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LOTKA-VOLTERRA SYSTEMS WITH DELAY
遅れをもつロトカーヴォルテラシステム

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ABSTRACT

Sufficient conditions for a Lotka-Volterra competitive delay system to be permanent and its positive equilibrium point to be a global attractor are given.

Key words: Lotka-Volterra systems, competition, delay effects, permanence.

1 INTRODUCTION

Stability of the Lotka-Volterra delay systems has been studied by a lot of authors. And most of the papers consider the situation at which undelayed intraspecific competitions present[1-3,6,8-10,12]. In these cases, either a Liapunov-Razumikhin functional is used[7,8,10,12] or comparison theorems can be applied[2,9] to obtain global attractivity of a positive equilibrium point. Essentially, the point is a global attractor if the undelayed intraspecific competition dominates over the delayed intra- (and inter-) specific competition.

If the system has no undelayed intraspecific competitions, in general case, the global attractivity of a positive equilibrium or even the weaker concept of stability—permanence or uniform persistence of the system (defined in Section 2) is not easy to investigate. For a discrete delay logistic equation modeling a single species growth, Wright[15] proved global attractivity of a positive equilibrium. Recently, by adopting a similar proof method to Wright[15], Kuang[6] gave some sufficient conditions for a positive equilibrium point of a nonautonomous delay equation to be a global attractor. And in[4], Gopalsamy proposed a method for constucting sign-definite functionals to obtain sufficient conditions for the trivial solution of a non-autonomous vector-matrix delay system to be asymptotically stable and applied the result to derive sufficient conditions for the global attractivity of the positive equilibrium point of a Lotka-Volterra delayed competition system.

In another aspect, permanence or uniform persistence which is a more important concept from the viewpoint of mathematical ecology concerning the survival of population and easier to allow a detailed analysis, has also been investgated[5,13,14] for Lotka-Volterra delay systems. By using continuous functional method, Wang and Ma[14] proved that a two-species prey-predator system with a finite number of discrete delay is permanent provided the undelayed system has a globally stable positive equilibrium point. By developing persistence theory for infinite-dimesional systems, Hale and Waltman[5,13] obtained a uniform persistence result for a two-species competition delayed system.

In this paper, by considering a similar continuous functional to one used in [14] and by adopting a modified approach of Wright[15], we extend Hale and Waltman's result[5,13] in the following sense: their conditions for permanence of a two-species competition delayed system are actually sufficient for its positive equlibrium point to be a global attractor. We also give a sufficent condition for permanence of the system which is weaker than one obtained in [5,13]. Our global attractivity condition (which is identical with Hale and Waltman's permanence condition) is also much weaker than Gopalsamys'[4] and is optimal in the sense that if there is no delay in the system, our condition is sufficient and necessary for the system to have a globally stable equlibrium point.

2 MAIN RESULTS

In this paper, we consider permanence and global attractivity of a positive equilibrium for the following two-species Lotka-Volterra competition delayed system

$$\dot{x}_1(t) = r_1 x_1(t)[1 - x_1(t - \tau_{11}) - \mu_1 x_2(t - \tau_{12})],$$
$$\dot{x}_2(t) = r_2 x_2(t)[1 - x_2(t - \tau_{22}) - \mu_2 x_1(t - \tau_{21})], \quad (1)$$

with initial conditions

$$x_i(t) = \phi_i(t) \geq 0, t \in [-\tau_0, 0]; \phi_i(0) > 0, i = 1, 2, \quad (2)$$

where x_i represents the density of species i, and $r_i > 0$ the reproduction rate, $\mu_i > 0$ the competition coefficient, $\tau_{ij} \geq 0$ $(i, j = 1, 2)$ the constant time lag, and $\tau_0 = \max\{\tau_{ij}\}$. $\phi_i(t)$ is continuous on $[-\tau_0, 0]$.

It is known that system (1) satisfying $\tau_{ij} = 0$ for $i, j = 1, 2$ (i.e. undelayed system) has a globally stable positive equilibrium point x^* if and only if the following condition (C) holds.

CONDITION (C). $\mu_1 < 1$ and $\mu_2 < 1$.

Definition. System (1) is permanent (uniform persistent) if there is a compact region K in the interior of $R_+^2 = \{x | x_i \geq 0; i = 1, 2\}$ such that all solutions $x(t) = (x_1(t), x_2(t))$ of system (1) with initial conditions (2) ultimately enter K.

Our first main result is as follows.

THEOREM 1. If condition (C) holds, then system (1) is permanent.

REMARK 1. In[5], it is proved for system (1) with $\tau_{ii} = 1, \tau_{ij} = 0 (i, j = 1, 2, i \neq j)$ that if (C) holds and both r_1 and r_2 are sufficiently small, then the system is permanent. From Theorem 1, we know that the smallness for r_1 and r_2 is not necessary to ensure permanence of (1).

REMARK 2. Although condition (C) implies permanence of system (1), in general, it cannot ensure global attractivity of the positive equilibrium point of the system, if $\tau_{11}^2 + \tau_{22}^2 \neq 0$[11].

By denoting $\tau = \max\{\tau_{11}, \tau_{22}\}$, $r = \max\{r_1, r_2\}$ and $\eta = r\tau$, we have

THEOREM 2. If condition (C) holds, then for sufficiently small $\eta = r\tau$, the positive equilibrium point x^* of (1) is a global attractor.

REMARK 3. Under the same conditions of Theorem 2, Hale and Waltman[5] proved permanence of system (1). Clearly, our result is much stronger than theirs.

REMARK 4. If $\tau_{11} = \tau_{22} = 0$ and condition (C) holds, then x^* is always a global attractor for any τ_{12} and τ_{21}. This was proved by Gopalsamy[2] and extended to more than two species cases by Martin and Smith[9]. In fact, they considered systems with distributed delays.

REMARK 5. By using a Liapunov functional method, Gopalsamy[4] obtained some sufficient conditions for x^* to be a global attractor of the system. But in [4], (i) all the τ_{ij} $(i = 1, 2)$ must be small; (ii) if $\tau_{ij} = 0$ $(i, j = 1, 2)$, then the sufficient conditions of Theorem 2 given in [4] is much stronger than our condition (C). And condition (C) is optimal in the sense that if it does not hold and $\tau_{ii} = 0$ for $i = 1, 2$, x^* will not be a global attractor[9].

3 DISCUSSION

In this paper, we have shown that a two-species Lotka-Volterra competition delayed system is always permanent under any delay effect, if the corresponding undelayed one has a globally stable positive equlibrium (which is equivalent to permanence), i.e., if the system satisfies condition (C). This means that delays cannot destroy permanence, although the global attractivity of the system may be lost for large $\tau_{ii}, i = 1, 2$[11]. In this case, the delays are called harmless ones[1,3].

We have also discussed global attractivity of the positive equilibrium and given a sufficient condition to guarantee it. Our condition is much weaker than that in [4] and is optimal in the sense that if our condition does not hold and $\tau_{ii} = 0, i = 1, 2$, then x^* will not be a global attractor[9].

In [11], a numerical example was given to show that if, for system (1),

η is less than 1.8, then x^* is locally stable. In the proof of Theorem 2, we can obtain an upper estimate for η ensuring for x^* to be a global attractor. But Theorem 2 of [4] cannot be applied to this example, since in condition (iv) of the theorem, the quadratic form $Q(y_1, y_2) = y_1^2 + 4y_1y_2 + y_2^2$ is not non-negative on the set $\{(y_1, y_2) \in R \times R | 1 + y_1 > 0, 1 + y_2 > 0\}$.

For general n-dimensional Lotka-Volterra undelayed cooperative systems and a two-dimensional undelayed prey-predator system, we know sufficient and necessary conditions to have a globally stable positive equilibrium point. We remain, as future problems, the questions whether these conditions (or somewhat stronger ones) can ensure permanence and global attractivity results for the former systems with delay, and global attractivity for the latter system with delay.

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