z)$ は有限$)\}$

Here, Z is the set of integer.

(3) $L_{n+1} = \{1, \cdots, n+2\}$,　$H_{n+1} = \Pi(C_j \mid j \in L_{n+1})$

(4) $\varphi_{n+1} : C_{n+1} \times M_{n+1} \times K \to C_{n+1}$; $(c, m, (t, t'))$　$c + \sum_{z \in Z} z \ m^{-1}(z)$

(5) $\mu_{n+1} : M_{n+1} \times H_{n+1} \times T \to M_{n+1}$

$$; (m, c, t) \mapsto \begin{cases} m|^t \circ \eta_{n+1}(m, c, t) & \text{if } c \in G_{n+1} \\ m & \text{otherwize} \end{cases}$$

(6) (G_{n+1}, η_{n+1})

(a) $G_{n+1} = G_A \cup G_B$

$G_A = \{c \in H_{n+1} \mid c(n+1)>0 \ \& \ (\exists j)(1 \leqq j \leqq n \ \& \ c(j)=0)\}$

$G_B = \{c \in H_{n+1} \mid c(n+2)=1\}$

(b) $\eta_{n+1}: M_{n+1} \times H_{n+1} \times T \rightarrow M_{n+1}|_t$; $(m, c, t) \mapsto \eta_{n+1}(m, c, t): T|_t \rightarrow Z$

$$\eta_{n+1}(m,c,t)(\tau) = \begin{cases} m(\tau)-1 & \text{if } \tau=t \ \& \ c \in G_A - G_B \\ m(\tau)+1 & \text{if } \tau=t \ \& \ c \in G_B - G_A \\ m(\tau) & \text{otherwize} \end{cases}$$

i=n+2

(1) $C_{n+2} = \{0, 1\}$

(2) $M_{n+2} = \{m: T \rightarrow \{0, 1\} \mid m^{-1}(1) \text{ is finite }\}$

(3) $L_{n+2} = \{n+2\}$, $H_{n+2} = C_{n+2}$

(4) $\varphi_{n+2}: C_{n+2} \times M_{n+2} \times K \rightarrow C_{n+2}$; $(c, m, (t, t'))$ $m(t')$

(5) $\mu_{n+2}: M_{n+2} \times C_{n+2} \times T \rightarrow M_{n+2}$

$$;(m,c,t) \mapsto \begin{cases} m|^t \cdot \eta_{n+2}[t] & \text{if } c \in G_{n+2} \\ m & \text{otherwize} \end{cases}$$

(6) (G_{n+2}, η_{n+2})

(a) $G_{n+2} = \{1\} \subset C_{n+2}$

(b) $\eta_{n+2}: M_{n+2} \times C_{n+2} \times T \rightarrow M_{n+2}|_t$; $(m, c, t) \mapsto \eta_{n+2}(m, c, t): T|_t \rightarrow \{0, 1\}$

$$\eta_{n+2}(m,c,t) = \begin{cases} 1 & \text{if } \tau = t - \ln(\text{rand})/\text{arrival ratio} \\ m(\tau) & \text{otherwize} \end{cases}$$

Conclusion

In this short paper, we tried to formulate a complex dynamical system model from a standpoint of hierarchical systems theory. The formulation should give not only a kind of guideline for making simulation program but a theoretical foundation of discrete type simulation technique. Due to limitation of space, we can not enter into details about acctual simulation modeling based on our formulation. Instead of it, we gave a simple example of formulation about queing system which is the most typical subject of discrete type simulation technique.

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