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Fields of definition of Teichmüller modular function fields and Oda's problem

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In this lecture, we shall explain moduli theoretic approach to the theory of exterior Galois representations in the pro-l fundamental groups of algebraic curves. First, we discuss T.Oda's idea connecting the exterior Galois representations for curves of a given topological type into a single universal monodromy representaion via the moduli space. Then, the geometry of the Deligne-Mumford-Knudsen compactification of the moduli spaces enables us further to relate the universal monodromy representations of different topological types. As a consequence, we obtain a fundamental conclusion that the exterior Galois image for $\mathbf{P}^1-\{0,1,\infty\}$ 'universally' appear 'in proper positions' of the exterior Galois image for every hyperbolic affine curve.

§1. Motivation

(1.1) Let X be a complete nonsingular algebraic curve of genus g defined over a number field k, S a finite subset of k-rational points of X with cardinality n, and put $C=X\setminus S$. We assume that C is of hyperbolic type, i.e., its Euler characteristic $2-2g-n$ is negative. Fix a rational prime l, and denote the maximal pro-l quotient of π_1 of $\overline{C}=C\otimes\bar{k}$ as follows:

$$\begin{aligned}\pi_1&=\pi_1(\overline{C})(l)\\&\cong\left\langle \begin{matrix}x_1,\cdots,x_{2g}\\ z_1,\cdots,z_n\end{matrix}\,\middle|\,[x_1,x_{g+1}]\cdots[x_g,x_{2g}]z_1\cdots z_n=1\right\rangle_{pro-l}.\end{aligned}$$

Then we have the following commutative diagram of exact sequences

$$\begin{array}{ccccccccc} 1 & \longrightarrow & \pi_1(\overline{C}) & \longrightarrow & \pi_1(C) & \longrightarrow & G_k & \longrightarrow & 1 \\ & & \downarrow & & \downarrow & & \| & & \\ 1 & \longrightarrow & \pi_1 & \longrightarrow & \pi_1'(C) & \longrightarrow & G_k & \longrightarrow & 1, \end{array}$$

where G_k denotes the absolute Galois group of k. From the latter sequence, there arises an exterior Galois representation

$$\varphi_C : G_k \to \mathrm{Out}\pi_1$$

whose image lies in the pro-l mapping class group $\Gamma_{g,n} \subset \mathrm{Out}\pi_1$ consisting of the outer automorphisms preserving the conjugacy classes of the inertia groups $\langle z_1 \rangle, \ldots, \langle z_n \rangle$.

One of the basic questions in the theory of exterior Galois representations motivating a series of papers by Ihara since 1986 ([Ih],[Ih2]) is the following:

(1.2) Problem. *Describe the image of* φ_C.

In fact, there is a natural weight filtration by normal subgroups of the pro-l mapping class group $\Gamma_{g,n}$:

$$\Gamma_{g,n} = \Gamma_{g,n}(0) \supset \Gamma_{g,n}(1) \supset \Gamma_{g,n}(2) \supset \ldots$$

such that $\bigcap_m \Gamma_{g,n}(m) = \{1\}$ which is equipped with the following properties for the graded quotients $\mathrm{gr}^m \Gamma_{g,n} = \Gamma_{g,n}(m)/\Gamma_{g,n}(m+1)$ $(m \geq 0)$:

(1) $\mathrm{gr}^0 \Gamma_{g,n} \cong \mathrm{GSp}(2g)(= \mathrm{GSp}(T_l J_X)$ where $T_l J_X$ is the l-adic Tate module of the Jacobian variety of $X)$;

(2) $\mathrm{gr}^m \Gamma_{g,n}$ $(m \geq 1)$ is a free $\mathbb{Z}_l$-module of finite rank.

So we shall make the following definition.

(1.3) Definition.

$$\begin{aligned} \mathrm{Im}\varphi_C(m) &:= \mathrm{Im}(\varphi_C) \cap \Gamma_{g,n}(m), \\ k_C(m) &:= \text{ the fixed field of } \varphi_C^{-1}(\Gamma_{g,n}(m)), \\ \mathcal{G}_C &:= \bigoplus_{m=1}^{\infty} (\mathrm{Im}\varphi_C(m)/\mathrm{Im}\varphi_C(m+1)) \otimes_{\mathbb{Z}_l} \mathbb{Q}_l \\ &= \bigoplus_{m=1}^{\infty} \mathrm{Gal}(k_C(m+1)/k_C(m)) \otimes_{Z_l} \mathbb{Q}_l. \end{aligned}$$

Since $\mathcal{G}_C$ has a natural graded Lie algebra structure, we call $\mathcal{G}_C$ the graded Lie algebra of pro-l Galois images of C. In this lecture, we interpret Ihara's question as the problem of describing the structure of $\mathcal{G}_C$.

§2. Moduli consideration (A.Grothendick [G], T.Oda [O2])

(2.1) The curve C over k is actually nothing but the morphism of schemes $C \to \mathrm{Spec}\,(k)$, and if a numbering of the deleted points in S is given, then it is naturally squeezed into the following diagram

$$\begin{array}{ccc} C & \longrightarrow & M_{g,n+1} \\ \downarrow & & \downarrow \\ \mathrm{Spec}\,(k) & \longrightarrow & M_{g,n} \end{array}$$

where $M_{g,n}$ is the moduli stack of n-pointed smooth projective curves of genus g, and the right vertical arrow is the forgetful morphism with respect to the $(n+1)$-th marked point. In a similar way to §2, we have the commutative diagram of exact sequences

$$\begin{array}{ccccccccc} 1 & \longrightarrow & \pi_1(\overline{C}) & \longrightarrow & \pi_1(M_{g,n+1}) & \longrightarrow & \pi_1(M_{g,n}) & \longrightarrow & 1 \\ & & \downarrow & & \downarrow & & \| & & \\ 1 & \longrightarrow & \pi_1 & \longrightarrow & \pi_1'(M_{g,n+1}) & \longrightarrow & \pi_1(M_{g,n}) & \longrightarrow & 1. \end{array}$$

and get the pro-l exterior monodromy representation

$$\varphi_{g,n} : \pi_1(M_{g,n}) \to \Gamma_{g,n} \subset \mathrm{Out}\pi_1.$$

(2.2) Definition.

$M_{g,n}(m) :=$ the profinite etale cover of $M_{g,n}$ correspoinding to $\varphi_{g,n}^{-1}(\Gamma_{g,n}(m)) \subset \pi_1(M_{g,n})$
$=$ the moduli of punctured curves with weight $-m$ structures on π_1^{pro-l}

(2.3) Definition.

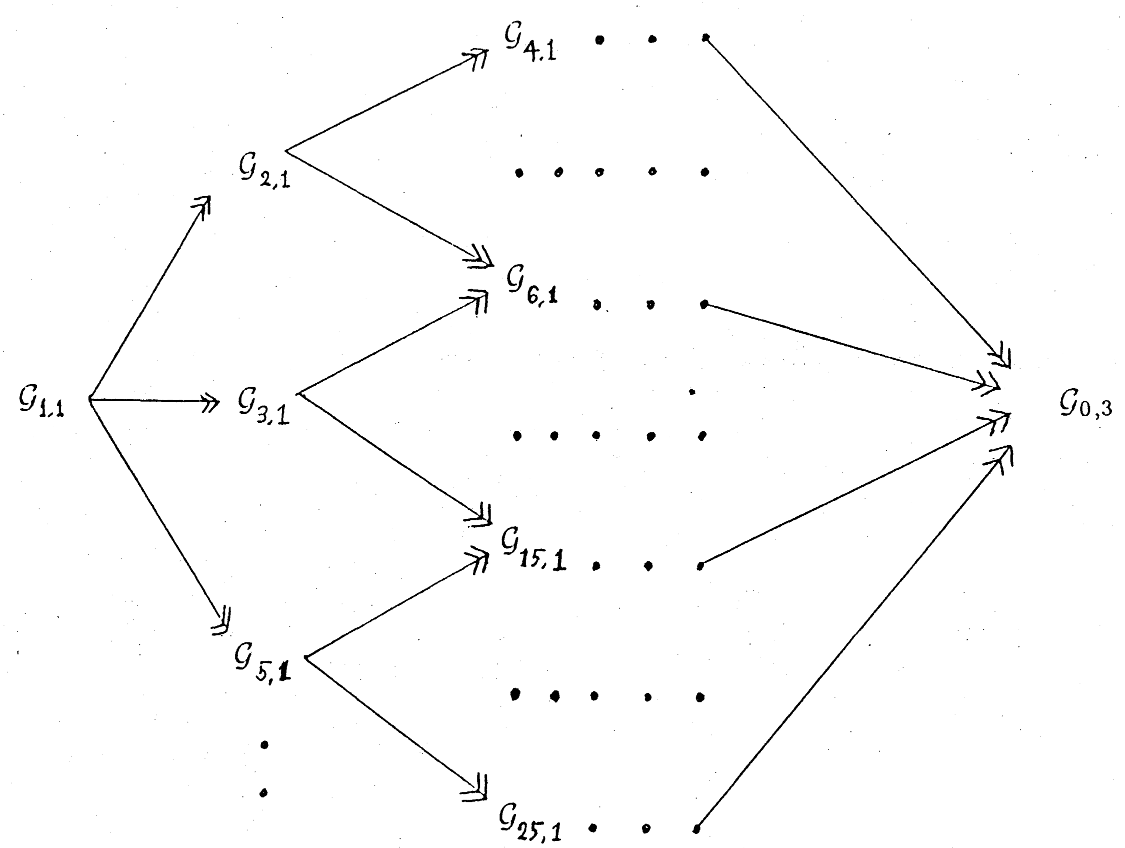
$$\mathbb{Q}_{g,n}(m) := \text{the field of definition of } M_{g,n}(m)$$

$$\mathcal{G}_{g,n} := \bigoplus_{m=1}^{\infty} Gal(\mathbb{Q}_{g,n}(m+1)/\mathbb{Q}_{g,n}(m)) \otimes_{\mathbb{Z}_l} \mathbb{Q}_l$$

We call $\mathcal{G}_{g,n}$ the graded Lie algebra of the universal Galois images of type (g, n).

We can give the following fundamental relations among the graded Lie algebras defined in the above paragraphs.

(2.4) Theorem ([NTU],[N2]). *Assume $n \geq 1$. Then*
(1) $\mathcal{G}_{g,n}$ is independent of n.
(2) There is a canonical surjection $\mathcal{G}_{g,n} \to \mathcal{G}_{rg,n}$ ($r \geq 0$, $2-2gr-n < 0$).
(3) There is a canonical surjection $\mathcal{G}_C \to \mathcal{G}_{g,n}$.



In particular, all the graded Lie algebras $\mathcal{G}_C$, $\mathcal{G}_{g,n}$ have surjections onto $\mathcal{G}_{0,3}$. The latter primitive Lie algebra $\mathcal{G}_{0,3}$ is expected to be a free Lie algebra with free generators in degrees $4k+2$ ($k = 1, 2, \ldots$) each of which corresponds to Soule's cyclotomic element of the K-theory $K_{4k+1}(\mathbb{Z}[l^{-1}], \mathbb{Z}_l)$ (cf. Deligne [De], Ihara [Ih], see also Matsumoto's report in this volume). On the other hand, there are explicit upper bounds for the ranks of graded quotients of $\mathcal{G}_{1,1}$ (cf. Nakamura-Tsunogai [NT]).

In the next section, we will explain how to get the result (1) of the above. By this result, we can let the number of punctures sufficiently large, which often makes problems easy to approach. In §4, we explain the result (2) relating defferent genera. Geometry of the Deligne-Mumford-Knudsen compactifications of the moduli of curves, especially the clutching morphisms among them enable us to couple the universal monodromy representations φ_{g_1,n_1}, φ_{g_2,n_2} into $\varphi_{g_1+g_2,n_1+n_2-2}$. The result (3) above is a consequence of a certain weight argument involving the Weil-Riemann conjecture and Bogomolov's theorem on homothety Galois images ([N2]).

§3. Universal monodromy braid representations

(3.1) We can generalize the exterior Galois representations in the pro-l fundamental groups as follows. For the hyperbolic curve C as in §2, we can define its r-th configuration power $C^{(r)}$ by

$$C^{(r)} = \{(a_1, \ldots, a_r) \in C^r \mid a_i \neq a_j \, (i \neq j)\} \quad (r \geq 2).$$

The fundamental group $\pi_1(C^{(r)})$ is nothing but the braid group on C with r strings. The forgetful homomorphism for the k-th component of $C^{(r)}$ induces the surjection $p_k : \pi_1(C^{(r)}) \to \pi_1(C^{(r-1)})$ $(1 \leq k \leq r)$ whose kernel

$$\Phi_k := \ker(p_k)$$

is isomorphic to π_1 of C minus $r-1$ points. We define $\Gamma_{g,n}^{(r)}$ to be the subgroup of $\mathrm{Out}\pi_1(\overline{C}^{(r)})(l)$ consisting of all the outer automorphisms preserving each $\Phi_k(l)$ $(1 \leq k \leq r)$ together with the conjugacy classes of the inertia subgroups in each of $\Phi_k(l)$ regarded as π_1 of an open curve as above. Then there arises an exterior Galois representation

$$\varphi_C^{(r)} : G_k \to \Gamma_{g,n}^{(r)} \subset \mathrm{Out}\pi_1(\overline{C}^{(r)})(l).$$

(3.2) The above construction can also be generalized in the moduli setting of §2 simply by replacing the forgetful morphism $M_{g,n+1} \to M_{g,n}$ by $M_{g,n+r} \to M_{g,n}$ to obtain the pro-l monodromy braid representation

$$\varphi_{g,n}^{(r)} : \pi_1(M_{g,n}) \to \Gamma_{g,n}^{(r)}.$$

It is also possible to define a natural weight filtration by normal subgroups in $\Gamma_{g,n}^{(r)}$:

$$\Gamma_{g,n}^{(r)} = \Gamma_{g,n}^{(r)}(0) \supset \Gamma_{g,n}^{(r)}(1) \supset \Gamma_{g,n}^{(r)}(2) \supset \dots$$

with similar properties to $\Gamma_{g,n}$ in §1. Therefore we can define the field tower $\{k_{C^{(r)}}(m)\}_m$ and $\{\mathbb{Q}_{g,n}^{(r)}(m)\}_m$ and the graded Lie algebras $\mathcal{G}_C^{(r)}$, $\mathcal{G}_{g,n}^{(r)}$ in exactly similar manners to the cases of $r = 1$ (see 1.3 and 2.3), but in fact we know their invariances with respect to r by the following theorem

(3.3) Theorem([NTU] generalizing [I], [IK]). *The naturally induced mapping*

$$\bigoplus_{m=1}^{\infty} \mathrm{gr}^m \Gamma_{g,n}^{(r)} \to \bigoplus_{m=1}^{\infty} \mathrm{gr}^m \Gamma_{g,n}^{(r-1)}$$

is injective at least for $n \geq 1$. From this follows that the field tower $\{k_{C^{(r)}}(m)\}_m$ and $\{\mathbb{Q}_{g,n}^{(r)}(m)\}_m$ are independent of r. In particular, the graded Lie algebras $\mathcal{G}_C^{(r)}$, $\mathcal{G}_{g,n}^{(r)}$ $(n \geq 1)$ are independent of the choice of r.

(3.4) On the other hand, it is possible to see that there are natural surjections

$$\mathcal{G}_{g,n+s}^{(r)} \to \mathcal{G}_{g,n}^{(r)},$$
$$\mathcal{G}_{g,n}^{(r)} \to \mathcal{G}_{g,n+s}^{(r-s)}$$

by precisely observing definitions, functorialities, and by applying certain weight arguments ([N2]). Combining these results, we obtain a chain of surjections

$$\mathcal{G}_{g,1} = \mathcal{G}_{g,1}^{(n+r-1)} \to \mathcal{G}_{g,n}^{(r)} \to \mathcal{G}_{g,n+r-1} \to \mathcal{G}_{g,1},$$

and hence their equalities.

Thus we conclude (1) of our Theorem.

§4. Coupling of universal monodromies

(4.1) A key relation leading to (2) of our Theorem (2.4) is the following inclusion

$$\mathbb{Q}_{g,n}(m) \subset \mathbb{Q}_{g_1,n_1}(m)\mathbb{Q}_{g_2,n_2}(m)$$

where $g = g_1 + g_2$, $n = n_1 + n_2 - 2$, $n_1, n_2, n \geq 1$, $2 - 2g - n < 0$ and $2 - 2g_i - n_i < 0$ for $i = 1, 2$. To see this, we need to couple two monodromy representations

$$\varphi_{g_1,n_1} : \pi_1(M_{g_1,n_1}) \to \Gamma_{g_1,n_1},$$
$$\varphi_{g_2,n_2} : \pi_1(M_{g_2,n_2}) \to \Gamma_{g_2,n_2}$$

into the third one

$$\varphi_{g,n} : \pi_1(M_{g,n}) \to \Gamma_{g,n}.$$

Consider two families of punctured curve

$$M_{g_1,n_1+1} \times M_{g_2,n_2} \to M_{g_1,n_1} \times M_{g_2,n_2}$$
$$M_{g_1,n_1} \times M_{g_2,n_2+1} \to M_{g_1,n_1} \times M_{g_2,n_2}$$

yielding

$$\varphi : \pi_1(M_{g_1,n_1} \times M_{g_2,n_2}) \to \Gamma_{g_1,n_1} \times \Gamma_{g_2,n_2}.$$

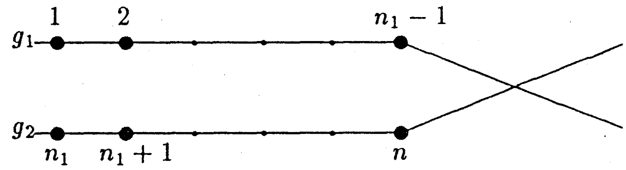
(4.2) Definition. We define

$$\pi_1(M_{g_1,n_1} \times M_{g_2,n_2})(m_1, m_2) := \varphi^{-1}(\Gamma_{g_1,n_1}(m_1) \times \Gamma_{g_2,n_2}(m_2))$$

for $m_1, m_2 \geq 1$.

(4.3) Let $\mathcal{H}_g$ $(g \geq 2)$ be the moduli scheme over $\mathbb{Q}$ of all the tri-canonically embedded stable curves introduced by Deligne-Mumford [DM]. It is known to be a smooth irreducible scheme of finite type over $\mathbb{Q}$. Let $\mathcal{M}_{g,n}$ be the moduli stack of n-pointed stable curves of genus g studied by Deligne-Mumford [DM] and Knudsen [K] (for simplicity we write $\mathcal{M}_{g,0} = \mathcal{M}_g$.) The forgetful morphism $h_g : \mathcal{H}_g \to \mathcal{M}_g$ is representable, smooth and surjective with fibres $PGL(5g-6)$-torsors. Put $\mathcal{H}_{g,n} = \mathcal{H}_g \times_{\mathcal{M}_g} \mathcal{M}_{g,n}$ and denote by $h_{g,n} : \mathcal{H}_{g,n} \to \mathcal{M}_{g,n}$ the canonical projection. We write $H_{g,n} \subset \mathcal{H}_{g,n}$ for the pullback of $M_{g,n} \subset \mathcal{M}_{g,n}$ via

$h_{g,n}$. It is an open subscheme of $\mathcal{H}_{g,n}$ with its complement being normal crossing divisors. Let $H^{\sim}$ be $\mathcal{H}_{g,n}$ minus the singular divisors other than that representing stable curves of type



Then,

$$H^{\sim} = H_{g,n} \cup D$$

where D is a divisor of $H^{\sim}$ of stable curves of the above type. We have a diagram

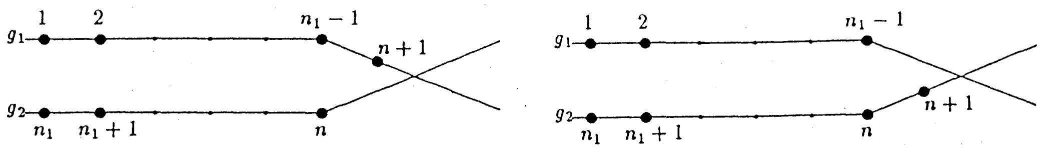
$$\begin{tikzcd}
1 \arrow[r] & \hat{\mathbb{Z}}(1) \arrow[r] & \pi_1^D(H^{\wedge}_{/D}) \arrow[r, "\varepsilon_D"] \arrow[d, "\partial_D"'] & \pi_1(D) \arrow[r] \arrow[d, "\mu_D"] & 1 \\
 & & \pi_1(H_{g,n}) \arrow[d, "\mu_{g,n}"'] & \pi_1(M_{g_1,n_1} \times M_{g_2,n_2}) & \\
 & & \pi_1(M_{g,n}) & &
\end{tikzcd}$$

where $H^{\wedge}_{/D}$ is the formal completion of $H^{\sim}$ along the divisor D, π_1^D denotes the fundamental group (tamely) ramified along D, and the first horizontal sequence is exact by the theory of Grothendieck-Murre [GM].

(4.4) Coupling Theorem [N2]. *There exists a closed subgroup N of $\pi_1^D(H^{\wedge}_{/D})$ which is surjectively mapped onto $\pi_1(D)(3,3)$ by ε_D making the following diagram commutes:*

$$\begin{tikzcd}
N \arrow[r, "\mu_D \circ \varepsilon_D|_N"] \arrow[d, "\partial_D|_N"'] & \pi_1(M_{g_1,n_1} \times M_{g_2,n_2})(3,3) \arrow[r, "\phi_D"] & \Gamma_{g_1,n_1}(3) \times \Gamma_{g_2,n_2}(3) \arrow[d, "\partial"] \\
\pi_1(H_{g,n}) \arrow[r, "\mu_{g,n}"] & \pi_1(M_{g,n}) \arrow[r, "\phi_{g,n}"] & \Gamma_{g,n}.
\end{tikzcd}$$

A key idea of the proof of the above coupling theorem is to consider the open subscheme $H^{\sim}_{g,n+1} = H_{g,n+1} \cup D_1 \cup D_2$ of $H_{g,n}$, where D_1 and D_2 are loci of stable curves of the following types respectively



which is a family of curves over $H^{\sim}_{g,n}$, and to combine Grothendieck-Murre exact sequences vertically and horizontally in suitable ways.

(4.5) Corollary([N2]). *We have*

$$\mathbb{Q}_{rg,n}(m) \subset \mathbb{Q}_{g,n}(m) \qquad (r \geq 0).$$

In particular,

$$\mathbb{Q}_{0,3}(m) \subset \mathbb{Q}_{g,n}(m) \subset \mathbb{Q}_{1,1}(m).$$

This corollary together with suitable weight arguments ([N2]) induce the surjective map $\mathcal{G}_{g,n} \to \mathcal{G}_{gr,n}$ of (2) of our Theorem.

Remark. M.Matsumoto also showed an alternative interesting method for relating $g = 0$ and $g > 0$ (see his report in this volume or forthcoming [M2]).

§5. Applications

(5.1) Application to anabelian Tate conjecture:.

Let $\mathrm{Out}_{G_k}\pi_1(\overline{C}^{(r)})(l)$ be the centralizer of the Galois image $\varphi_C^{(r)}(G_k)$ in $\mathrm{Out}\pi_1(\overline{C}^{(r)})(l)$ (called the 'Galois centralizer'), and let

$$\Phi_C^{(r)} : \mathrm{Aut}_k C^{(r)} \to \mathrm{Out}_{G_k}\pi_1(\overline{C}^{(r)})(l)$$

be the natural mapping. It is expected that $\Phi_C^{(r)}$ gives a bijection, and conjectured that the Galois centralizer $\mathrm{Out}_{G_k}\pi_1(\overline{C}^{(r)})(l)$ is at least finite. These types of problems were studied in our previous works [N], [NT] etc. where some affirmative examples making $\Phi_C^{(r)}$ bijective were given .

When $r = 1$, we can apply the results of Theorem (2.4) to show that $\mathrm{Out}_{G_k}\pi_1(\overline{C})(l)$ can be embedded into $\mathrm{Sp}(2g, \mathbb{Z}_l) \times S_n$ and hence that, if the endomorphism ring of the Jacobian of X is isomorphic to $\mathbb{Z}$, then the Galois centralizer is embedded into $\{\pm 1\} \times S_n$.

Moreover for general $r \geq 1$, it is possible to show the existence of natural injective homomorphisms:

$$\mathrm{Aut}_k C \times S_r \hookrightarrow \mathrm{Aut}_k C^{(r)} \hookrightarrow \mathrm{Out}_{G_k}\pi_1(\overline{C}^{(r)})(l) \hookrightarrow \mathrm{Out}_{G_k}\pi_1(\overline{C})(l) \times S_r.$$

for hyperbolic affine curves C of non-exceptional topological types. This result will appear elsewhere.

(5.2) Application to Topology:.

Let $\Gamma_{g,n}^{top}$ be the topological mapping class group of a genus g Riemann surface with n marking points. There is a natural weight filtration in $\Gamma_{g,n}^{top}$, and the problem of determining the graded quotients $\mathrm{gr}^m\Gamma_{g,n}^{top}$ is still open. Each graded quotient $\mathrm{gr}^m\Gamma_{g,n}^{top}$ can be embedded into an explicit module (via the so-called Johnson homomorphism), and several researches for estimating the images have been done (cf. Johnson, Morita, Asada-Nakamura, Oda). Especially, S.Morita [Mo] showed that there are nontrivial cokernels of Johnson homomorphisms of odd degrees greater than 1. As an application of our above Theorem, we can complement to his result by obtaining nontrivial cokernels of Johnson homomorphisms of even degrees except 2,4,8,12.

(5.3) In [G], A.Grothendieck proposed mysteriously and hypothetically an existence of the anabelian dictionary involving "profinite paradigm" which firstly sounded like "profinite paradise" to the first author. Since then, the first author has been charmed by the idea of profinite paradise where a good deal of arithmetic number fields are controled by systems of finitely presented profinite groups, as is indicated mysteriously in [G]. Here in the present article, we considered fields of definition of certain profinite towers over the moduli stacks of curves which are controled by Galois-Teichmüller profinite groups. Related with this point of view, in the second conference of the present volume, Professor M.Fried called our attention to another type of interesting towers over the moduli of genus 0 curves (Hurwitz spaces) which arise from the notion of universal Frattini covers (cf. [FJ], [F]). It seems to be hoped that various kinds of realizations of Grothendieck's dream shall be possible, producing branches and interactions among them. Anyway we should like to thank him for very encouraging discussions around these ideas.

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