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On Interval Analysis of AC Network Equation

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1. Introduction

Using interval Gaussian algorithm[2], this paper presents a method for obtaining the interval solutions of the linear equation derived from the AC network in which the parameters are given by complex interval numbers. The inherent difficulty of interval computation lies in over-estimation of numerical results[1]. Hence the problem is to develope a technique to overcome it.

In the analysis of the linear network of which parameters are given by complex numbers, a cutset or tieset equation is usually used. However, when the network parameters are the complex interval parameters, this type of equation is not necessarily acceptable because the elements of the coefficient matrix of the equation have a large width due to the addition and subtraction of the interval parameters. This leads us to the over-estimation of the interval solutions.

For the linear network with interval resistive parameters a mehtod is proposed to avoid the over-estimation, in which the network is formulated by the hybrid equation and the aids of Hansen's preconditioning and maximally distant trees provide us with well-estimated interval solution of branch voltage and currents[4,5,6,8]. Here we try to extend this method to the AC network equation in which reactive elements are expressed by complex interval parameters. The complex hybrid equation is derived and is equivalently expressed by the real interval equation to which interval Gaussian algorithm(abbreviated as IGA) can be applied. As an example, we give the numerical computation of the frequency characteristics of a notch filter circuit and compare with Monte Carlo method[3,7].

2. Formulation of Interval Hybrid Equation

We consider a linear AC network. We define the branch of the network to be a single element: a resistive element, a reactive element, an ideal voltage and current source. The network is connected and is assumed to contain neither cutsets of current sources nor tiesets of voltage sources. The current source is connected across the element and the voltage source through it. The values of the resistive and reactive elements are given by the real and complex interval numbers, respectively. The current and voltage sources are also given by the complex interval numbers. We consider the graph associated with the network with the current source open and voltage sources short. We suppose that the graph has b branches and n nodes. We choose a tree in the graph. The numbers of twigs and links are denoted by $\rho(= n-1)$ and $\mu(= b-n+1)$, respectively. Let the fundamental cutset and loop matrices be Q and B respectively. The dot on vector and matrix denotes the complex interval number.

Kirchhoff's current and voltage laws become

$$\dot{\mathbf{I}}_t + Q_l \dot{\mathbf{I}}_l = Q\dot{\mathbf{J}} \tag{1}$$

$$B_t \dot{\mathbf{V}}_t + \dot{\mathbf{V}}_l = B\dot{\mathbf{E}} \tag{2}$$

where $Q = [1_t, Q_l]$, $\dot{\mathbf{I}} = [\dot{\mathbf{I}}_t, \dot{\mathbf{I}}_l]^T$, $B = [B_t, 1_l]$ and $\dot{\mathbf{V}} = [\dot{\mathbf{V}}_t, \dot{\mathbf{V}}_l]^T$. The symbol T means the transpose. The matrix 1_t is ρ dimensional unit matrix and Q_l is $\rho \times \mu$ matrix. The matrix 1_l is $\mu \times \mu$ unit matrix and B_t is $\mu \times \rho$ matrix. The vectors $\dot{\mathbf{I}}$ and $\dot{\mathbf{V}}$ are the branch current and voltage vectors, $\dot{\mathbf{I}}_t$ and $\dot{\mathbf{V}}_t$ is the twig current and voltage vectors and $\dot{\mathbf{I}}_l$ and $\dot{\mathbf{V}}_l$ are the link current voltage vectors. The vectors $\dot{\mathbf{J}}$ and $\dot{\mathbf{E}}$ is the current and voltage source vectors, respectively.

Ohm's law is expressed by

$$\dot{\mathbf{I}}_{tG} = \mathbf{G}\dot{\mathbf{V}}_G, \dot{\mathbf{I}}_{tB} = j\mathbf{B}\dot{\mathbf{V}}_B \tag{3}$$

$$\dot{\mathbf{V}}_{lR} = \mathbf{R}\dot{\mathbf{I}}_R, \dot{\mathbf{V}}_{lX} = j\mathbf{X}\dot{\mathbf{I}}_X, j = \sqrt{-1} \tag{4}$$

where

$$\dot{\mathbf{I}}_t = [\dot{\mathbf{I}}_{tG}, \dot{\mathbf{I}}_{tB}]^T$$

$$\dot{\mathbf{V}}_l = [\dot{\mathbf{V}}_{lR}, \dot{\mathbf{V}}_{lX}]^T$$

$$\mathbf{G} = diag(G_1, G_2, ..., G_{\rho_1}),\ \mathbf{B} = diag(B_1, B_2, ..., B_{\rho_2}),$$

$$\mathbf{R} = diag(R_1, R_2, ..., R_{\mu_1}),\ \mathbf{X} = diag(X_1, X_2, ..., X_{\mu_2}),$$

$$\rho = \rho_1 + \rho_2,\ \mu = \mu_1 + \mu_2.$$

Substituting Eqs.(3) and (4) into Eqs.(1) and (2), we have the complex interval hybrid equation, which is equivalently expressed by the real interval equation

$$\mathbf{Ax} = \mathbf{c} \tag{5}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{G} & \cdot & \cdot & \cdot & Q_{lGR} & \cdot & Q_{lGX} & \cdot \\ \cdot & \mathbf{G} & \cdot & \cdot & \cdot & Q_{lGR} & \cdot & Q_{lGX} \\ \cdot & \cdot & \mathbf{B} & \cdot & \cdot & Q_{lBR} & \cdot & Q_{lBX} \\ \cdot & \cdot & \cdot & -\mathbf{B} & Q_{lBR} & \cdot & Q_{lBX} & \cdot \\ B_{tRG} & \cdot & B_{tRB} & \cdot & \mathbf{R} & \cdot & \cdot & \cdot \\ \cdot & B_{tRG} & \cdot & B_{tRB} & \cdot & \mathbf{R} & \cdot & \cdot \\ \cdot & B_{tXG} & \cdot & B_{tXB} & \cdot & \cdot & \mathbf{X} & \cdot \\ B_{tXG} & \cdot & B_{tXB} & \cdot & \cdot & \cdot & \cdot & -\mathbf{X} \end{bmatrix}$$

$$\mathbf{x} = [\mathbf{V}_{Gr}, \mathbf{V}_{Gi}, \mathbf{V}_{Br}, \mathbf{V}_{Bi},$$

$$\mathbf{I}_{Rr}, \mathbf{I}_{Ri}, \mathbf{I}_{Xr}, \mathbf{I}_{Xi}]^T$$

$$\mathbf{c} = [\mathbf{J}_{sGr}, \mathbf{J}_{sGi}, \mathbf{J}_{sBi}, \mathbf{J}_{sBr},$$

$$\mathbf{E}_{sRr}, \mathbf{E}_{sRi}, \mathbf{E}_{sXi}, \mathbf{E}_{sXr}]^T$$

$$\mathbf{V}_{Gr} = Re(\dot{\mathbf{V}}_G),\ \mathbf{V}_{Gi} = Im(\dot{\mathbf{V}}_G),\ \mathbf{V}_{Br} = Re(\dot{\mathbf{V}}_B),\ \mathbf{V}_{Br} = Im(\dot{\mathbf{V}}_B),$$

$$\mathbf{I}_{Rr} = Re(\dot{\mathbf{I}}_R),\ \mathbf{I}_{Ri} = Im(\dot{\mathbf{I}}_R),\ \mathbf{I}_{Rr} = Re(\dot{\mathbf{I}}_R),\ \mathbf{I}_{Ri} = Im(\dot{\mathbf{I}}_R),$$

$$\mathbf{J}_{sGr} = Re(\dot{\mathbf{J}}_{sG}), \mathbf{J}_{sGi} = Im(\dot{\mathbf{J}}_{sG}), \mathbf{J}_{sBr} = Re(\dot{\mathbf{J}}_{sB}), \mathbf{J}_{sBi} = Im(\dot{\mathbf{J}}_{sB}),$$

$$\mathbf{E}_{sGr} = Re(\dot{\mathbf{E}}_{sG}), \mathbf{E}_{sGi} = Im(\dot{\mathbf{E}}_{sG}), \mathbf{E}_{sBr} = Re(\dot{\mathbf{E}}_{sB}), \mathbf{E}_{sBi} = Im(\dot{\mathbf{E}}_{sB}).$$

The submatrix Q_{lGR} is the submatrix of the fundamental cutset matrix Q associated with the conductive twig and the resistive link. The suffices of the other submatrices have the same meaning. The interval matrix $\mathbf{A}$ has neither addition nor subtraction of interval parameters. If an interval matrix $\mathbf{A}$ is strictly diagonally dominant, then IGA can be implemented for the interval matrix $\mathbf{A}$ without row or column interchanges[2]. However, when the matrix $\mathbf{A}$ does not hold this condition, Eq.(5) has possibility to be solved by a transformation given by Hansen, which tries to transform the matrix $\mathbf{A}$ into a strongly diagonally dominant interval matrix $\tilde{\mathbf{A}}$, which is called the modified interval matrix of $\mathbf{A}$[4].

3. Introducing Maximally Distant Tree

In order to obtain the better-estimated interval solutions concerning all the branches, we solve several network equations deriving from the different trees. To do so, we introduce the maximally distant trees[6,8].

We pick up a tree T_1 and select the maximally distant tree T_2 from T_1 and examine whether the pair of the trees $\{T_1, T_2\}$ covers all the branches of the network. If not, we choose another tree T_3 and examine whether the tree set $\{T_1, T_2, T_3\}$ cover all the branches. We continue this procedure until the set of the trees $\{T_1, T_2, ..., T_K\}$ covers all the branches where the integer K is the minimum number of the trees.

For the tree T_k $(k = 1, 2, ..., K)$ we formulate the modified hybrid equation

$$\tilde{\mathbf{A}}^{(k)}\mathbf{x}^{(k)} = \tilde{\mathbf{c}}^{(k)}, \ k = 1, 2, ..., K \tag{6}$$

where the elements of the interval matrix $\tilde{\mathbf{A}}^{(k)}$ takes the values determined by the tree T_k. The interval solution $\mathbf{x}^{(k)}$ for each tree T_k is computed by IGA. Ohm's law hence gives us the branch voltage and current vectors

$$\mathbf{V}^{(k)} = [\mathbf{V}_{Gr}^{(k)}, \mathbf{V}_{Gi}^{(k)}, \mathbf{V}_{Br}^{(k)}, \mathbf{V}_{Bi}^{(k)}, \mathbf{V}_{Rr}^{(k)}, \mathbf{V}_{Ri}^{(k)}, \mathbf{V}_{Xr}^{(k)}, \mathbf{V}_{Xi}^{(k)}]^T, \ k = 1, 2, ..., K \tag{7}$$

$$\mathbf{I}^{(k)} = [\mathbf{I}_{Gr}^{(k)}, \mathbf{I}_{Gi}^{(k)}, \mathbf{I}_{Br}^{(k)}, \mathbf{I}_{Bi}^{(k)}, \mathbf{I}_{Rr}^{(k)}, \mathbf{I}_{Ri}^{(k)}, \mathbf{I}_{Xr}^{(k)}, \mathbf{I}_{Xi}^{(k)}]^T, \ k = 1, 2, ..., K \tag{8}$$

We represent the true voltage and current vectors solutions as $\mathbf{V}_{true}$ and $\mathbf{I}_{true}$, respectively. Then we have

$$\mathbf{V}_{true} \subseteq \mathbf{V}^{(k)}, \ \mathbf{I}_{true} \subseteq \mathbf{I}^{(k)}, \ k = 1, 2, ..., K \tag{9}$$

where the relation $\subseteq$ denotes the inclusion of two interval vectors elementwise. Hence we have

$$\mathbf{V}_{true} \subseteq (\cap_{k=1}^{K}\mathbf{V}^{(k)}) \subseteq \mathbf{V}^{(k)}, \ k = 1, 2, ..., K \tag{10}$$

$$\mathbf{I}_{true} \subseteq (\cap_{k=1}^{K}\mathbf{I}^{(k)}) \subseteq \mathbf{I}^{(k)}, \ k = 1, 2, ..., K \tag{11}$$

where the relation $\cap$ denotes the intersection of two interval vectors elementwise. Hence it is reasonable to consider the branch voltage vector $\mathbf{V} = \cap_{k=1}^{K}\mathbf{V}^{(k)}$ and the branch current vector $\mathbf{I} = \cap_{k=1}^{K}\mathbf{I}^{(k)}$ as the nearest interval voltage and current vectors to the true solutions of the interval network equation. Hence we construct the complex interval branch voltasge and current vectors

$$\dot{\mathbf{V}} = \mathbf{V}_r + j\mathbf{V}_i \tag{12}$$

$$\dot{\mathbf{I}} = \mathbf{I}_r + j\mathbf{I}_i \tag{13}$$

where

$$\mathbf{V}_r = [\mathbf{V}_{Gr}, \mathbf{V}_{Br}, \mathbf{V}_{Rr}, \mathbf{V}_{Xr}]^T, \mathbf{V}_i = [\mathbf{V}_{Gi}, \mathbf{V}_{Bi}, \mathbf{V}_{Ri}, \mathbf{V}_{Xi}]^T$$

$$\mathbf{I}_r = [\mathbf{I}_{Gr}, \mathbf{I}_{Br}, \mathbf{I}_{Rr}, \mathbf{I}_{Xr}]^T, \mathbf{I}_i = [\mathbf{I}_{Gi}, \mathbf{I}_{Bi}, \mathbf{I}_{Ri}, \mathbf{I}_{Xi}]^T$$

The vectors $\dot{\mathbf{V}}$ and $\dot{\mathbf{I}}$ are complex interval vectors with b components. Each component constitutes rectangle in the complex plane with sides parallel to the coordinates axes.

4. Application to Notch Filter Circuit

We deal with the practical circuit of the notch filter as shown in Fig.1. The parameters of the circuit are given in the reference[7]. The maximally distant tree pair is $T_1 = \{1,3,4,5\}$ and $T_2 = \{2,6,7,8\}$. The set $T_1 \cup T_2$ covers all the branches. The result compared with MC method is shown in Fig.2. Notations Re and Im are real and Imaginary parts of the output voltage V_3. The real and dashed lines show respectively the ranges of real and imaginary parts of the complex voltages obtained by Monte Carlo method(shaded ranges). The chain lines give respectively the results of real and imaginary parts computed by the method proposed here. Both results are fairly in a good agreement.

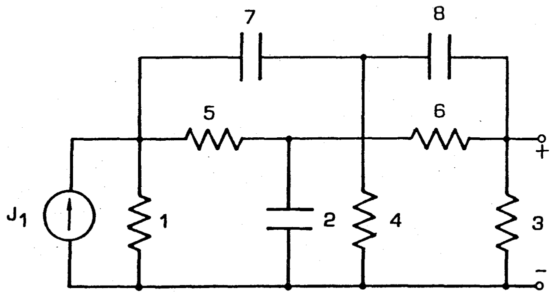

Fig. 1 Notch filter circuit.

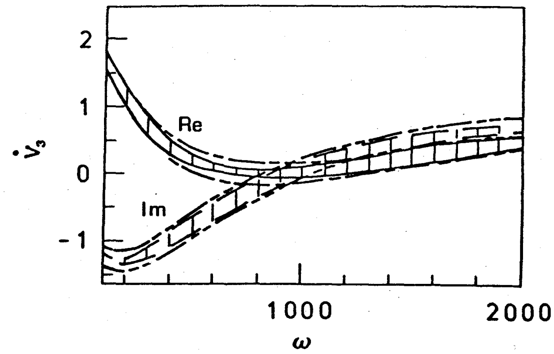

Fig. 2 The frequency characteristics of the voltage $\dot{V}_3$.

5. Conclusion

From the interval mathematical point of view the cutset or tieset equation is not pertinent to the equation for linear AC network with interval parameters. Instead we propose to formulate the interval hybrid equation. In order to have a well-estimated interval solution, two techniques such as Hansen's preconditioning and the maximally distant trees are used. Combining these techniques, we have well-estimated frequecy characteristics of the notch filter circuit and have compared with those by Monte Carlo method. As is expected, the proposed method is much faster than Monte Carlo method. This method can be basically applied to not only passive networks but also active networks if the maximally distant trees are found. Two-graph method would be effective for active networks[9].

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