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SIMPLE STABLE MAPS OF 3-MANIFOLDS INTO SURFACES

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1. Introduction

Let M be a closed orientable 3-manifold and N a 2-manifold (or a surface) with $\partial N = \emptyset$. A (C^∞-)stable map $f : M \to N$ is said to be *simple* if f has no cusp point and if every component of the f-fiber $f^{-1}(a)$ contains at most one singular point for all $a \in N$. In this paper, we consider the following problems.

PROBLEM 1.1. Determine the diffeomorphism types of those 3-manifolds which admit simple stable maps into 2-manifolds.

PROBLEM 1.2. Classify simple stable maps up to right-left equivalence.

Recall that if $f : M \to N$ is stable, then its singularities consist of three types: definite fold points, indefinite fold points, and cusp points. Furthermore, $f^{-1}(a)$ contains at most two singular points for all $a \in N$ ([**7**, **9**]). In the terminology of Kushner-Levine-Porto [**7**, **9**], the simple stable maps are precisely the stable maps without vertices. This class of stable maps were first studied by Kobayashi [**6**].

A stable map $f : M \to N$ is said to be *special generic* if it has only definite fold points as its singularities. Burlet-de Rham [**1**] have shown that a closed orientable 3-manifold admits a special generic map into $\mathbf{R}^2$ if and only if it is diffeomorphic to the 3-sphere or the connected sum of some copies of $S^1 \times S^2$. Furthermore, by Levine [**8**], every closed orientable 3-manifold admits a stable map into $\mathbf{R}^2$ without cusp points. Since special generic maps are simple, the

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class of simple stable maps is an intermediate class between the class of special generic maps and that of stable maps without cusp points.

One of the main results of this paper is an answer to Problem 1.1 as follows: *a closed orientable 3-manifold admits a simple stable map into a 2-manifold if and only if it is a graph manifold.* Here, a graph manifold is a 3-manifold built up of S^1-bundles over compact surfaces attached along their torus boundaries. Note that the class of graph manifolds has been investigated by several authors [**4, 5, 10, 11, 15, 18**] and that they can be completely classified by certain coded finite graphs ([**10**]). Thus, summarizing the above results, we have the situation as follows.

$$
\begin{array}{ccc}
\text{Class of stable maps} & & \text{Class of 3-manifolds} \\
& & \\
\{\text{special generic maps}\} & \longleftrightarrow & \{S^3, \sharp S^1 \times S^2\} \\
\bigcap & & \bigcap \\
\{\text{simple stable maps}\} & \longleftrightarrow & \{\text{graph manifolds}\} \\
\bigcap & & \bigcap \\
\{\text{stable maps without cusp points}\} & \longleftrightarrow & \{\text{3-manifolds}\} \\
\bigcap & & || \\
\{\text{stable maps}\} & \longleftrightarrow & \{\text{3-manifolds}\}
\end{array}
$$

Here we note that the class of graph manifolds is very large; for example, it contains the Seifert fibered 3-manifolds [**16**], the link 3-manifolds which arise around isolated singularities of complex surfaces [**10**], etc.. Nevertheless, it is relatively easy to handle.

As to Problem 1.2, we do not know a complete answer yet. Instead, we define a weaker equivalence relation, called quasi-equivalence, and classify simple stable maps up to this weakened equivalence. For a simple stable map $f : M \to N$, consider the quotient map $q_f : M \to W_f$ which identifies points in M

belonging to the same component of an f-fiber. Then two simple stable maps are quasi-equivalent if their quotient maps are right-left equivalent in a certain sense. Our classification is based on the graph link $L(f)$ associated with every simple stable map $f : M \to N$, where $L(f)$ consists of the singular set of f and some components of regular f-fibers. Here, a graph link is a link in a 3-manifold whose exterior is a graph manifold. Since graph links have been classified [**2, 10, 17**], we can determine whether two simple stable maps are quasi-equivalent or not.

In this paper, we will not give precise proofs to the theorems. Readers who are interested in more details should refer to [**12**].

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2. A characterization of 3-manifolds admitting simple stable maps

THEOREM 2.1. *For a closed orientable 3-manifold M, the following three are equivalent.*

(1) There exists a stable map $g : M \to \mathbf{R}^2$ such that $g|S(g) : S(g) \to \mathbf{R}^2$ is a smooth embedding.

(2) There exists a simple stable map $f : M \to N$ for some 2-manifold N.

(3) M is a graph manifold.

Since a stable map as in (1) is simple, the part (1) obviously implies the part (2). In order to show that if M admits a simple stable map as in (2), then M is a graph manifold, we need the notion of the Stein factorization as follows. For a simple stable map $f : M \to N$ and $p, p' \in M$, we define $p \sim p'$ if $f(p) = f(p')(= a)$ and p, p' belong to the same connected component of $f^{-1}(a)$. We denote by W_f the quotient space of M by this equivalence relation and by

$q_f : M \to W_f$ the quotient map. We have a unique map $\bar{f} : W_f \to N$ such that $f = \bar{f} \circ q_f$. The space W_f or the commutative diagram

$$\begin{array}{ccc} M & \xrightarrow{f} & N \\ & q_f \searrow \quad \nearrow \bar{f} & \\ & W_f & \end{array}$$

is called the *Stein factorization* of f. It is known that a point x in W_f has a neighborhood as in Figure 1 ([7, 9]).

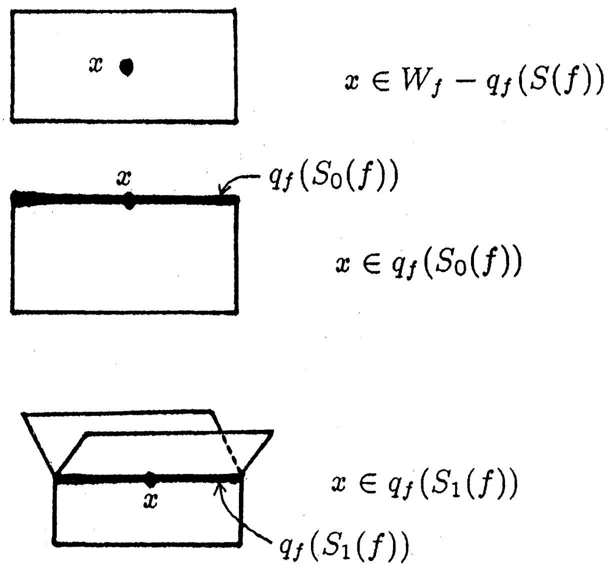

Figure 1

Now we show that if $f : M \to N$ is a simple stable map, then M is a graph manifold. Set $R = W_f - \mathrm{Int}N(\Sigma)$, where $\Sigma = q_f(S(f))$ and $N(\Sigma)$ is a regular neighborhood of Σ in W_f. Since $\bar{f}|R : R \to N$ is a local homeomorphism, it induces a natural smooth structure on R. With this smooth structure, $q_f|q_f^{-1}(R) : q_f^{-1}(R) \to R$ is a proper submersion with S^1-fibers, and hence $q_f^{-1}(R)$ is the total space of an S^1-bundle over the surface R. Furthermore let $N(\Sigma_0)$ and $N(\Sigma_1)$ be regular neighborhoods of Σ_0 and Σ_1 respectively, where $\Sigma_0 = q_f(\{\text{definite fold points}\})$ and $\Sigma_1 = q_f(\{\text{indefinite fold points}\})$. It is not difficult to see that $q_f^{-1}(N(\Sigma_0))$ is diffeomorphic to a disjoint union of some copies of the solid torus $S^1 \times D^2$ and that $q_f^{-1}(N(\Sigma_1))$ is a Seifert fibered space, which is a graph manifold. Since M is the union of $q_f^{-1}(R), q_f^{-1}(N(\Sigma_0))$ and $q_f^{-1}(N(\Sigma_1))$ attached along their torus boundaries, M is a graph manifold. Thus, we have shown that the part (2) implies the part (3) in Theorem 2.1.

COROLLARY 2.2. *(1) Let M be a homotopy 3-sphere. If there exists a simple stable map $f : M \to N$ for some 2-manifold N, then M is diffeomorphic to the 3-sphere.*

(2) Let M be a closed orientable hyperbolic 3-manifold. Then M admits no simple stable map into a 2-manifold.

Proof. The part (1) follows from the fact that a homotopy 3-sphere is diffeomorphic to the 3-sphere if it is a graph manifold ([**11, 15**]). The part (2) is a consequence of the well-known fact that a hyperbolic 3-manifold is never a graph manifold. □

In order to show that the part (3) implies the part (1) in Theorem 2.1, we construct a stable map as in (1) for each graph manifold. Since the proof is long, we omit it here. For details, see [**12**].

3. A classification of simple stable maps

Definition 3.1. Let $f : M \to N$ and $g : M' \to N'$ be simple stable maps of closed orientable 3-manifolds into 2-manifolds. We say that f and g are *quasi-equivalent* if there exist a diffeomorphism $\Phi : M \to M'$ and a homeomorphism $\varphi : W_f \to W_g$ such that the following diagram is commutative:

$$\begin{array}{ccc} M & \xrightarrow{\Phi} & M' \\ {\scriptstyle q_f}\downarrow & & \downarrow{\scriptstyle q_g} \\ W_f & \xrightarrow{\varphi} & W_g. \end{array}$$

Remark 3.2. We say that a homeomorphism $\varphi : W_f \to W_g$ is *admissible* if it is a "diffeomorphism" with respect to the "smooth structures" on W_f and W_g induced by $\bar{f} : W_f \to N$ and $\bar{g} : W_g \to N'$ respectively (a precise definition will be given in the appendix). We will show in the appendix that a homeomorphism φ as in Definition 3.1 is necessarily admissible.

Remark 3.3. It is easy to see that if simple stable maps $f : M \to N$ and $g : M' \to N'$ are right-left equivalent, then they are quasi-equivalent. Conversely, if f and g are quasi-equivalent, then $g = \bar{g}' \circ q_f \circ \Phi^{-1}$, where $\Phi : M \to M'$ is the diffeomorphism as in Definition 3.1 and $\bar{g}' : W_f \to N'$ is an "immersion".

For special generic maps, we have the following.

Theorem 3.4. (Burlet-de Rham [**1**]) *Let $f : M \to \mathbf{R}^2$ and $g : M' \to \mathbf{R}^2$ be special generic maps of closed orientable 3-manifolds into the plane. Then f and g are quasi-equivalent if and only if $b_1(M) = b_1(M')$ and $\sharp S(f) = \sharp S(g)$, where $b_1(M)$ (resp. $b_1(M')$) is the first betti number of M (resp. M') and $\sharp S(f)$ (resp. $\sharp S(g)$) is the number of the connected components of $S(f)$ (resp. $S(g)$).*

Theorem 3.4 is based on the fact that there are very few special generic maps. For simple stable maps, the situation is much more complicated.

Let $f : M \to N$ be a simple stable map of a closed orientable 3-manifold into an orientable surface. We define the associated link $L(f)$ of f in M as follows. Set $W_f - q_f(S(f)) = \cup_{i=1}^{s} R_i$, where R_i are the components. Note that each R_i is a (smooth) surface. If R_i is homeomorphic to $\mathrm{Int}D^2$, take distinct two points x_i and y_i in R_i. If R_i is not homeomorphic to $\mathrm{Int}D^2$, take a point x_i in R_i. Define

$$\tilde{S}_\infty(f) = \left(\bigcup_{i=1}^{s} q_f^{-1}(x_i)\right) \cup \left(\bigcup_{R_i \approx \mathrm{Int}D^2} q_f^{-1}(y_i)\right),$$

and

$$L(f) = S(f) \cup \tilde{S}_\infty(f).$$

We call $L(f)$ the *link associated with* f. Note that $S(f)$ is a closed 1-dimensional submanifold of M and that each $q_f^{-1}(x_i)$ or $q_f^{-1}(y_i)$ is diffeomorphic to the circle. Thus $L(f)$ is a link in M. Note also that the isotopy class of $L(f)$ does not depend on the choice of the points x_i and y_i.

We define an equivalence between associated links as follows. Let $f : M \to N$ and $g : M' \to N'$ be simple stable maps of closed orientable 3-manifolds into orientable surfaces. We orient M, N, M' and N' arbitrarily. Then each regular fiber of q_f and q_g inherits a natural orientation. We say that $L(f)$ and $L(g)$ are *equivalent* if there exists a diffeomorphism $\Phi : M \to M'$ such that

$$\Phi(\{\text{definite fold points of } f\}) = \{\text{definite fold points of } g\},$$

$$\Phi(\{\text{indefinite fold points of } f\}) = \{\text{indefinite fold points of } g\},$$

$$\Phi(\tilde{S}_\infty(f)) = \tilde{S}_\infty(g),$$

and that Φ preserves the orientations of the components of $\tilde{S}_\infty(f)$ and $\tilde{S}_\infty(g)$ simultaneously or reverses the orientations simultaneously. Note that this does not depend on the choice of the orientations of M, N, M' and N'.

THEOREM 3.5. *Let $f : M \to N$ be a simple stable map of a closed orientable 3-manifold into an orientable surface. Then the quasi-equivalence class of f*

determines and is determined by the equivalence class of the associated link $L(f)$.

REMARK 3.6. By an argument similar to that in §2, we see that $L(f)$ is a graph link; i.e., $M - \mathrm{Int}N(L(f))$ is a graph manifold. Furthermore, as in the case of graph manifolds, graph links have been classified by certain coded finite graphs ([**2, 10, 17**]).

For the proof of Theorem 3.5, we use the torus decomposition theorem of Jaco-Shalen-Johannson [**4, 5**], which states that a set of disjointedly embedded tori in a 3-manifold is uniquely determined up to isotopy if it satisfies certain good conditions. Our idea is to show that the set of the tori associated with the canonical decomposition of a quotient space as in §2 satisfies the good conditions. For this reason, we need two regular fibers over each component R_i of $W_f - q_f(S(f))$ which is homeomorphic to $\mathrm{Int}D^2$. In fact, Theorem 3.5 does not hold if we take only one regular fiber for each R_i (see Remark 5.13 (3) of [**12**]). For details of the proof of Theorem 3.5, see [**12**].

Appendix

In this appendix, we prove some important facts about smooth structures on quotient spaces in Stein factorizations.

DEFINITION A. Let $f : M \to N$ be a simple stable map of a closed orientable 3-manifold into a 2-manifold and let C be a component of $q_f(\{\text{indefinite fold points}\})$ $(\subset W_f)$. Note that the regular neighborhood $N(C)$ of C in W_f is homeomorphic to $Y \times S^1$ or $Y \times_\tau S^1$ ([9]), where $Y = \{re^{i\theta} \in \mathbf{C}; r \leq 1, \theta = 0, \pm 2\pi/3\}$, $\tau : Y \to Y$ is the complex conjugation restricted to Y $(\subset \mathbf{C})$ and $Y \times_\tau S^1 = Y \times [0,1]/(y,1) \sim (\tau(y), 0)$. We define $\sigma(C)$ to be the subspace of $N(C)$ which corresponds to $\{re^{i\theta} \in Y; \theta = 0\} \times S^1$ by the above homeomorphisms and call it the *stem* of C.

DEFINITION B. Let $f : M \to N$ and $g : M' \to N'$ be simple stable maps of closed orientable 3-manifolds into 2-manifolds. Set $\Sigma_0(W_f) = q_f(\{\text{definite fold points}\})$, $\Sigma_1(W_f) = q_f(\{\text{indefinite fold points}\})$ and $\Sigma(W_f) = \Sigma_0(W_f) \cup \Sigma_1(W_f)$ $(= q_f(S(f)))$. A homeomorphism $\varphi : W_f \to W_g$ is *admissible* if, for all $x \in W_f$, there exists an open neighborhood U of x in W_f such that

(1) if $x \in W_f - \Sigma(W_f)$, then $U \subset W_f - \Sigma(W_f)$, $\bar{f}|U$ and $\bar{g}|V$ $(V = \varphi(U))$ are homeomorphisms onto open subsets of N and N' respectively, and the composition

$$\bar{f}(U) \xrightarrow{\bar{f}^{-1}} U \xrightarrow{\varphi} V \xrightarrow{\bar{g}} \bar{g}(V)$$

is a diffeomorphism,

(2) if $x \in \Sigma_0(W_f)$, then $U \subset W_f - \Sigma_1(W_f)$, $\bar{f}|U$ and $\bar{g}|V$ are homeomorphisms onto subsets in N and N' respectively diffeomorphic to $\mathbf{R}^2_+ = \{(x_1, x_2) \in \mathbf{R}^2; x_1 \geq 0\}$, and the composition

$$\bar{f}(U) \xrightarrow{\bar{f}^{-1}} U \xrightarrow{\varphi} V \xrightarrow{\bar{g}} \bar{g}(V)$$

is a diffeomorphism,

(3) if $x \in \Sigma_1(W_f)$, then $U \subset N(C)$, $\bar{f}|U_i$ and $\bar{g}|V_i$ ($V_i = \varphi(U_i)$) are homeomorphisms onto open subsets in N and N' respectively, and the composition

$$\bar{f}(U_i) \xrightarrow{\bar{f}^{-1}} U_i \xrightarrow{\varphi} V_i \xrightarrow{\bar{g}} \bar{g}(V_i)$$

is a diffeomorphism ($i = 1, 2$), where C is the component of $\Sigma_1(W_f)$ containing x, $U_i = (U \cap \sigma(C)) \cup U_i'$, and $U - \sigma(C)$ has exactly two connected components U_1' and U_2' (see Figure 2).

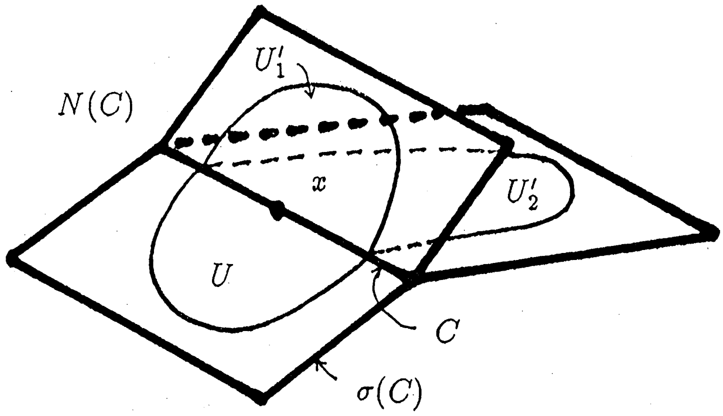

Figure 2

LEMMA C. *Let $f : M \to N$ and $g : M' \to N'$ be simple stable maps of closed orientable 3-manifolds into 2-manifolds. A homeomorphism $\varphi : W_f \to W_g$ is admissible if, for every $x \in W_f$, there exist an open neighborhood $\tilde{U}$ of x in W_f and a diffeomorphism $\Phi : q_f^{-1}(\tilde{U}) \to q_g^{-1}(\varphi(\tilde{U}))$ such that the following diagram is commutative:*

$$\begin{array}{ccc} q_f^{-1}(\tilde{U}) & \xrightarrow{\Phi} & q_g^{-1}(\varphi(\tilde{U})) \\ {\scriptstyle q_f}\downarrow & & \downarrow{\scriptstyle q_g} \\ \tilde{U} & \xrightarrow{\varphi|\tilde{U}} & \varphi(\tilde{U}). \end{array}$$

Proof. We show that there exists an open neighborhood U of x in W_f as in Definition B.

Case 1. $x \in W_f - q_f(S(f))$.

There exists an open contractible neighborhood U of x in W_f such that $U \subset (W_f - q_f(S(f))) \cap \tilde{U}$ and that $\bar{f}|U : U \to N$ and $\bar{g}|V : V \to N'$ are embeddings onto open subsets, where $V = \varphi(U)$. Since $\bar{f} \circ q_f : q_f^{-1}(U) \to \bar{f}(U)$ is the trivial S^1-bundle, there exists a smooth map $s : \bar{f}(U) \to q_f^{-1}(U)$ such that $\bar{f} \circ q_f \circ s = \mathrm{id}_{\bar{f}(U)}$. Then we see that $\bar{g} \circ \varphi \circ \bar{f}^{-1}|\bar{f}(U) = g \circ \Phi \circ s$, which implies that it is a smooth map. By a similar argument, we see that $\bar{f} \circ \varphi^{-1} \circ \bar{g}^{-1}|\bar{g}(V)$ is also a smooth map. Thus $\bar{g} \circ \varphi \circ \bar{f}^{-1}|\bar{f}(U)$ is a diffeomorphism.

Case 2. $x \in q_f(\{\text{definite fold points}\})$.

There exists an open neighborhood U of x in W_f such that $U \subset \tilde{U}$, that $\bar{f}|U : U \to N$ and $\bar{g}|V : V \to N'$ $(V = \varphi(U))$ are embeddings onto subsets diffeomorphic to $\mathbf{R}^2_+$, and that there exist diffeomorphisms Ψ and ψ which satisfy the following commutative diagram:

$$\begin{array}{ccc} q_f^{-1}(U) & \xrightarrow{\Psi} & \mathbf{R}^2 \times \mathbf{R} \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle l \times \mathrm{id}} \\ \bar{f}(U) & \xrightarrow{\psi} & \mathbf{R}^2_+, \end{array}$$

where $l : \mathbf{R}^2 \to \mathbf{R}$ is the map defined by $l(x_1, x_2) = x_1^2 + x_2^2$. Set $h = \bar{g} \circ \varphi \circ \bar{f}^{-1} \circ \psi^{-1} : \mathbf{R}^2_+ \to N'$. Then we see easily that $h \circ (l \times \mathrm{id}) = g \circ \Phi \circ \Psi^{-1}$ is a smooth map. Since $h(x_1^2 + x_2^2, x_3) = h \circ (l \times \mathrm{id})(x_1, x_2, x_3)$, we see that $h(x_1^2 + x_2^2, x_3)$ is smooth with respect to x_1, x_2 and x_3. To show that h is smooth, we need the following lemma.

LEMMA D. *Suppose that $F : \mathbf{R}^2_+ \to \mathbf{R}$ is a function such that $F(x_1^2, x_2)$ is smooth with respect to x_1 and x_2. Then F itself is smooth.*

Proof. By an argument similar to that in Example (A) of [**3**, p.108], using the Malgrange Preparation Theorem, we see that, for every $y \in \mathbf{R}$, there exists a smooth function germ F_1 at $(0, y) \in \mathbf{R}^2$ such that $F_1(x_1^2, x_2) = F(x_1^2, x_2)$ on a neighborhood of $(0, y)$. Thus, F is smooth near $0 \times \mathbf{R}$, and hence it is smooth on $\mathbf{R}^2_+$. □

By the above lemma, we see that h is smooth. Since ψ is a diffeomorphism, we see that $\bar{g} \circ \varphi \circ \bar{f}^{-1}|\bar{f}(U)$ is a smooth map. By a similar argument, we see that $\bar{f} \circ \varphi^{-1} \circ \bar{g}^{-1}|\bar{g}(V)$ is also a smooth map. Thus $\bar{g} \circ \varphi \circ \bar{f}^{-1}|\bar{f}(U)$ is a diffeomorphism.

Case 3: $x \in q_f(\{\text{indefinite fold points}\})$.

Let C be the component of $q_f(\{\text{indefinite fold points}\})$ containing x. There exists an open neighborhood U of x in W_f such that $U \subset N(C) \cap \tilde{U}$, that $\bar{f}|U_i : U_i \to N$ and $\bar{g}|V_i : V_i \to N'$ are embeddings onto open subsets ($i = 1, 2$), where U_i (resp. V_i) are the subsets of U (resp. $V = \varphi(U)$) constructed as in Definition B using $\sigma(C)$ (resp. $\sigma(\varphi(C))$). By the commutativity of the diagram

$$\begin{array}{ccc} q_f^{-1}(U) & \xrightarrow{\Phi|q_f^{-1}(U)} & q_g^{-1}(V) \\ {\scriptstyle q_f}\downarrow & & \downarrow{\scriptstyle q_g} \\ U & \xrightarrow{\varphi|U} & V, \end{array}$$

we see easily that $\varphi(U \cap \sigma(C)) = V \cap \sigma(\varphi(C))$. Thus we may assume that $\varphi(U_i) = V_i$. Taking a smaller neighborhood if necessary, we have a smooth map $s : \bar{f}(U) \to q_f^{-1}(U)$ such that $f \circ s = \mathrm{id}_{\bar{f}(U)}$. We can construct such a map using the normal form of f near $q_f^{-1}(U) \cap S(f)$. Since $\bar{g} \circ \varphi \circ \bar{f}^{-1} = g \circ \Phi \circ s$ on $\bar{f}(U_i)$, we see that $\bar{g} \circ \varphi \circ \bar{f}^{-1}|\bar{f}(U_i)$ is a smooth map. By a similar argument, we see that $\bar{f} \circ \varphi^{-1} \circ \bar{g}^{-1}|\bar{g}(V_i)$ is also a smooth map. Thus $\bar{g} \circ \varphi \circ \bar{f}^{-1}|\bar{f}(U_i)$ is a diffeomorphism ($i = 1, 2$).

By the three arguments as above, we conclude that φ is admissible. This completes the proof of Lemma C. □

As an immediate corollary, we have the following.

COROLLARY E. *Let $f : M \to N$ and $g : M' \to N'$ be simple stable maps of closed orientable 3-manifolds into surfaces. Then f and g are quasi-equivalent if and only if there exist a diffeomorphism $\Phi : M \to M'$ and an admissible homeomorphism $\varphi : W_f \to W_g$ such that $q_g = \varphi \circ q_f \circ \Phi^{-1}$.*

Using similar arguments, we can characterize the smooth functions on a quotient space as follows. Here, by definition, a function on a quotient space is smooth if it is smooth in a sense similar to Definition B.

PROPOSITION F. *Let $f : M \to N$ be a simple stable map of a closed orientable 3-manifold into a 2-manifold. Then a function $h : W_f \to \mathbf{R}$ is smooth if and only if $h \circ q_f : M \to \mathbf{R}$ is smooth.*

We can also characterize smooth maps between quotient spaces using the ring of the smooth functions as follows. For a simple stable map $f : M \to N$, denote by $C^\infty(W_f)$ the ring of the smooth functions on the quotient space W_f. For a map $\varphi : W_f \to W_g$ between quotient spaces of simple stable maps, we define $\varphi^* h = h \circ \varphi : W_f \to \mathbf{R}$ for $h \in C^\infty(W_g)$.

PROPOSITION G†. *Let $f : M \to N$ and $g : M' \to N'$ be simple stable maps of closed orientable 3-manifolds into 2-manifolds. A map $\varphi : W_f \to W_g$ is an admissible homeomorphism if and only if $\varphi^* C^\infty(W_g) = C^\infty(W_f)$.*

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