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CERTAIN SUBCLASSES OF MEROMORPHICALLY MULTIVALENT FUNCTIONS

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Abstract. Let $\sum_p$ be the class of functions of the form

$$f(z)=\frac{1}{z^p}+\frac{a_0}{z^{p-1}}+\cdots+a_{k+p-1}z^k+\cdots \quad (p\in N=\{1,2,\cdots\})$$

which are analytic in the punctured open unit disk. In this paper, a new subclass $\sum_{n,p}(\alpha,\delta,\mu,\lambda)$ of $\sum_p$ is introduced. For functions $f\in\sum_{n,p}(\alpha,\delta,\mu,\lambda)$, we find a sufficient condition on the function $g\in\sum_p$ which can guarantee that

$$Re\left\{\frac{D^{n+p}f(z)}{D^{n+p}g(z)}\right\}>\alpha \quad (0\leq\alpha<1, z\in E=\{z:|z|<1\})$$

implies

$$Re\left\{\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\right\}>\beta \quad (0\leq\alpha<\beta<1, z\in E).$$

Further, some applications of this result are given.

1. Introduction and Preliminaries

Let $\sum_p$ denote the class of functions of the form

$$f(z)=\frac{1}{z^p}+\frac{a_0}{z^{p-1}}+\cdots+a_{k+p-1}z^k+\cdots \quad (p\in N=\{1,2,\cdots\}) \tag{1.1}$$

which are analytic in the annulus $D=\{z:0<|z|<1\}$. The Hadamard product or convolution of two functions f and g in $\sum_p$ will be denoted by $f*g$. Following Uralegaddi and Path [6], we define

$$\begin{aligned} D^{n+p-1}f(z) &=\frac{1}{z^p(1-z)^{n+p}}*f(z)\\ &=\frac{1}{z^p}\left(\frac{z^{n+2p-1}f(z)}{(n+p-1)!}\right)^{(n+p-1)}, \end{aligned} \tag{1.2}$$

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where n is any integer greater than $-p$. The symbol D^{n+p-1} when $p=1$ was introduced by Uralegaddi and Ganigi [5]. It follows from (1.2) that

$$z(D^{n+p-1}f(z))' = (n+p)D^{n+p}f(z) - (n+2p)D^{n+p-1}f(z). \tag{1.3}$$

We denote by $\sum_{n,p}(\alpha,\delta,\mu,\lambda)$ the class of all functions $f \in \sum_p$ such that

$$Re\Big\{(1-\lambda)\Big(\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\Big)^{\mu} + \lambda\frac{D^{n+p}f(z)}{D^{n+p}g(z)}\Big(\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\Big)^{\mu-1}\Big\} > \alpha,$$

where $g \in \sum_p$ satisfies the condition

$$Re\Big\{\frac{D^{n+p-1}g(z)}{D^{n+p}g(z)}\Big\} > \delta \quad (0 \le \delta < 1, z \in E = \{z : |z| < 1\}),$$

where α and μ are real numbers such that $0 \le \alpha < 1, \mu > 0$, n is any integer greater than $-p$ and λ is a complex number such that $Re\{\lambda\} > 0$.

The object of the present paper is to investigate some interesting properties and applications for the class $\sum_{n,p}(\alpha,\delta,\mu,\lambda)$.

To establish our main results we need the following lemmas.

LEMMA 1. *Let Ω be a set in the complex plane $\mathbb{C}$ and let the function $\psi : \mathbb{C}^2 \to \mathbb{C}$ satisfy the condition $\psi(ir_2, s_1) \notin \Omega$, for all real $r_2, s_1 \le -\frac{1+r_2^2}{2}$. If $q(z)$ is analytic in E with $q(0)=1$ and $\psi(q(z), zq'(z)) \in \Omega, z \in E$, then $Re\{q(z)\} > 0$ in $E = \{z : |z| < 1\}$.*

A more general form of this lemma may be found in [3].

LEMMA 2([4]). *If $q(z)$ is analytic in E with $q(0)=1$, and if λ is a complex number satisfying $Re\{\lambda\} \ge 0 \quad (\lambda \ne 0)$, then $Re\{q(z)+\lambda zq'(z)\} > \alpha \quad (0 \le \alpha < 1)$ implies $Re\{q(z)\} > \alpha + (1-\alpha)(2\gamma - 1)$, where γ is given by*

$$\gamma = \gamma(Re\lambda) = \int_0^1 (1+t^{Re\lambda})^{-1}dt$$

which is increasing function of $Re\{\lambda\}$ *and* $\frac{1}{2} \leq \gamma < 1$. *The estimate is sharp in the sense that the bound can not be improved.*

For real numbers a, b, c with $c \neq 0, -1, -2, \cdots$ and $c > b > 0$, the hypergeometric series

$$F(a, b; c; z) = 1 + \frac{ab}{c}\frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}\frac{z^2}{2!} + \cdots$$

represents an analytic function in E [1]. The following identities are well known [1].

LEMMA 3. *For real numbers* a, b, c *with* $c \neq 0, -1, -2, \cdots$ *and* $c > b > 0$, *we have*

$$\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}dt = \frac{\Gamma(b)\Gamma(c-a)}{\Gamma(c)}F(a, b; c; z), \tag{1.4}$$

$$F(a, b; c; z) = (1-z)^{-a}F(a, c-b; c; \frac{z}{z-1}) \tag{1.5}$$

and

$$F(1, 1; 2; \frac{1}{2}) = 2ln2. \tag{1.6}$$

2. Main Results

THEOREM 1. *Let* $f \in \sum_{n,p}(\alpha, \delta, \mu, \lambda), \quad \lambda \geq 0$. *Then*

$$Re\Big(\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\Big)^{\mu} > \frac{2(n+p)\alpha\mu + \delta\lambda}{2(n+p)\mu + \delta\lambda} \quad (0 \leq \alpha < 1, \mu > 0, z \in E), \tag{2.1}$$

where the function $g \in \sum_p$ *satisfies the condition*

$$Re\Big(\frac{D^{n+p-1}g(z)}{D^{n+p}g(z)}\Big) > \delta \quad (0 \leq \delta < 1, z \in E). \tag{2.2}$$

Proof. Let $\gamma = \frac{2(n+p)\alpha\mu+\delta\lambda}{2(n+p)\mu+\delta\lambda}$ and we define the function $q(z)$ by

$$q(z) = (1-\gamma)^{-1}\Big(\Big(\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\Big)^{\mu} - \gamma\Big).$$

Then $q(z)$ is analytic in E and $q(0)=1$. If we set $\beta(z)=\frac{D^{n+p-1}g(z)}{D^{n+p}g(z)}$, then by the hypothesis, $Re\{\beta(z)\}>\delta$. Differentiating $q(z)$ and using the identity (1.3), we have

$$(1-\lambda)\Big(\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\Big)^{\mu}+\lambda\frac{D^{n+p}f(z)}{D^{n+p}g(z)}\Big(\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\Big)^{\mu-1}$$
$$=\gamma+(1-\gamma)q(z)+\frac{(1-\gamma)\lambda}{\mu(n+p)}\beta(z)zq'(z).$$

Let us define the function $\psi(r,s)$ by

$$\psi(r,s)=\gamma+(1-\gamma)r+\frac{\lambda(1-\gamma)}{\mu(n+p)}\beta(z)s. \tag{2.3}$$

Using (2.3) and the fact that $f\in\sum_{n,p}(\alpha,\delta,\mu,\lambda)$, we obtain

$$\{\psi(q(z),zq'(z));z\in E\}\subset\Omega=\{\omega\in\mathbb{C};Re\{\omega\}>\alpha\}.$$

Now for all real $r_2,s_1\leq-\frac{1+r_2^2}{2}$, we have

$$\begin{aligned}Re\{\psi(ir_2,s_1)\}&=\gamma+\frac{\lambda(1-\gamma)s_1}{\mu(n+p)}Re\{\beta(z)\}\\&\leq\gamma-\frac{\lambda(1-\gamma)\delta(1+r_2^2)}{2\mu(n+p)}\\&\leq\gamma-\frac{\lambda(1-\gamma)\delta}{2\mu(n+p)}=\alpha.\end{aligned}$$

Hence for each $z\in E$, $\psi(ir_2,s_1)\notin\Omega$. Thus by Lemma 1, $Re\{q(z)\}>0$ in E and hence

$$Re\Big\{\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\Big\}^{\mu}>\gamma$$

in E. This proves our theorem.

COROLLARY 1. *Let $f(z)$ and $g(z)$ be in $\sum_p$ and $g(z)$ satisfy the condition (2.2). If $\lambda\geq1$ and*

$$Re\Big\{(1-\lambda)\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}+\lambda\frac{D^{n+p}f(z)}{D^{n+p}g(z)}\Big\}>\alpha\quad(0\leq\alpha<1,z\in E), \tag{2.4}$$

then

$$Re\Big\{\frac{D^{n+p}f(z)}{D^{n+p}g(z)}\Big\}>\gamma=\frac{\alpha(2n+2p+\delta)+\delta(\lambda-1)}{2(n+p)+\lambda\delta}.$$

Proof. We have

$$\lambda\frac{D^{n+p}f(z)}{D^{n+p}g(z)} = \left[(1-\lambda)\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}+\lambda\frac{D^{n+p}f(z)}{D^{n+p}g(z)}\right]+(\lambda-1)\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)} \quad (z \in E).$$

Since $\lambda \geq 1$, making use of (2.4) and (2.1) (for $\mu = 1$), we deduce that

$$Re\left\{\frac{D^{n+p}f(z)}{D^{n+p}g(z)}\right\} > \gamma = \frac{\alpha(2(n+p)+\delta)+\delta(\lambda-1)}{2(n+p)+\lambda\delta}.$$

COROLLARY 2. *Let λ be a complex number with $Re\{\lambda\} \geq 0$ $(\lambda \neq 0)$. If $f \in \sum_p$ satisfies*

$$Re\{(1-\lambda)(z^p D^{n+p-1}f(z))^\mu + \lambda z^p D^{n+p}f(z)(z^p D^{n+p-1}f(z))^{\mu-1}\} > \alpha$$

for $0 \leq \alpha < 1$ and $\mu > 0$, then

$$Re\{z^p D^{n+p-1}f(z)\}^\mu > \frac{2\mu(n+p)\alpha + Re\{\lambda\}}{2\mu(n+p)+Re\{\lambda\}} \quad (z \in E). \tag{2.5}$$

Further, if $\lambda \geq 1$ and $f \in \sum_p$ satisfies

$$Re\{(1-\lambda)z^p D^{n+p-1}f(z) + \lambda z^p D^{n+p}f(z)\} > \alpha \quad (z \in E),$$

then

$$Re\{z^p D^{n+p}f(z)\} > \frac{(2n+2p+1)\alpha+\lambda-1}{2(n+p)+\lambda} \quad (0 \leq \alpha < 1, z \in E). \tag{2.6}$$

Proof. The result (2.5) and (2.6) follows by putting $g(z) = \frac{1}{z^p}$ in Theorem 1 and Corollary 1, respectively.

Remarks. Choosing n, μ, p and λ appropriately in Corollary 2, we obtain the following results.

(i) For $\lambda = 1$, $n = 0$ and $p = 1$ in Corollary 2, we have ;

$$Re\left\{(2+\frac{zf'(z)}{f(z)})(zf(z))^\mu\right\} > \alpha \quad (0 \leq \alpha < 1, z \in E) \tag{2.7}$$

implies

$$Re\{zf(z)\}^\mu > \frac{2\mu\alpha+1}{2\mu+1} \quad (z \in E).$$

(ii) For a complex number λ satisfying $Re\{\lambda\} \geq 0$ $(\lambda \neq 0)$ and $n = 0$, $p = 1$ and $\mu = 1$ in Corollary 2, we have ;

$$Re\{(1+\lambda)zf(z) + \lambda z^2 f'(z)\} > \alpha \quad (0 \leq \alpha < 1, z \in E)$$

implies

$$Re\{zf(z)\} > \frac{2\alpha + Re\{\lambda\}}{2 + Re\{\lambda\}} \quad (z \in E).$$

(iii) Replacing $f(z)$ by $-zf'(z)$ in the result (ii), we have ;

$$-Re\{(1+2\lambda)z^2 f'(z) + \lambda z^3 f''(z)\} > \alpha \quad (0 \leq \alpha < 1, z \in E)$$

implies

$$-Re\{z^2 f'(z)\} > \frac{2\alpha + Re\{\lambda\}}{2 + Re\{\lambda\}} \quad (z \in E).$$

(iv) For real λ with $\lambda \geq 1$ and $n = 0$, $p = 1$ and $\mu = 1$ in Corollary 2, we have ;

$$Re\{(1+\lambda)zf(z) + \lambda z^2 f'(z)\} > \alpha \quad (0 \leq \alpha < 1, z \in E)$$

implies

$$Re\{zf(z)\} > \frac{3\alpha + \lambda - 1}{2 + \lambda} \quad (z \in E).$$

THEOREM 2. *Let λ be a complex number satisfying $Re\{\lambda\} > 0$. Let $f \in \sum_p$ satisfy the condition*

$$(2.8) \qquad Re\{(1-\lambda)(z^p D^{n+p-1} f(z))^\mu + \lambda z^p D^{n+p} f(z)(z^p D^{n+p-1} f(z))^{\mu-1}\} > \alpha$$

for some $\alpha (0 \leq \alpha < 1)$ and $\mu > 0$. Then

$$(2.9) \qquad Re\{z^p D^{n+p-1} f(z)\}^\mu > \alpha + (1-\alpha)(2\rho - 1),$$

where

$$\rho = \frac{1}{2} F(1, 1; 1 + \frac{\mu(n+p)}{Re\{\lambda\}}; \frac{1}{2}).$$

Proof. Let $q(z) = (z^p D^{n+p-1} f(z))^\mu$. Then $q(z)$ is analytic in E with $q(0) = 1$. Differentiating $q(z)$ and using the identity (1.3), we get

$$\begin{aligned}&(1-\lambda)(z^p D^{n+p-1} f(z))^\mu + \lambda z^p D^{n+p} f(z)(z^p D^{n+p-1} f(z))^{\mu-1}\\ &= q(z) + \frac{\lambda z q'(z)}{\mu(n+p)},\end{aligned}$$

so that by the hypothesis (2.8), we have

$$Re\Big\{q(z) + \frac{\lambda z q'(z)}{\mu(n+p)}\Big\} > \alpha \quad (z \in E).$$

In view of Lemma 2, this implies that

$$Re\{q(z)\} > \alpha + (1-\alpha)(2\rho - 1),$$

where

$$\rho = \rho(Re\{\lambda\}) = \int_0^1 (1 + t^{Re \frac{\lambda}{\mu(n+p)}})^{-1} dt.$$

Setting $Re\{\lambda\} = \lambda_1 > 0$, we have

$$\begin{aligned}\rho &= \int_0^1 (1 + t^{\frac{\lambda_1}{\mu(n+p)}})^{-1} dt\\ &= \frac{\mu(n+p)}{\lambda_1} \int_0^1 u^{\frac{\mu(n+p)}{\lambda_1}-1} (1+u)^{-1} du.\end{aligned}$$

Using (1.4) and (1.5), we get

$$\begin{aligned}\rho &= F(1, \frac{\mu(n+p)}{\lambda_1}; 1 + \frac{\mu(n+p)}{\lambda_1}; -1)\\ &= \frac{1}{2} F(1, 1; 1 + \frac{\mu(n+p)}{\lambda_1}; \frac{1}{2}).\end{aligned}$$

COROLLARY 3. *Let λ be a real number satisfying $\lambda \geq 1$. If $f \in \sum_p$ satisfies*

$$Re\{(1-\lambda) z^p D^{n+p-1} f(z) + \lambda z^p D^{n+p} f(z)\} > \alpha \quad (z \in E)$$

for $\alpha (0 \leq \alpha < 1)$, then

$$Re(z^p D^{n+p} f(z)) > \alpha + (1-\alpha)(2\rho' - 1)(1 - \lambda^{-1}) \quad (z \in E),$$

where

$$\rho' = \frac{1}{2}F(1,1;1+\frac{n+p}{\lambda};\frac{1}{2}).$$

Proof. The result follows by using the identity

$$\lambda z^p D^{n+p} f(z) = [(1-\lambda)z^p D^{n+p-1} f(z) + \lambda z^p D^{n+p} f(z)] + (\lambda - 1) z^p D^{n+p-1} f(z).$$

Remark. We note that if $n = 0,\ p = 1,\ \mu = \lambda > 0$ in Corollary 2, that is,

(2.10) $$Re\{(1-\lambda)(zf(z))^\lambda + \lambda z^2 f'(z)(zf(z))^{\lambda-1}\} > \alpha \quad (0 \le \alpha < 1, z \in E),$$

then (2.5) implies that

(2.11) $$Re(zf(z))^\lambda > \frac{2\alpha+1}{3} \quad (z \in E).$$

Whereas, if $f \in \sum_1$ satisfies the condition (2.10), then by using Theorem 2,

$$Re(zf(z))^\lambda > 2(1 - ln2)\alpha + (2ln2 - 1) \quad (z \in E),$$

which is better than (2.11).

Similarly, if $n = p = 1,\ \lambda = 2$ in (2.6) and

$$Re\{2zD^2 f(z) - zDf(z)\} > \alpha \quad (z \in E),$$

then we have

$$Re(zD^2 f(z)) > \frac{5\alpha+1}{6} \quad (z \in E).$$

Whereas, by using (1.6), Corollary 3 implies that

$$Re(zD^2 f(z)) > \frac{1}{2}[(3 - 2ln2) + (2ln2 - 1)] > \frac{5\alpha+1}{6}.$$

We observe that if λ is a real number satisfying $\lambda > 0$ and

$$h(z) = \frac{D^{n+p} f(z)}{D^{n+p} g(z)} + \left(\frac{1}{\lambda} - 1\right) \frac{D^{n+p-1} f(z)}{D^{n+p-1} g(z)} \quad (z \in E),$$

then from Theorem 1 (for $\mu = 1$), we have ;

$$Reh(z) > \frac{\alpha}{\lambda} \text{ implies } Re\Big(\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\Big) > \frac{2(n+p)\alpha + \lambda\delta}{2(n+p)+\lambda\delta}, \tag{2.12}$$

whenever

$$Re\Big(\frac{D^{n+p-1}g(z)}{D^{n+p}g(z)}\Big) > \delta \quad (0 \leq \delta < 1, z \in E).$$

Let $\lambda \to +\infty$, then from (2.12) we have ;

$$Reh(z) \geq 0 \text{ in } E \text{ implies } Re\Big(\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\Big) \geq 1 \text{ in } E,$$

whenever

$$Re\Big(\frac{D^{n+p-1}g(z)}{D^{n+p}g(z)}\Big) > \delta \quad (0 \leq \delta < 1, z \in E).$$

In the following theorem, we shall extend the above results as follows.

THEOREM 3. *Suppose $f(z)$ and $g(z)$ are in $\sum_p$ and $g(z)$ satisfies the condition (2.2). If*

$$Re\Big\{\frac{D^{n+p}f(z)}{D^{n+p}g(z)} - \frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\Big\} > -\frac{(1-\alpha)\delta}{2(n+p)} \quad (z \in E) \tag{2.13}$$

for some $\alpha(0 \leq \alpha < 1)$, then

$$Re\Big\{\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\Big\} > \alpha \quad (z \in E) \tag{2.14}$$

and

$$Re\Big\{\frac{D^{n+p}f(z)}{D^{n+p}g(z)}\Big\} > \frac{(2n+2p+\delta)\alpha - \delta}{2(n+p)} \quad (z \in E). \tag{2.15}$$

Proof. Let

$$q(z) = (1-\alpha)^{-1}\Big[\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)} - \alpha\Big].$$

Then $q(z)$ is analytic in E with $q(0) = 1$. Setting

$$\beta(z) = \frac{D^{n+p-1}g(z)}{D^{n+p}g(z)} \quad (z \in E),$$

we observe that by hypothesis, $Re\{\beta(z)\} > \delta$ in E. A simple computation shows that

$$\begin{aligned}\frac{(1-\alpha)zq'(z)\beta(z)}{n+p} &= \frac{D^{n+p}f(z)}{D^{n+p}g(z)} - \frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)} \\ &= \psi(q(z), zq'(z)),\end{aligned}$$

where

$$\psi(r,s) = \frac{(1-\alpha)\beta(z)s}{n+p}.$$

Using the hypothesis (2.13), we get

$$\{\psi(q(z), zq'(z)); z \in E\} \subset \Omega = \{\omega \in \mathbb{C} : Re\omega > -\frac{(1-\alpha)\delta}{2(n+p)}\}.$$

Now, for all real $r_2, s_1 \leq -\frac{(1+r_2^2)}{2}$, we have

$$\begin{aligned}Re\{\psi(ir_2, s_1)\} &= \frac{s_1(1-\alpha)Re\{\beta(z)\}}{n+p} \\ &\leq -\frac{(1-\alpha)\delta(1+r_2^2)}{2(n+p)} \\ &\leq -\frac{(1-\alpha)\delta}{2(n+p)}.\end{aligned}$$

This shows that $\psi(ir_2, s_1) \notin \Omega$ for each $z \in E$. Hence by Lemma 1, $Re\{q(z)\} > 0$ in E. This proves (2.14). The proof of (2.15) follows by using (2.13) and (2.14) in the identity

$$Re\left\{\frac{D^{n+p}f(z)}{D^{n+p}g(z)}\right\} = Re\left\{\frac{D^{n+p}f(z)}{D^{n+p}g(z)} - \frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\right\} + Re\left\{\frac{D^{n+p-1}f(z)}{D^{n+p-1}g(z)}\right\}.$$

This completes the proof of Theorem 3.

Remarks. (i) For $n = 0,\ p = 1$ and $g(z) = \frac{1}{z}$ in Theorem 3, we have ;

$$Re(zf(z) + z^2 f'(z)) > -\frac{(1-\alpha)}{2} \quad (z \in E)$$

implies

$$Re(zf(z)) > \alpha \quad (z \in E)$$

and

$$Re(2zf(z) + z^2 f'(z)) > \frac{3\alpha - 1}{2} \quad (z \in E).$$

(ii) For $n = p = 1$, $g(z) = \frac{1}{z}$ in Theorem 3, we have ;

$$Re(zD^2 f(z) - zDf(z)) > -\frac{1-\alpha}{4} \quad (z \in E)$$

implies

$$Re(zDf(z)) > \alpha \quad (z \in E)$$

and

$$Re(zD^2 f(z)) > \frac{5\alpha - 1}{4} \quad (z \in E).$$

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