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Church-Rosser Property and Unique Normal Form Property of Non-Duplicating Term Rewriting Systems – DRAFT*–

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1 Introduction

The original idea of the conditional linearization of non-left-linear term rewriting systems was introduced by De Vrijer [4], Klop and De Vrijer [7] for giving a simpler proof of Chew's theorem [2, 10]. They developed an interesting method for proving the unique normal form property for some non-Church-Rosser, non-left-linear term rewriting system R. The method is based on the fact that the unique normal form property of the original non-left-linear term rewriting system R follows the Church-Rosser property of an associated left-linear conditional term rewriting system R^L which is obtained form R by *linearizing* the non-left-linear rules. In Klop and Bergstra [1] it is proven that non-overlapping left-linear conditional term rewriting systems are Church-Rosser. Hence, combining these two results, Klop and De Vrijer [4, 7, 6] showed that the term rewriting system R has the unique normal form property if R^L is non-overlapping. However, as their conditional linearization technique is based on the Church-Rosser property for the traditional conditional term rewriting system R^L, its application is restricted in non-overlapping R^L (though this limitation may be slightly relaxed with R^L containing only trivial critical pairs).

In this paper, we introduce a new conditonal linearization based on a left-right separated conditional term rewriting system R_L. The point of our linearization is that by replacing a traditional conditional system R^L with a left-right separated conditional system R_L we can

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easily relax the non-overlapping limitation of conditional systems originated from Klop and Bergstra [1].

By developing a new concept of weighted reduction systems we present a sufficient condition for the Church-Rosser property of a left-right separated conditional term rewriting system R_L which may have overlapping rewrite rules. Applying this result to our conditional linearization, we show a sufficient condition for the unique normal form property of a non-duplicating non-left-linear overlapping term rewriting system R.

Moreover, our result can be naturally applied to proving the Church-Rosser property of some non-duplicating non-left-linear overlapping term rewriting systems such as right-ground term rewriting systems. Oyamaguch and Ota [8] proved that non-E-overlapping right-ground term rewriting systems are Church-Rosser by using the joinability of E-graphs, and Oyamaguch extended this result into some overlapping systems [9]. The results by conditonal linearization in this paper strengthen some part of Oyamaguchi's results by E-graphs [8, 9], and vice verse. Hence, we believe that both approach should be working together for developing the potential of non-left-linear term rewriting system theory.

2 Reduction Systems

Assuming that the reader is familiar with the basic concepts and notations concerning reduction systems in [3, 5, 6], we briefly explain notations and definitions.

A reduction system (or an abstract reduction system) is a structure $A = \langle D, \rightarrow \rangle$ consisting of some set D and some binary relation $\rightarrow$ on D (i.e., $\rightarrow \subseteq D \times D$), called a reduction relation. A reduction (starting with x_0) in A is a finite or infinite sequence $x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \cdots$. The identity of elements x, y of D is denoted by $x \equiv y$. $\xrightarrow{\equiv}$ is the reflexive closure of $\rightarrow$, $\leftrightarrow$ is the symmetric closure of $\rightarrow$, $\xrightarrow{*}$ is the transitive reflexive closure of $\rightarrow$, and $\xleftrightarrow{*}$ is the equivalence relation generated by $\rightarrow$ (i.e., the transitive reflexive symmetric closure of $\rightarrow$).

If $x \in D$ is minimal with respect to $\rightarrow$, i.e., $\neg\exists y \in D[x \rightarrow y]$, then we say that x is a normal form; let NF be the set of normal forms. If $x \xrightarrow{*} y$ and $y \in NF$ then we say x has a normal form y and y is a normal form of x.

Definition 2.1 *$A = \langle D, \rightarrow \rangle$ is Church-Rosser (or confluent) iff*
$\forall x, y, z \in D[x \xrightarrow{*} y \wedge x \xrightarrow{*} z \Rightarrow \exists w \in A, y \xrightarrow{*} w \wedge z \xrightarrow{*} w]$.

Definition 2.2 *$A = \langle D, \rightarrow \rangle$ has unique normal forms iff*
$\forall x, y \in NF[x \xleftrightarrow{*} y \Rightarrow x \equiv y]$.

The following fact observed by Klop and De Vrijer [7] plays an essential role in our linearization too.

Proposition 2.3 *[Klop and De Vrijer] Let $A_0 = \langle D, \underset{0}{\rightarrow} \rangle$ and $A_1 = \langle D, \underset{1}{\rightarrow} \rangle$ be two reduction systems with the sets of normal forms NF_0 and NF_1 respectively. Then A_0 has unique normal forms if each of the following conditions holds:*

(i) $\underset{1}{\rightarrow}$ *extends* $\underset{0}{\rightarrow}$,

(ii) A_1 *is Church-Rosser,*

(iii) NF_1 *contains* NF_0.

3 Weight Decreasing Joinability

This section introduces the new concept of weight decreasing joinability. In the later sections this concept is used for analyzing the Church-Rosser property of conditional term rewriting systems with extra variables occurring in conditional parts of rewriting rules.

Let N^+ be the set of positive integers. $A = \langle D, \rightarrow \rangle$ is a weighted reduction system if $\rightarrow = \bigcup_{w \in N^+} \rightarrow_w$, that is, positive integers (weights w) are assigned to each reduction to represent costs.

A proof of $x \overset{*}{\leftrightarrow} y$ is a sequence $\mathcal{P}$: $x_0 \leftrightarrow_{w_1} x_1 \leftrightarrow_{w_2} x_2 \cdots \leftrightarrow_{w_n} x_n$ such that $x \equiv x_0$ and $y \equiv x_n$. The weight $w(\mathcal{P})$ of the proof $\mathcal{P}$ is $\sum_{i=1}^{n} w_i$. We usually abbreviate a proof $\mathcal{P}$ of $x \overset{*}{\leftrightarrow} y$ by $\mathcal{P}$: $x \overset{*}{\leftrightarrow} y$. The form of a proof may be indicated by writing, for example, $\mathcal{P}$: $x \overset{*}{\rightarrow} \cdot \overset{*}{\leftarrow} y$, $\mathcal{P}'$: $x \leftarrow \cdot \overset{*}{\rightarrow} \cdot \leftarrow y$, etc. We use the symbols $\mathcal{P}, \mathcal{Q}, \cdots$ for proofs.

Definition 3.1 *A weighted reduction system* $A = \langle D, \rightarrow \rangle$ *is weight decreasing joinable iff* $\forall x, y \in D$ *[for any proof* $\mathcal{P}$: $x \overset{*}{\leftrightarrow} y$ *there exists some proof* $\mathcal{P}'$: $x \overset{*}{\rightarrow} \cdot \overset{*}{\leftarrow} y$ *such that* $w(\mathcal{P}) \geq w(\mathcal{P}')$ *].*

It is clear that if a weighted reduction system A is weight decreasing joinable then A is Church-Rosser. We will now show a sufficient condition for the weight decreasing joinability.

Lemma 3.2 *Let A be a weighted reduction system. Then A is weight decreasing joinable if the following condition holds:*
for any $x, y \in D$ [for any proof $\mathcal{P}$: $x \leftarrow \cdot \rightarrow y$ there exists a proof $\mathcal{P}'$: $x \overset{*}{\leftrightarrow} y$ such that **(i)** $w(\mathcal{P}) > w(\mathcal{P}')$, *or* **(ii)** $w(\mathcal{P}) \geq w(\mathcal{P}')$ *and* $\mathcal{P}'$: $x \overset{\equiv}{\rightarrow} \cdot \overset{\equiv}{\leftarrow} y$*].*

Proof. The lemma can be easily proven by induction on the weight of a proof of $x \overset{*}{\leftrightarrow} y$. □

The following lemma is used to show the Church-Rosser property of non-duplicating systems.

Lemma 3.3 *Let* $A_0 = \langle D, \underset{0}{\rightarrow} \rangle$ *and* $A_1 = \langle D, \underset{1}{\rightarrow} \rangle$. *Let* $\mathcal{P}_i$: $x_i \underset{1}{\overset{*}{\leftrightarrow}} y$ $(i = 1, \cdots n)$ *and let* $w = \sum_{i=1}^{n} w(\mathcal{P}_i)$. *Assume that for any* $a, b \in D$ *and any proof* $\mathcal{P}$: $a \underset{1}{\overset{*}{\leftrightarrow}} b$ *such that* $w(\mathcal{P}) \leq w$ *there exists proofs* $\mathcal{P}'$: $a \underset{1}{\overset{*}{\rightarrow}} c \underset{1}{\overset{*}{\leftarrow}} b$ *with* $w(\mathcal{P}') \leq w(\mathcal{P})$ *and* $a \underset{0}{\overset{*}{\rightarrow}} c \underset{0}{\overset{*}{\leftarrow}} b$ *for some* $c \in D$. *Then, there exist proofs* $\mathcal{P}'_i$: $x_i \underset{0}{\overset{*}{\rightarrow}} z$ $(i = 1, \cdots n)$ *and* $\mathcal{Q}$: $y \underset{1}{\overset{*}{\leftrightarrow}} z$ *with* $w(\mathcal{Q}) \leq w$ *for some* z.

Proof. By induction on w. *Base step* $w = 0$ is trivial. *Induction step*: From I.H., we have proofs $\tilde{\mathcal{P}}_i$: $x_i \xrightarrow[0]{*} z'$ $(i = 1, \cdots n-1)$ and $\tilde{\mathcal{Q}}$: $y \overset{*}{\underset{1}{\leftrightarrow}} z'$ for some z' such that $\sum_{i=1}^{n-1} w(\mathcal{P}_i) \geq w(\tilde{\mathcal{Q}})$. By connecting the proofs $\tilde{\mathcal{Q}}$ and $\mathcal{P}_n$ we have a proof $\hat{\mathcal{P}}$: $z' \overset{*}{\underset{1}{\leftrightarrow}} y \overset{*}{\underset{1}{\leftrightarrow}} x_n$. Since $\sum_{i=1}^{n-1} w(\mathcal{P}_i) \geq w(\tilde{\mathcal{Q}})$ and $w(\hat{\mathcal{P}}) = w(\tilde{\mathcal{Q}}) + w(\mathcal{P}_n)$, it follows that $w \geq w(\hat{\mathcal{P}})$. By the assumption, we have proofs $\check{\mathcal{P}}$: $z' \xrightarrow[1]{*} z \xleftarrow[1]{*} x_n$ with $w \geq w(\hat{\mathcal{P}}) \geq w(\check{\mathcal{P}})$ and $z' \xrightarrow[0]{*} z \xleftarrow[0]{*} x_n$ for some z. Thus we obtain proofs $\mathcal{P}'_i$: $x_i \xrightarrow[0]{*} z$ $(i = 1, \cdots, n)$.

By combining subproofs of $\hat{\mathcal{P}}$: $z' \overset{*}{\underset{1}{\leftrightarrow}} y \overset{*}{\underset{1}{\leftrightarrow}} x_n$ and $\check{\mathcal{P}}$: $z' \xrightarrow[1]{*} z \xleftarrow[1]{*} x_n$, we can make $\mathcal{Q}'$: $y \overset{*}{\underset{1}{\leftrightarrow}} z' \xrightarrow[1]{*} z$ and $\mathcal{Q}''$: $y \overset{*}{\underset{1}{\leftrightarrow}} x_n \xrightarrow[1]{*} z$. Note that $w + w \geq w(\hat{\mathcal{P}}) + w(\check{\mathcal{P}}) = w(\mathcal{Q}') + w(\mathcal{Q}'')$. Thus $w \geq w(\mathcal{Q}')$ or $w \geq w(\mathcal{Q}'')$. Take $\mathcal{Q}'$ as Q if $w \geq w(\mathcal{Q}')$; otherwise, take $\mathcal{Q}''$ as Q. □

4 Term Rewriting Systems

In the following sections, we briefly explain the basic notions and definitions concerning term rewriting systems [3, 5, 6].

Let $\mathcal{F}$ be an enumerable set of function symbols denoted by $f, g, h, \cdots$, and let $\mathcal{V}$ be an enumerable set of variable symbols denoted by $x, y, z, \cdots$ where $\mathcal{F} \cap \mathcal{V} = \phi$. By $T(\mathcal{F}, \mathcal{V})$, we denote the set of terms constructed from $\mathcal{F}$ and $\mathcal{V}$. The term set $T(\mathcal{F}, \mathcal{V})$ is sometimes denoted by T.

A substitution θ is a mapping from a term set $T(\mathcal{F}, \mathcal{V})$ to $T(\mathcal{F}, \mathcal{V})$ such that for a term t, $\theta(t)$ is completely determined by its values on the variable symbols occurring in t. Following common usage, we write this as $t\theta$ instead of $\theta(t)$.

Consider an extra constant □ called a hole and the set $T(\mathcal{F} \cup \{\square\}, \mathcal{V})$. Then $C \in T(\mathcal{F} \cup \{\square\}, \mathcal{V})$ is called a context on $\mathcal{F}$. We use the notation $C[\ , \ldots, \ ]$ for the context containing n holes $(n \geq 0)$, and if $t_1, \ldots, t_n \in T(\mathcal{F}, \mathcal{V})$, then $C[t_1, \ldots, t_n]$ denotes the result of placing $t_1, \ldots, t_n$ in the holes of $C[\ , \ldots, \ ]$ from left to right. In particular, $C[\ ]$ denotes a context containing precisely one hole. s is called a subterm of $t \equiv C[s]$. If s is a subterm occurrence of t, then we write $s \subseteq t$. If a term t has an occurrence of some (function or variable) symbol e, we write $e \in t$. The variable occurrences $z_1, \cdots, z_n$ of $C[z_1, \cdots, z_n]$ are fresh if $z_1, \cdots, z_n \notin C[\ , \cdots, \ ]$ and $z_i \not\equiv z_j$ $(i \neq j)$.

A rewriting rule is a pair $\langle l, r \rangle$ of terms such that $l \notin \mathcal{V}$ and any variable in r also occurs in l. We write $l \to r$ for $\langle l, r \rangle$. A redex is a term $l\theta$, where $l \to r$. In this case $r\theta$ is called a contractum of $l\theta$. The set of rewriting rules defines a reduction relation $\to$ on T as follows:

$$t \to s \text{ iff } t \equiv C[l\theta],\ s \equiv C[r\theta] \quad \text{for some rule } l \to r, \text{ and some } C[\ ],\ \theta.$$

When we want to specify the redex occurrence $\Delta \equiv l\theta$ of t in this reduction, we write $t \xrightarrow{\Delta} s$.

Definition 4.1 *A term rewriting system R is a reduction system $R = \langle T(\mathcal{F}, \mathcal{V}), \to \rangle$ such that the reduction relation $\to$ on $T(\mathcal{F}, \mathcal{V})$ is defined by a set of rewriting rules. If R has $l \to r$ as a* *rewriting rule, we write* $l \to r \in R$.

We say that R is left-linear if for any $l \to r \in R$, l is linear (i.e., every variable in l occurs only once). If R has critical pair then we say that R is overlapping: otherwise non-overlapping [5, 6].

A rewriting rule $l \to r$ is duplicating if r contains more occurrences of some variable then l; otherwise, $l \to r$ is non-duplicating. We say that R is non-duplicating if every $l \to r \in R$ is non-duplicating.

5 Left-Right Separated Conditional Systems

In this section we introduce a new conditional term rewriting system R in which l and r of any rewrite rule $l \to r$ do not share the same variable; every variable in r is connected to some variable in l thorough an equational condition. A decidable sufficient condition for the Church-Rosser property of R is presented.

$V(t)$ denotes the set of variables occurring in a term t.

Definition 5.1 *A left-right separated conditional term rewriting system is a conditional term rewriting system with extra variables in which every conditional rewrite rule has the form:*

$$l \to r \;\Leftarrow\; x_1 = y_1, \cdots, x_n = y_n$$

with $l, r \in T(\mathcal{F}, \mathcal{V})$, $V(l) = \{x_1, \cdots, x_n\}$ *and* $V(r) \subseteq \{y_1, \cdots, y_n\}$ *such that* **(i)** l *is left-linear,* **(ii)** $\{x_1, \cdots, x_n\} \cap \{y_1, \cdots, y_n\} = \phi$, **(iii)** $x_i \not\equiv x_j$ *if* $i \neq j$, **(iv)** r *does not contain more occurrences of some variables than the conditional part* $x_1 = y_1, \cdots, x_n = y_n$.

Definition 5.2 *Let* R *be a left-right separated conditional term rewriting system. We inductively define term rewriting systems* R_i *for* $i \geq 1$ *as follows:*

$$R_1 = \{l\theta \to r\theta \mid l \to r \;\Leftarrow\; x_1 = y_1, \cdots, x_n = y_n \;\in R \text{ and } x_j\theta \equiv y_j\theta \;(j = 1, \cdots, n)\},$$

$$R_{i+1} = \{l\theta \to r\theta \mid l \to r \;\Leftarrow\; x_1 = y_1, \cdots, x_n = y_n \;\in R \text{ and } x_j\theta \overset{*}{\underset{R_i}{\longleftrightarrow}} y_j\theta \;(j = 1, \cdots, n)\}.$$

In R_{i+1}, *proofs of* $x_j\theta \overset{*}{\underset{R_i}{\longleftrightarrow}} y_j\theta$ $(j = 1, \cdots, n)$ *are called subproofs associating with one step reduction by* $l\theta \to r\theta$. *Note that* $R_i \subseteq R_{i+1}$ *for all* $i \geq 1$. *We have* $s \underset{R}{\longrightarrow} t$ *if and only if* $s \underset{R_i}{\longrightarrow} t$ *for some* i.

The weight $w(s \underset{R}{\longrightarrow} t)$ of one step reduction $s \underset{R}{\longrightarrow} t$ is inductively defined as follows:

(i) $w(s \underset{R}{\longrightarrow} t) = 1$ if $s \underset{R_1}{\longrightarrow} t$,

(ii) $w(s \underset{R}{\longrightarrow} t) = 1 + w(\mathcal{P}_1) + \cdots + w(\mathcal{P}_n)$ if $s \underset{R_{i+1}}{\longrightarrow} t$ $(i \geq 1)$, where $\mathcal{P}_1, \cdots, \mathcal{P}_m$ $(m \geq 0)$ are subproofs associating with one step reduction $s \underset{R_{i+1}}{\longrightarrow} t$.

Let $l \to r \Leftarrow x_1 = y_1, \cdots, x_m = y_m$ and $l' \to r'\ x'_1 = y'_1, \cdots, x'_n = y'_n$ be two rules in a left-right separated conditonal term rewriting system R. Assume that we have renamed the variables appropriately, so that two rules share no variables. Assume that $s \notin V$ is a subterm occurrence in l, i.e., $t \equiv C[s]$, such that s and l' are unifiable, i.e., $s\theta \equiv l'\theta$, with a minimal unifier θ. Note that $r\theta \equiv r$, $r'\theta \equiv r'$, $y_i\theta \equiv y_i$ $(i = 1, \cdots, m)$ and $y'_j\theta \equiv y'_j$ $(j = 1, \cdots, n)$ as $\{x_1, \cdots, x_m\} \cap \{y_1, \cdots, y_m\} = \phi$ and $\{x'_1, \cdots, x'_n\} \cap \{y'_1, \cdots, y'_n\} = \phi$. Thus, from $l\theta \equiv C[s]\theta \equiv C\theta[l'\theta]$, two reductions starting with $l\theta$, i.e., $l\theta \to C\theta[r']$ and $l\theta \to r$, can be obtained by using $l \to r \Leftarrow x_1 = y_1, \cdots, x_m = y_m$ and $l' \to r' \Leftarrow x'_1 = y'_1, \cdots, x'_n = y'_n$ if we have subproofs of $x_1\theta \overset{*}{\leftrightarrow} y_1, \cdots, x_m\theta \overset{*}{\leftrightarrow} y_m$ and $x'_1\theta \overset{*}{\leftrightarrow} y'_1, \cdots, x'_n\theta \overset{*}{\leftrightarrow} y'_n$. Then we say that $l \to r \Leftarrow x_1 = y_1, \cdots, x_m = y_m$ and $l' \to r'\ x'_1 = y'_1, \cdots, x'_n = y'_n$ are overlapping, and

$E \vdash \langle C\theta[r'], r\rangle$

is a conditional critical pair associated with the multiset of equations $E = [x_1\theta = y_1, \cdots, x_m\theta = y_m, x'_1\theta = y'_1, \cdots, x'_n\theta = y'_n]$ in R. We may choose $l \to r \Leftarrow x_1 = y_1, \cdots, x_m = y_m$ and $l' \to r'\ x'_1 = y'_1, \cdots, x'_n = y'_n$ to be the same rule, but in this case we shall not consider the case $s \equiv l$. If R has no critical pair, then we say that R is non-overlapping.

$E \sqcup E'$ denotes the union of multisets E and E'. We write $E \sqsubseteq E'$ if no elements in E occur more than E'.

Definition 5.3 *Let E be a multiset of equations $t' = s'$ and a fresh constant $\bullet$. Then relations $t \underset{E}{\sim} s$ and $t \underset{E}{\rightsquigarrow} s$ on terms is inductively defined as follows:*

(i) $t \underset{\phi}{\sim} t$,

(ii) $t \underset{[t=s]}{\sim} s$,

(iii) *If* $t \underset{E}{\sim} s$, *then* $s \underset{E}{\sim} t$,

(iv) *If* $t \underset{E}{\sim} r$ *and* $r \underset{E'}{\sim} s$, *then* $t \underset{E \sqcup E'}{\sim} s$,

(v) *If* $t \underset{E}{\sim} s$, *then* $C[t] \underset{E}{\sim} C[s]$,

(vi) *If* $l \to r \Leftarrow x_1 = y_1, \cdots, x_n = y_n \in R$ *and* $x_i\theta \underset{E_i}{\sim} y_i\theta$ $(i = 1, \cdots, n)$, *then* $C[l\theta] \underset{E}{\rightsquigarrow} C[r\theta]$ *where* $E = E_1 \sqcup \cdots \sqcup E_n$,

(vii) *If* $t \underset{E}{\rightsquigarrow} r$, *then* $t \underset{E \sqcup [\bullet]}{\sim} s$.

Lemma 5.4 *Let $E = [p_1 = q_1, \cdots, p_m = q_m, \bullet, \cdots, \bullet]$ be a multiset in which $\bullet$ occurs n times $(n \geq 0)$, and let $\mathcal{P}_i$: $p_i\theta \overset{*}{\leftrightarrow} q_i\theta$ $(i = 1, \cdots, m)$.*

(1) *If* $t \underset{E}{\sim} s$ *then there exists a proof* $\mathcal{Q}$: $t\theta \overset{*}{\leftrightarrow} s\theta$ *with* $w(\mathcal{Q}) \leq \sum_{i=1}^{m} w(\mathcal{P}_i) + n$.

(2) *If* $t \underset{E}{\rightsquigarrow} s$ *then there exists a proof* $\mathcal{Q}'$: $t\theta \to s\theta$ *with* $w(\mathcal{Q}') \leq \sum_{i=1}^{m} w(\mathcal{P}_i) + n + 1$.

Proof. By induction on the construction of $t \underset{E}{\sim} s$ and $t \underset{E}{\rightsquigarrow\!\!\triangleright} s$ in Definition 5.3, we prove (1) and (2) simultaneously.

Base Step: Trivial as (i) $t \underset{\phi}{\sim} s \equiv t$ or (ii) $t \underset{[t=s]}{\sim} s$ of Definition 5.3.

Induction Step: If we have $t \underset{E}{\sim} s$ by (iii) (iv) (v) and $t \underset{E}{\rightsquigarrow\!\!\triangleright} s$ by (vi) of Definition 5.3, then from the induction hypothesis (1) and (2) clearly follow. Assume that $t \underset{E}{\rightsquigarrow\!\!\triangleright} s$ by (v) of Definition 5.3. Then we have a rule $l \rightarrow r \Leftarrow x_1 = y_1, \cdots, x_k = y_k$ such that $t \equiv C[l\theta']$, $s \equiv C[r\theta']$, $x_i\theta' \underset{E_i}{\sim} y_i\theta'$ $(i = 1, \cdots, k)$ for some θ' and $E = E_1 \sqcup \cdots \sqcup E_k$. From the induction hypothesis and $E = E_1 \sqcup \cdots \sqcup E_k$, it can be easily shown that $\mathcal{Q}_i$: $x_i\theta'\theta \overset{*}{\leftrightarrow} x_i\theta'\theta$ $(i = 1, \cdots, k)$ and $\sum_{i=1}^{k} w(\mathcal{Q}_i) \leq \sum_{i=1}^{m} w(\mathcal{P}_i) + n$. Therefore we have a proof $\mathcal{Q}'$: $t\theta \rightarrow s\theta$ with $w(\mathcal{Q}') \leq \sum_{i=1}^{m} w(\mathcal{P}_i) + n + 1$. □

Theorem 5.5 *Let R be a left-right separated conditional term rewriting system. Then R is weight decreasing joinable if for any conditional critical pair $E \vdash \langle q, q' \rangle$ one of the following conditions holds:*

(i) $q \underset{E'}{\sim} q'$ *for some E' such that $E' \sqsubseteq E \sqcup [\bullet]$ or,*

(ii) $q \underset{E_1}{\rightsquigarrow\!\!\triangleright} \cdot \underset{E_2}{\triangleleft\!\!\leftsquigarrow} q'$ *for some E_1 and E_2 such that $E_1 \sqcup E_2 \sqsubseteq E$ or,*

(iii) $q \underset{E'}{\rightsquigarrow\!\!\triangleright} q'$ *(or $q' \underset{E'}{\rightsquigarrow\!\!\triangleright} q$) and $E' \sqsubseteq E \sqcup [\bullet]$.*

Note. *The above conditions (i) (ii) (iii) are decidable if R has finite rewrite rules. Thus, the theorem presents a decidable condition for guaranteeing the Church-Rosser property of R.*

Proof. The theorem follows from Lemma 3.2 if for any $\mathcal{P}$: $t \leftarrow p \rightarrow s$ $(t \not\equiv s)$ there exists some proof $\mathcal{Q}$: $t \overset{*}{\leftrightarrow} s$ such that (i) $w(\mathcal{P}) > w(\mathcal{Q})$, or (ii) $w(\mathcal{P}) \geq w(\mathcal{Q})$ and $\mathcal{Q}$: $t \overset{\equiv}{\rightarrow} \cdot \overset{\equiv}{\leftarrow} s$. Hence we will show a proof $\mathcal{Q}$ satisfying (i) or (ii) for a given proof $\mathcal{P}$: $t \leftarrow p \rightarrow s$.

Let $\mathcal{P}$: $t \overset{\Delta}{\leftarrow} p \overset{\Delta'}{\rightarrow} s$ where two redexes $\Delta \equiv l\theta$ and $\Delta' \equiv l'\theta'$ are associated with two rules $\mathbf{r}_1$: $l \rightarrow r \Leftarrow x_1 = y_1, \cdots, x_m = y_m$ and $\mathbf{r}_2$: $l' \rightarrow r' \Leftarrow x'_1 = y'_1, \cdots, x'_{m'} = y'_{m'}$ respectively.

Case 1. Δ and Δ' are disjoint. Then $p \equiv C[\Delta, \Delta']$ for some context $C[\ ,\ ]$ and $\mathcal{P}$: $t \equiv C[t', \Delta'] \overset{\Delta}{\leftarrow} C[\Delta, \Delta'] \overset{\Delta'}{\rightarrow} C[\Delta, s'] \equiv s$ for some t' and s'. Thus, we can take $\mathcal{Q}$: $t \equiv C[t', \Delta'] \overset{\Delta'}{\rightarrow} C[t', s'] \overset{\Delta}{\leftarrow} C[\Delta, s'] \equiv s$ with $w(\mathcal{Q}) = w(\mathcal{P})$.

Case 2. Δ' occurs in θ of $\Delta \equiv l\theta$ (i.e., Δ' occurs below the pattern l). Without loss of generality we may assume that $\mathbf{r}_1$: $C_L[x_1, \cdots, x_m] \rightarrow C_R[y_1, \cdots, y_n] \Leftarrow x_1 = y_1, \cdots, x_m = y_m$ (all the variable occurrences are displayed and $n \leq m$), $\mathcal{P}'$: $p \equiv C[C_L[p_1, \cdots, p_m]] \overset{\Delta}{\rightarrow} t \equiv C[C_R[t_1, \cdots, t_n]]$ with subproofs $\mathcal{P}_i$: $p_i \overset{*}{\leftrightarrow} t_i$ $(i = 1, \cdots, m)$, and $\mathcal{P}''$: $p \equiv C[C_L[p_1, p_2, \cdots, p_m]] \overset{\Delta'}{\rightarrow} s \equiv C[C_L[p'_1, p_2, \cdots, p_m]]$ by $p_1 \overset{\Delta'}{\rightarrow} p'_1$. Thus $w(\mathcal{P}) = w(\mathcal{P}') + w(\mathcal{P}'')$ and $w(\mathcal{P}') = 1 + \sum_{i=1}^{m} w(\mathcal{P}_i)$. Since we have a proof $\mathcal{Q}'$: $p'_1 \overset{\Delta'}{\leftarrow} p_1 \overset{*}{\leftrightarrow} t_1$ with $w(\mathcal{Q}') = w(\mathcal{P}'') + w(\mathcal{P}_1)$, we can apply $\mathbf{r}_1$ to $s \equiv C[C_L[p'_1, p_2, \cdots, p_m]]$ too. Then, we have a proof $\mathcal{Q}$: $s \equiv C[C_L[p'_1, \cdots, p_m]] \rightarrow t \equiv C[C_R[t_1, \cdots, t_n]]$ with $w(\mathcal{Q}) = 1 + w(\mathcal{Q}') + \sum_{i=2}^{m} w(\mathcal{P}_i) = w(\mathcal{P})$.

Case 3. Δ and Δ' coincide by the application of the same rule, i.e., $\mathbf{r} = \mathbf{r}_1 = \mathbf{r}_2$. (*Note. In a left-right separated conditional term rewriting system the application of the same rule at* *the same position does not imply the same result as the variables occurring in the left-hand side of a rule does not cover that in the right-hand side. Thus this case is necessary even if the system is non-overlapping.)* Let the rule applied to Δ and Δ' be $\mathbf{r}$: $C_L[x_1,\cdots,x_m] \to C_R[y_1,\cdots,y_n] \Leftarrow x_1 = y_1,\cdots,x_m = y_m$ (all the variable occurrences are displayed and $n \leq m$), and let $\mathcal{P}'$: $p \equiv C[C_L[p_1,\cdots,p_m]] \xrightarrow{\Delta} t \equiv C[C_R[t_1,\cdots,t_n]]$ with subproofs $\mathcal{P}'_i$: $p_i \overset{*}{\leftrightarrow} t_i$ $(i = 1,\cdots,m)$ and $\mathcal{P}''$: $p \equiv C[C_L[p_1,\cdots,p_m]] \xrightarrow{\Delta'} s \equiv C[C_R[s_1,\cdots,s_n]]$ with subproofs $\mathcal{P}''_i$: $p_i \overset{*}{\leftrightarrow} s_i$ $(i = 1,\cdots,m)$. Here $w(\mathcal{P}) = w(\mathcal{P}') + w(\mathcal{P}'') = 1 + \sum_{i=1}^{m} w(\mathcal{P}_i) + 1 + \sum_{i=1}^{m} w(\mathcal{P}'_i)$. Thus we have a proof $\mathcal{Q}$: $t \equiv C[C_R[t_1,\cdots,t_n]] \overset{*}{\leftrightarrow} C[C_R[p_1,\cdots,p_n]] \overset{*}{\leftrightarrow} C[C_R[s_1,\cdots,s_n]] \equiv s$ with $w(\mathcal{Q}) = \sum_{i=1}^{m} w(\mathcal{P}_i) + \sum_{i=1}^{m} w(\mathcal{P}'_i) < w(\mathcal{P})$.

Case 4. Δ' occurs in Δ but neither Case 2 nor Case 3 (i.e., Δ' overlaps with the pattern l of $\Delta \equiv l\theta$). Then, there exists a conditional critical pair $[p_1 = q_1,\cdots,p_m = q_m] \vdash \langle q, q'\rangle$ between $\mathbf{r}_1$ and $\mathbf{r}_2$, and we can write $\mathcal{P}$: $t \equiv C[q\theta] \xleftarrow{\Delta} p \equiv C[\Delta] \xrightarrow{\Delta'} s \equiv C[q'\theta]$ with subproofs $\mathcal{P}_i$: $p_i\theta \overset{*}{\leftrightarrow} q_i\theta$ $(i = 1,\cdots,m)$. Thus $w(\mathcal{P}) = \sum_{i=1}^{m} w(\mathcal{P}_i) + 2$. From the assumption about critical pairs the possible relations between q and q' are give in the following subcases.

Subcase 4.1. $q \underset{E'}{\sim} q'$ for some E' such that $E' \sqsubseteq E \sqcup [\bullet]$. By Lemma 5.4 and $E' \sqsubseteq E \sqcup [\bullet]$, we have a proof $\mathcal{Q}'$: $q\theta \overset{*}{\leftrightarrow} q'\theta$ with $w(\mathcal{Q}') = \sum_{i=1}^{m} w(\mathcal{P}_i) + 1 < w(\mathcal{P})$. Hence it is obtained that $\mathcal{Q}$: $t \equiv C[q\theta] \overset{*}{\leftrightarrow} s \equiv C[q'\theta]$ with $w(\mathcal{Q}) < w(\mathcal{P})$.

Subcase 4.2. $q \underset{E_1}{\rightsquigarrow} \cdot \underset{E_2}{\leftsquigarrow} q'$ for some E_1 and E_2 such that $E_1 \sqcup E_2 \sqsubseteq E$. By Lemma 5.4 and $E_1 \sqcup E_2 \sqsubseteq E$, we have a proof $\mathcal{Q}'$: $q\theta \to \cdot \leftarrow q'\theta$ with $w(\mathcal{Q}') = \sum_{i=1}^{m} w(\mathcal{P}_i) + 2 \leq w(\mathcal{P})$. Hence we can take $\mathcal{Q}$: $t \equiv C[q\theta] \to \cdot \leftarrow s \equiv C[q'\theta]$ with $w(\mathcal{Q}) \leq w(\mathcal{P})$.

Subcase 4.3. $q \underset{E'}{\rightsquigarrow} q'$ (or $q' \underset{E'}{\rightsquigarrow} q$) and $E' \sqsubseteq E \sqcup [\bullet]$. By Lemma 5.4 and $E' \sqsubseteq E \sqcup [\bullet]$, we have a proof $\mathcal{Q}'$: $q\theta \to q'\theta$ with $w(\mathcal{Q}') = \sum_{i=1}^{m} w(\mathcal{P}_i) + 2 \leq w(\mathcal{P})$. Hence we obtain $\mathcal{Q}$: $t \equiv C[q\theta] \to s \equiv C[q'\theta]$ with $w(\mathcal{Q}) \leq w(\mathcal{P})$. For the case of $q' \underset{E'}{\rightsquigarrow} q$ we can obtain $\mathcal{Q}$: $s \leftarrow t$ with $w(\mathcal{Q}) \leq w(\mathcal{P})$ similarly. □

Corollary 5.6 *Let R be a left-right separated conditional term rewriting system. Then R is weight decreasing joinable if R is non-overlapping.*

Example 5.7 *Let R_L be the left-right separated conditional term rewriting system with the following rewriting rules:*

$$R_L \quad \begin{cases} f(x', x'') \to h(x, f(x, b)) \Leftarrow x' = x, x'' = x \\ f(g(y'), y'') \to h(y, f(g(y), a)) \Leftarrow y' = y, y'' = y \\ a \to b \end{cases}$$

Here, we have a conditonal critical pair

$$[g(y') = x, y'' = x, y' = y, y'' = y] \vdash \langle h(x, f(x, b)), h(y, f(g(y), a))\rangle$$

Since $h(x, f(x, b)) \underset{[y''=x]}{\sim} h(y'', f(x, b)) \underset{[g(y')=x]}{\sim} h(y'', f(g(y'), b)) \underset{[y''=y, y'=y]}{\sim} h(y, f(g(y), b)) \underset{[\bullet]}{\sim} h(y, f(g(y), a))$, *we have* $h(x, f(x, b)) \underset{E'}{\sim} h(y, f(g(y), a))$ *where* $E' = [g(y') = x, y'' = x, y'' =$

$y, y' = y, \bullet]$. *Thus, from Theorem 5.5 it follows that R_L is weight decreasing joinable.* □

In Theorem 5.5 we request that every conditional critical pair $E \vdash \langle q, q' \rangle$ satisfies (i), (ii) or (iii). However, it is clear that we can ignore the conditional critical pairs which cannot appear in the actual proofs of R. Thus, we can strengthen Theorem 5.5 as follows.

Corollary 5.8 *Let R be a left-right separated conditional term rewriting system. Then R is weight decreasing joinable if any conditional critical pair $E \vdash \langle q, q' \rangle$ such that E is satisfiable in R satisfies (i), (ii) or (iii) in Theorem 5.5.*
Note. *The satisfiability of E is generally undecidable.*

6 Conditonal Linearization

The original idea of the conditional linearization of non-left-linear term rewriting systems was introduced by De Vrijer [4], Klop and De Vrijer [7] for giving a simpler proof of Chew's theorem [2, 10]. In this section, we introduce a new conditonal linearization based on left-right separated conditional term rewriting systems. The point of our linearization is that by replacing traditional conditional systems with left-right separated conditional systems we can easily relax the non-overlapping limitation because of the results of the previous section.

Now we explain a new linearization of non-left-linear rules. For instance, let consider a non-duplicating non-left-linear rule $f(x, x, x, y, y, z) \to g(x, x, x, z)$. Then, by replacing all the variable occurrences x, x, x, y, y, z from left to right in the left handside with distinct fresh variable occurrences $x', x'', x''', y', y'', z'$ respectively and connecting every fresh variable to corresponding original one with equation, we can make a left-right separated conditional rule $f(x', x'', x''', y', y'', z') \to g(x, x, x, z) \Leftarrow x' = x, x'' = x, x''' = x, y' = y, y' = y, z' = z$. More formally we have the following definition, the framework of which originates essentially from De Vrijer [4], Klop and De Vrijer [7].

Definition 6.1 (i) *If $\mathbf{r}$ is a non-duplicating rewrite rule $l \to r$, then the (left-right separated) conditional linearization of $\mathbf{r}$ is a left-right separated conditional rewrite rule $\mathbf{r}_L$: $l' \to r \Leftarrow x_1 = y_1, \cdots, x_m = y_m$*
such that $l'\theta \equiv l$ for the substitution $\theta = [x_1 := y_1, \cdots, x_m := y_m]$.

(ii) *If R is a non-duplicating term rewriting system, then R_L, the conditional linearization of R, is defined as the set of the rewrite rules $\{\mathbf{r}_L | \mathbf{r} \in R\}$.*

Note. The non-duplicating limitation of R in the above definition is necessary to guarantee that R_L is a left-right separated conditional term rewriting system.

Note. The above conditional linearization is different form the original one by Klop and De Vrijer [4, 7] in which the left-linear version of a rewrite rule $\mathbf{r}$ is a traditional conditonal rewrite rule without extra variables in the right handside and the conditional part. Hence, in the case **r** is already left-linear, Klop and De Vrijer [4, 7] can take **r** itself as its conditional linearization. On the other hand, in our definition we cannot take **r** itself as its conditional linearization because **r** must be translated into a left-right separated rewrite rule.

Theorem 6.2 *If a conditional linearization R_L of a non-duplicating term rewriting system R is Church-Rosser, then R has unique normal forms.*

Proof. By Propsiton 2.3, similar to Klop and De Vrijer [4, 7]. □

Example 6.3 *Let R be the non-duplicating term rewriting system with the following rewriting rules:*

$$R \quad \begin{cases} f(x,x) \to h(x, f(x,b)) \\ f(g(y),y) \to h(y, f(g(y),a)) \\ a \to b \end{cases}$$

Note that R is non-left-linear and non-terminating. Then we have the following R_L as the linearization of R:

$$R_L \quad \begin{cases} f(x',x'') \to h(x, f(x,b)) \Leftarrow x' = x, x'' = x \\ f(g(y'),y'') \to h(y, f(g(y),a)) \Leftarrow y' = y, y'' = y \\ a \to b \end{cases}$$

In Example 5.7 the Church-Rosser property of R_L has already been shown. Thus, form Theorem 6.2 it follows that R has unique normal forms. □

7 Church-Rosser Property of Non-Duplicating Systems

In the previous section we have shown a general method based on the conditional linearization technique to prove the unique normal form property for non-left-linear overlapping non-duplicating term rewriting systems. In this section we show that the same conditional linearization technique can be used as a general method for proving the Church-Rosser property of some class of non-duplicating term rewriting systems.

Theorem 7.1 *Let R be a right-ground (i.e., no variables occur in the right handside of rewrite rules) term rewriting system. If the conditional linearization R_L of R is weight decreasing joinable then R is Church-Rosser.*

Proof. Let R and R_L have reduction relations $\to$ and $\underset{L}{\to}$ respectively. Since $\underset{L}{\to}$ extends $\to$ and R_L is weight decreasing joinable, the theorem clearly holds if we show the claim: for any t, s and $\mathcal{P}$: $t \overset{*}{\underset{L}{\leftrightarrow}} s$ there exist proofs $\mathcal{Q}$: $t \overset{*}{\underset{L}{\to}} r \overset{*}{\underset{L}{\leftarrow}} s$ with $w(\mathcal{P}) \geq w(\mathcal{Q})$ and $t \overset{*}{\to} r \overset{*}{\leftarrow} s$

for some term r. We will prove this claim by induction on $w(\mathcal{P})$. *Base Step* $w(\mathcal{P}) = 0$ is trivial. *Induction Step* $w(\mathcal{P}) = w$ $(w > 0)$: Form the weight decreasing joinability of R_L, we have a proof $\mathcal{P}'$: $t \xrightarrow[L]{*} \cdot \xleftarrow[L]{*} s$ with $w \geq w(\mathcal{P}')$. Let $\mathcal{P}'$ have the form $t \xrightarrow[L]{} s' \xrightarrow[L]{*} \cdot \xleftarrow[L]{*} s$. Without loss of generality we may assume that $C_L[x_1, \cdots, x_m] \to C_R \Leftarrow x_1 = x, \cdots, x_m = x$ (all the variable occurrences are displayed) is a linearization of $C_L[x, \cdots, x] \to C_R$ and $\mathcal{P}''$: $t \equiv C[C_L[t_1, \cdots, t_m]] \xrightarrow[L]{} s' \equiv C[C_R]$ with subproofs $\mathcal{P}_i$: $t_i \xleftrightarrow[L]{*} t'$ $(i = 1, \cdots, m)$ for some t'. Then, from Lemma 3.3 and the induction hypothesis we have proofs $t_i \xrightarrow{*} t''$ $(i = 1, \cdots m)$. Hence we can take the reduction $t \equiv C[C_L[t_1, \cdots, t_m]] \xrightarrow{*} C[C_L[t'', \cdots, t'']] \to s' \equiv C[C_R]$. Let $\hat{\mathcal{P}}$: $s' \xrightarrow[L]{*} \cdot \xleftarrow[L]{*} s$. From $w > w(\hat{\mathcal{P}})$ and I.H., we have $\hat{\mathcal{Q}}$: $s' \xrightarrow[L]{*} r \xleftarrow[L]{*} s$ with $w(\hat{\mathcal{P}}) \geq w(\hat{\mathcal{Q}})$ and $s' \xrightarrow{*} r \xleftarrow{*} s$ for some r. Thus, the theorem follows. □

The following corollary is originally proven by Oyamaguchi [8].

Corollary 7.2 *[Oyamaguchi] Let R be a right-ground term rewriting system having a non-overlapping conditional linearization R_L. Then R is Church-Rosser.*

Next we relax the right-ground limitation of R in Theorem 7.1.

Theorem 7.3 *Let R be a term rewriting system in which every rewrite rule $l \to r$ is right-linear and no non-linear variables in l occur in r. If the conditional linearization R_L of R is weight decreasing joinable then R is Church-Rosser.*

Proof. The proof is similar to that of Theorem 7.1. Let R and R_L have reduction relations $\to$ and $\xrightarrow[L]{}$ respectively. Since $\xrightarrow[L]{}$ extends $\to$ and R_L is weight decreasing joinable, the theorem clearly holds if we show the claim: for any t, s and $\mathcal{P}$: $t \xleftrightarrow[L]{*} s$ there exist proofs $\mathcal{Q}$: $t \xrightarrow[L]{*} r \xleftarrow[L]{*} s$ with $w(\mathcal{P}) \geq w(\mathcal{Q})$ and $t \xrightarrow{*} r \xleftarrow{*} s$ for some term r. We will prove this claim by induction on $w(\mathcal{P})$. *Base Step* $w(\mathcal{P}) = 0$ is trivial. *Induction Step* $w(\mathcal{P}) = w$ $(w > 0)$: Form the weight decreasing joinability of R_L, we have a proof $\mathcal{P}'$: $t \xrightarrow[L]{*} \cdot \xleftarrow[L]{*} s$ with $w \geq w(\mathcal{P}')$. Let $\mathcal{P}'$ have the form $t \xrightarrow[L]{} \hat{s} \xrightarrow[L]{*} \cdot \xleftarrow[L]{*} s$. Without loss of generality we may assume that $C_L[x_1, \cdots, x_m, y_1] \to C_R[y] \Leftarrow x_1 = x, \cdots, x_m = x, y_1 = y$ (all the variable occurrences are displayed) is the linearization of $C_L[x, \cdots, x, y] \to C_R[y]$ and $t \equiv C[C_L[t_1, \cdots, t_m, p_1]] \xrightarrow[L]{} \hat{s} \equiv C[C_R[p]]$ with subproofs $\mathcal{P}_i$: $t_i \xleftrightarrow[L]{*} t'$ $(i = 1, \cdots, m)$ for some t' and $p_1 \xleftrightarrow[L]{*} p$. Then, we can take $t \equiv C[C_L[t_1, \cdots, t_m, p_1]] \xrightarrow[L]{} s' \equiv C[C_R[p_1]] \xleftrightarrow[L]{*} \hat{s} \equiv C[C_R[p]] \xrightarrow[L]{*} \cdot \xleftarrow[L]{*} s$ with the weight $w(\mathcal{P}')$. Let $\mathcal{P}''$: $t \equiv C[C_L[t_1, \cdots, t_m, p_1]] \xrightarrow[L]{} s' \equiv C[C_R[p_1]]$. Then, from Lemma 3.3 and the induction hypothesis we have proofs $t_i \xrightarrow{*} t''$ $(i = 1, \cdots m)$. Hence we can take the reduction $t \equiv C[C_L[t_1, \cdots, t_m, p_1]] \xrightarrow{*} C[C_L[t'', \cdots, t'', p_1]] \to s' \equiv C[C_R[p_1]]$. Let $\hat{\mathcal{P}}$: $s' \xleftrightarrow[L]{*} \hat{s} \xrightarrow[L]{*} \cdot \xleftarrow[L]{*} s$. From $w > w(\hat{\mathcal{P}})$ and I.H., we have $\hat{\mathcal{Q}}$: $s' \xrightarrow[L]{*} r \xleftarrow[L]{*} s$ with $w(\hat{\mathcal{P}}) \geq w(\hat{\mathcal{Q}})$ and $s' \xrightarrow{*} r \xleftarrow{*} s$ for some r. Thus, the theorem follows. □

Corollary 7.4 *Let R be a term rewriting system in which every rewrite rule $l \longrightarrow r$ is right-linear and no non-linear variables in l occur in r. If the conditional linearization R_L of R is non-overlapping then R is Church-Rosser.*

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