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Trees and Terms in Relational Graph Rewriting System

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Abstract

We extend the graph structure introduced by Raoult. We defined another general category of graphs which always have pushouts. Under the framework, we proved an important theorem of rewriting systems concerned with critical pairs which is called critical pair lemma. Since our category is defined by relational equations and it always have pushouts, the proof of this lemma is very simple and clear. In our general framework, treelike graphs and Raoult-Graphs are defined by some relational conditions. We gave a sufficient condition of rewriting rules and matchings which guarantees the closedness of our rewritings under those graphs. These results show that the critical pair lemma is also held under some conditions for a graph rewriting system in which graphs are restricted to treelike graphs or Raoult-Graphs.

1 Introduction

The study of graph grammars was initiated in 1960's motivated by practical applications to syntactic pattern recognition. There are many researches [1–5,7–14] on graph grammars and graph rewritings which have a lot of applications including software specification, data bases, analysis of concurrent systems, developmental biology and many others.

Though a term rewriting is a special case of a graph rewriting, relations between the theory of graph rewritings and term rewritings are considered. Raoult[14] and Kennaway[4, 5] introduced original categories of graphs in order to model term rewritings by graph rewritings. They formulated rewritings using partial functions and pushouts in their categories and studied several properties of rewritings. Raoult[14] introduced the idea of proof of the critical pair lemma[6] using categorical framework, but their proof was insufficient because their category of graph does not always have pushouts.

In this paper, we extend the graph structure introduced by Raoult [14]. We defined another general category of graphs which always have pushouts [11]. Under the framework, we proved an important theorem of rewriting systems concerned with critical pairs which is called critical pair lemma in [6]. Since our category is defined by relational equations and it always have pushouts, the proof of this lemma is very simple and clear.

Related results for hyper graph rewritings are shown in [13] and [9]. Plump[13] have shown for double-pushout graph transformations in the sense of Ehrig et al.[1]. Löwe and Müller[9] have shown for single-pushout graph transformations. A hyper graph is an extension of a graph, but the representation of trees and terms are not always simple. Further, the term representation on a hyper graph is slightly different from the intuitional term representation on a tree.

In our general framework, treelike graphs and Raoult-Graphs are defined by some relational conditions. We gave a sufficient condition of rewriting rules and matchings which guarantees the closedness of our rewritings under those graphs. Most properties are shown by simple relational calculations. We showed our theorem concerned about critical pair is applicable to those graphs by restricting rewriting rules.

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2 Preliminary

A *relation* α of a set A into another set B is a subset of the cartesian product $A \times B$ and denoted by $\alpha : A \rightharpoonup B$. The *inverse relation* $\alpha^\sharp : B \rightharpoonup A$ of α is a relation such that $(b,a) \in \alpha^\sharp$ if and only if $(a,b) \in \alpha$. The *composite* $\alpha\beta : A \rightharpoonup C$ of $\alpha : A \rightharpoonup B$ followed by $\beta : B \rightharpoonup C$ is a relation such that $(a,c) \in \alpha\beta$ if and only if there exists $b \in B$ with $(a,b) \in \alpha$ and $(b,c) \in \beta$.

As a relation of a set A into a set B is a subset of $A \times B$, the inclusion relation, union, intersection and difference of them are available as usual and denoted by $\subset$, $\cup$, $\cap$ and $-$, respectively. The *identity relation* $\mathrm{id}_A : A \rightharpoonup A$ is a relation with $\mathrm{id}_A = \{(a,a) \in A \times A \mid a \in A\}$ (the diagonal set of A).

A *partial function* f of a set A into a set B is a relation $f : A \rightharpoonup B$ with $f^\sharp f \subset \mathrm{id}_B$ and it is denoted by $f : A \to B$. A *(total) function* f of a set A into a set B is a relation $f : A \rightharpoonup B$ with $f^\sharp f \subset \mathrm{id}_B$ and $\mathrm{id}_A \subset f f^\sharp$, and it is also denoted by $f : A \to B$. Clearly a function is a partial function. Note that the identity relation id_A of a set A is a function. The definitions of partial functions and (total) functions here coincide with ordinary ones. A function $f : A \to B$ is injective if and only if $f f^\sharp = \mathrm{id}_A$ and surjective if and only if $f^\sharp f = \mathrm{id}_B$.

In this paper, we denote the category of sets and functions by **Set**, and the category of sets and partial functions by **Pfn**. For a partial function $f : A \rightharpoonup B$, we denote the domain of f by $\mathrm{dom}(f)(\subset A)$, the image of f by $Im(f)$, and we define a relation $d(f) : A \rightharpoonup A$ by $d(f) = \{(a,a) \in A \times A | a \in \mathrm{dom}(f)\}$.

Fact 2.1 ([10]) *The category* **Pfn** *has pushouts.*

Definition 2.2 A *(simple) graph* $< A, \alpha >$ is a pair of a set A and a relation $\alpha : A \rightharpoonup A$. A *partial morphism* f of a graph $< A, \alpha >$ into a graph $< B, \beta >$, denoted by $f :< A, \alpha > \to < B, \beta >$, is a partial function $f : A \to B$ satisfying $d(f)\alpha f \subset f\beta$. A partial morphism f of graphs which satisfies $f f^\sharp = \mathrm{id}_A$ is called a *injective morphism* of graphs. We denote by **Graph** the category of graphs and their partial morphisms.

Theorem 2.3 ([11]) *The category* **Graph** *has pushouts.*

Definition 2.4 A *rewriting rule* $f :< A, \alpha > \to < B, \beta >$ is a partial morphism of a graph $< A, \alpha >$ into a graph $< B, \beta >$. An injective morphism $g :< A, \alpha > \to < G, \xi >$ of graphs is called a *matching*. In this situation, we also say that a rewriting rule f matches a graph $< G, \xi >$ by g. Let a square

$$\begin{array}{ccc} < A, \alpha > & \xrightarrow{f} & < B, \beta > \\ {\scriptstyle g}\downarrow & & \downarrow{\scriptstyle h} \\ < G, \xi > & \xrightarrow[k]{} & < H, \eta > \end{array}$$

be a pushout in the category **Graph**. Each pushout of a rule f and a matching g defines a direct derivation from a graph $< G, \xi >$ to a graph $< H, \eta >$ and is denoted by $< G, \xi > \Rightarrow_{f/g} < H, \eta >$. A *rewriting morphism* $k :< G, \xi > \to < H, \eta >$ of the rewriting $< G, \xi > \Rightarrow_{f/g} < H, \eta >$ and an *induced matching* $h :< B, \beta > \to < H, \eta >$ are denoted by $k = f/g$, $h = g/f$.

We note $\eta = h^\sharp \beta h \cup k^\sharp \xi k$ in Definition 2.4.

Lemma 2.5 *Let a square*

$$\begin{array}{ccc} < A, \alpha > & \xrightarrow{f} & < B, \beta > \\ {\scriptstyle g}\downarrow & & \downarrow{\scriptstyle h} \\ < G, \xi > & \xrightarrow[k]{} & < H, \eta > \end{array}$$

be a pushout in the category **Graph**.

(1) *If g is injective ($gg^\sharp = id_A$) then h is injective ($hh^\sharp = id_B$).*