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General Properties between Canonical Correlation and the Independent-Oscillator Model on a Partial $*$-Algebra

廣川 真男 (日立製作所 基礎研究所)

1 序

The independent-oscillator (IO) model is the model of the quantum particle surrounded by a large number of independent heat bath particles, each attached to the quantum particle by a spring. The Hamiltonian of the system is given by

$$H_{IO} \stackrel{\text{def}}{=} \frac{p^2}{2m} + V(x) + \sum_{j=1}^{\infty} \left[\frac{p_j^2}{2m_j} + \frac{1}{2} m_j \omega_j^2 (q_j - x)^2 \right]. \tag{1}$$

Here x and p are the coordinate and momentum operators of the quantum particle of mass m, while q_j and p_j are those of the jth heat bath particle of mass m_j. Of course, we have the usual commutation relations:

$$[x, p] = i\hbar, \qquad [q_j, p_{j'}] = i\hbar\delta_{jj'}. \tag{2}$$

$V(x)$ is the potential energy of the external force on the quantum particle. This model appeared in the literature [1, 2, 3, 4, 5]. Especially, Ford, Lewis and O'Connell found the IO model to be convenient since other heat bath models can generally be related to the IO model in Ref.[4]. They showed in §IV of Ref.[4] that from the IO model H_{IO} we can derive the generalized quantum Langevin equation:

$$\frac{d}{dt} p(t) + \int_{-\infty}^{t} ds \mu(t-s) \frac{p(s)}{m} + V'(x) = F(t), \tag{3}$$

which is the momentum operator version of (2.1) in Ref.[4], where the prime denotes the derivative with respect to x. $\mu(t)$ is the memory function given by

$$\mu(t) = \sum_{j=}^{\infty} m_j \omega_j^2 \cos(\omega_j t)\, \theta(t), \tag{4}$$

where $\theta(t)$ is the Heaviside step function, and $F(t)$ is an operator-valued random force with mean zero, and a mean force characterized by a memory function $\mu(t)$: The symmetrized correlation function of $F(t)$ is given by

$$\frac{1}{2} < F(t)F(s) + F(s)F(t) >_B$$
$$= \frac{1}{2}\sum_{j=1}^{\infty} \hbar m_j \omega_j^3 \coth(\hbar\omega_j/2kT) \cos\left[\omega_j\left(t-s\right)\right], \tag{5}$$

and the nonequal-time commutator of $F(t)$ is

$$[F(t), F(s)] = -i\sum_{j=1}^{\infty} \hbar m_j \omega_j^3 \sin\left[\omega_j\left(t-s\right)\right]. \tag{6}$$

Here, for operator O, $< O >_B$ means that $< O >_B \stackrel{\text{def}}{=} \text{tr}\left(Oe^{-H_B/kT}\right)/\text{tr}\left(e^{-H_B/kT}\right)$, where $H_B \stackrel{\text{def}}{=} \sum_j \left[\frac{1}{2m_j}p_j^2 + \frac{1}{2}m_j\omega_j^2 q_j^2\right]$, k is the Boltzmann constant, and T is absolute temperature. The Fourier-Laplace transform of the memory function is given as

$$[\mu](z) \stackrel{\text{def}}{=} \int_0^{\infty} dt e^{itz}\mu(t) = \frac{i}{2}\sum_{j=1}^{\infty} m_j\omega_j^2 \left[\frac{1}{z-\omega_j} + \frac{1}{z+\omega_j}\right] \tag{7}$$

for every $\text{Im}z > 0$.

Furthermore, Li, Ford and O'Connell investigated the symmetrized correlation of the coordinate operator and the quantum random force of the generalized quantum Langevin equation in Ref.[5].

Ford, Lewis and O'Connell showed that properties (5) and (6) are characterization of the operator-valued random force $F(t)$ by the memory function $\mu(t)$ (see (2.2), (2.3), (4.13) and (4.14) in Ref.[4]). And besides, in §3 in Ref.[1] Ford and Kac remarked that, in the generalized quantum Langevin equation, the correlation and commutator for the operator-valued random force must have the forms (5) and (6). Then, in this paper, we prove general properties including (5) and (6) between canonical correlation and the IO model on a partial $*$-algebra [6, 7, 8]. The partial $*$-algebra which we treat in this paper is given by a completion of a set of operators. The completion is done by the Bogoliubov scalar product which gives the canonical correlation. In order that we shall directly deal with bosonic operators which are unbounded, we will choice the partial $*$-algebra, not C^*-algebra, for unbounded operators.

We consider a quantum particle in thermal equilibrium with any quantum system in a finite volume under conditions (**A.1**)-(**A.4**) below. From now on, we set the Planck constant $\hbar = 1$. Let $H_{\text{q,p,s}}$ be an arbitrary total Hamiltonian which governs our system of the quantum particle with the quantum system such that $e^{-\beta H_{\text{q,p,s}}}$ is a trace class operator (where $\beta \equiv 1/kT$ denotes the inverse temperature). $H_{\text{q,p,s}}$ has the form of $H_{\text{q,p,s}} = p^2/2m + V(x) + H_{\text{q,s}} + H_{\text{int}}$, where $H_{\text{q,s}}$ denotes the Hamiltonian of a quantum system surrounding the quantum particle with (x, p), and

$H_{\rm int}$ is the interaction Hamiltonian between the quantum particle and the quantum system. Here, of course, the form of $H_{\rm q,s} + H_{\rm int}$ is unknown now. The canonical correlation function $R_p(t_1, t_2)$ for the momentum operator p is defined by

$$\begin{aligned} & R_p(t_1,t_2) \\ \stackrel{\rm def}{=} & \frac{1}{\beta {\rm tr}\left(e^{-\beta H_{\rm q,p,s}}\right)} \\ & \int_0^\beta d\lambda {\rm tr}\left(e^{-(\beta-\lambda)H_{\rm q,p,s}} e^{iH_{\rm q,p,s}t_1} p e^{-iH_{\rm q,p,s}t_1} e^{-\lambda H_{\rm q,p,s}} e^{iH_{\rm q,p,s}t_2} p e^{-iH_{\rm q,p,s}t_2}\right). \end{aligned}$$

For any p and $H_{\rm q,p,s}$ satisfying (**A.1**)-(**A.4**), we prove that, on a partial $*$-algebra $\mathbf{X}_c(H_{\rm q,p,s})$ which is called the Liouville space, the Heisenberg operator $p(t) \stackrel{\rm def}{=} e^{iH_{\rm q,p,s}t} p e^{-iH_{\rm q,p,s}t}$ satisfies a quantum Langevin equation with a quantum fluctuation $I(t)$, which has the similar form to (3) (see (17) in the main theorem). Here we note that we can not apply theories in Ref.[9] nor Ref.[10] to the momentum operator because of a condition. We show that the memory function $\mu(t)$ for the IO model characterizes a fluctuation-dissipation relation in our Langevin equation and the canonical correlation function $R_p(t_1, t_2)$ (see (19) and (20) in the main theorem), which means that $H_{\rm int}$ is characterized by $[\mu](z)$. Furthermore the symmetrized autocorrelation and nonequal-time commutator of $I(t)$ have the similar representation to (5) and (6), which are implied by our fluctuation-dissipation relation (see (21) and (22) in the main theorem). They are general results for p and $H_{\rm q,p,s}$ in mathematics, so they give one more mathematical evidence that the IO model represents the system of the quantum particle with the most general quantum system, which was indicated by Ford, Lewis and O'Connell in Ref.[4]. It is a symmetry with respect to the canonical correlation that derives the close relations between the canonical correlation and the distribution of the memory function of the IO model.

As mentioned above, some properties of the IO model was studied in Refs.[1, 2, 3, 4, 5]. Especially Ford and Kac say on p.808 in Ref.[1]: "*since we have derived the quantum Langevin equation only for very special oscillator models (i.e. the IO model), one might wonder to what extent we have demonstrated the universality of the equation. The answer, of course, is that we have not. Rather, the logic is reversed: if there is a universal description, then it must be of the form we have obtained.*" And, Ford, Lewis and O'Connell showed in Ref.[4] that a number of other heat-bath models within the framework of the general macroscopic description of the quantum Langevin equation are reduced to the IO model by physically adequate reasons. In this paper, for the momentum operator of our system we shall derive a quantum Langevin equation by the general theory by Mori[11, 12], and show general properties between canonical correlation and the IO model. The author thinks that our argument is valid over not only the momentum operator of our system but also observables which are realized as self-adjoint operators in some class, which gives a physical and mathematical proof for Ford and Kac's remark above.

2 主定理

In this section, in order to introduce canonical correlation functions defined by the Bogoliubov scalar product, the Liouville space, and explain our main theorem, we set up a general framework.

We consider a quantum particle in thermal equilibrium with any quantum system in the finite volume. So, we give a state space for our system by a separable infinite-dimensional Hilbert space, which is denoted by simply $\mathcal{F}_{\mathrm{q,p,s}}$. And we denote the inner product of $\mathcal{F}_{\mathrm{q,p,s}}$ by $(\ ,\ )_{\mathrm{q,p,s}}$.

Let x and p be the coordinate and momentum operators of the quantum particle of mass m, and $V(x)$ is the potential energy of the external force on the quantum particle. Let $V(x)$ be a potential energy of the external force on the quantum particle.

For our system, there exists a Hamiltonian $H_{\mathrm{q,p,s}}$ whose form is given by $H_{\mathrm{q,p,s}} = p^2/2m + V(x) + H_{\mathrm{q,s}} + H_{\mathrm{int}}$, where $H_{\mathrm{q,s}}$ denotes the Hamiltonian of the quantum system surrounding the quantum particle with (x, p), and H_{int} is the interaction Hamiltonian between the quantum particle and the quantum system. Here, of course, the form of $H_{\mathrm{q,s}} + H_{\mathrm{int}}$ is unknown now. So $H_{\mathrm{q,p,s}}$ may be non-quadratic, but must be realized as a self-adjoint operator acting in the Hilbert space $\mathcal{F}_{\mathrm{q,p,s}}$. Since we are now considering the thermal equilibrium quantum system, $H_{\mathrm{q,p,s}}$ is a self-adjoint operator acting in $\mathcal{F}_{\mathrm{q,p,s}}$, and

(A.1) $e^{-\tau H_{\mathrm{q,p,s}}}$ is a trace class operator on $\mathcal{F}_{\mathrm{q,p,s}}$ for every $\tau \in (0, \beta]$,

where $\beta \equiv 1/kT$ is the inverse temperature. This condition implies that the spectra of $H_{\mathrm{q,p,s}}$ are purely discrete and the eigenvectors $\{\varphi_n \mid n \in \mathbf{N}^*\}$ of $H_{\mathrm{q,p,s}}$ form a complete orthonormal system of $\mathcal{F}_{\mathrm{q,p,s}}$, where $\mathbf{N}^* \stackrel{\mathrm{def}}{=} \{0, 1, \cdots\}$. We count the eigenvalues λ_n $(n \in \mathbf{N}^*)$ of $H_{\mathrm{q,p,s}}$ in such a way that $H_{\mathrm{q,p,s}}\varphi_n = \lambda_n \varphi_n$ and $0 < \lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_n \leq \lambda_{n+1} \leq \cdots \nearrow \infty$.

For the Hamiltonian $H_{\mathrm{q,p,s}}$, we can construct a Liouville space $\mathbf{X}_c(H_{\mathrm{q,p,s}})$, which is a set of adequate operators acting in $\mathcal{F}_{\mathrm{q,p,s}}$ [9, 13]. We denote the linear hull of $\{\varphi_n \mid n \in \mathbf{N}^*\}$ by $\mathbf{D}_{\mathrm{q,p,s}}$, i.e., $\mathbf{D}_{\mathrm{q,p,s}} \stackrel{\mathrm{def}}{=} \mathbf{L.h.}[\{\varphi_n \mid n \in \mathbf{N}^*\}]$. From here on, we denote the linear hull of a set S by $\mathbf{L.h.}[S]$. Obviously $\mathbf{D}_{\mathrm{q,p,s}}$ is dense in $\mathcal{F}_{\mathrm{q,p,s}}$. Further, we denote by $\mathbf{B}(\mathbf{D}_{\mathrm{q,p,s}}, \mathcal{F}_{\mathrm{q,p,s}})$ the space of bounded linear operators from $\mathbf{D}_{\mathrm{q,p,s}}$ to $\mathcal{F}_{\mathrm{q,p,s}}$. Every element A in $\mathbf{B}(\mathbf{D}_{\mathrm{q,p,s}}, \mathcal{F}_{\mathrm{q,p,s}})$ has a unique extension to an element in $\mathbf{B}(\mathcal{F}_{\mathrm{q,p,s}})$, the space of bounded linear operators on $\mathcal{F}_{\mathrm{q,p,s}}$. We denote the extension of A by A^-, and $A^* \lceil \mathbf{D}_{\mathrm{q,p,s}}$ by A^+, which means that the domain of operator A^* is restricted to $\mathbf{D}_{\mathrm{q,p,s}}$.

We first define a class $\mathbf{T}(H_{\mathrm{q,p,s}})$ of operators, which is a set of operators A satisfying the following conditions: (**T.1**) the domain of each operator is equal to $\mathbf{D}_{\mathrm{q,p,s}}$, and the domain of the adjoint operator of each operator includes $\mathbf{D}_{\mathrm{q,p,s}}$ (i.e., $\mathrm{D}(A) = \mathbf{D}_{\mathrm{q,p,s}}$ and $\mathrm{D}(A^*) \supset \mathbf{D}_{\mathrm{q,p,s}}$, where $\mathrm{D}(B)$ denotes the domain of each operator B); (**T.2**) for all τ in $(0, \beta]$ operators $e^{-\tau H_{\mathrm{q,p,s}}}A$ and $Ae^{-\tau H_{\mathrm{q,p,s}}}$ are in $\mathbf{B}(\mathbf{D}_{\mathrm{q,p,s}}, \mathcal{F}_{\mathrm{q,p,s}})$, furthermore, $(e^{-\tau H_{\mathrm{q,p,s}}}A)^-$ and $(Ae^{-\tau H_{\mathrm{q,p,s}}})^-$ are Hilbert-Schmidt operators on $\mathcal{F}_{\mathrm{q,p,s}}$. We must now turn our attention to the unboundedness of operators because it is known that limits on the precision of the measurement of observables for bounded operators (e.g., fermion) and unbounded operators (e.g., boson) are different[14, 15, 16]. For unbounded operators, the problem of their domains is delicate, so we provide condition (**T.1**). Condition (**T.2**) addresses convergency with respect to the Bogoliubov scalar product[9, 13, 17]. We note here that $\mathbf{T}(H_{\mathrm{q,p,s}})$ is a linear space. We can then introduce the Bogoliubov (Kubo-Mori) scalar product $<\ ;\ >$ as

$$< A; B > \stackrel{\mathrm{def}}{=} \frac{1}{\beta Z(\beta)} \int_0^\beta d\lambda \mathrm{tr}((e^{-(\beta-\lambda)H_{\mathrm{q,p,s}}}A^*)^-(e^{-\lambda H_{\mathrm{q,p,s}}}B)^-),$$

for $A, B \in \mathbf{T}(H_{\mathrm{q,p,s}})$, where $Z(\beta) \stackrel{\mathrm{def}}{=} \mathrm{tr}(e^{-\beta H_{\mathrm{q,p,s}}})$. It can be easily proven that $<\ ;\ >$ is an inner product of $\mathbf{T}(H_{\mathrm{q,p,s}})$ (see Ref.[13]). The inner product introduces a norm: $\|A\|_{H_{\mathrm{q,p,s}}} \stackrel{\mathrm{def}}{=} < A; A >^{1/2}$. We can therefore obtain the Liouville space $\mathbf{X}_c(H_{\mathrm{q,p,s}})$ defined by a Hilbert space which is the completion of $\mathbf{T}(H_{\mathrm{q,p,s}})$ with respect to the norm $\|\ \ \|_{H_{\mathrm{q,p,s}}}$. It is interesting to note that $\mathbf{X}_c(H_{\mathrm{q,p,s}})$ is a partial $*$-algebra with a unit (see Proposition 3.14 in Ref.[13]). The definition of partial $*$-algebras is given in Refs.[6, 7, 8]. We also note here that an element in $\mathbf{X}_c(H_{\mathrm{q,p,s}})$ is not always an operator acting in $\mathcal{F}_{\mathrm{q,p,s}}$. It is noteworthy that Naudts et al. attempted to argue in general about linear response theory on the Hilbert space which is constructed by a completion of a von Neumann algebra with KMS-state[18]. Roughly speaking, the von Neumann algebra with KMS-state can be regarded as a set of operators which can be taken a statistical average with the KMS-condition, however the operators are bounded. So, for our purpose we do use the partial $*$-algebra instead of the von Neumann algebra because the operators we treat are unbounded. And we deal with Mori's theory on $\mathbf{X}_c(H_{\mathrm{q,p,s}})$, which is just the partial $*$-algebra constructed by the completion concerning the Bogoliubov scalar product.

In order to introduce the Heisenberg operator $p(t)$ of the momentum operator, we define here the Liouville operator $\mathcal{L}_{\mathrm{q,p,s}}$ determined by the Hamiltonian $H_{\mathrm{q,p,s}}$.

We can define, for adequate operators A, the Liouville operator $\mathcal{L}_{\mathrm{q,p,s}}$ by $\mathcal{L}_{\mathrm{q,p,s}}A \stackrel{\mathrm{def}}{=} [H_{\mathrm{q,p,s}}, A] = H_{\mathrm{q,p,s}}A - AH_{\mathrm{q,p,s}}$ (see Lemma 3.8 in Ref.[13]). The domain $\mathrm{D}(\mathcal{L}_{\mathrm{q,p,s}})$ of the Liouville operator $\mathcal{L}_{\mathrm{q,p,s}}$ then contains a dense subspace $\mathcal{D}_{q,p,s}$ of all elements $A \in \mathbf{T}(H_{q,p,s})$ satisfying that $H_{q,p,s}A$ and $AH_{q,p,s}\lceil \mathbf{D}_{q,p,s}$ are in $\mathbf{T}(H_{q,p,s})$; furthermore, $Ax, A^{+}x, H_{q,p,s}Ax, H_{q,p,s}A^{+}x$,

$AH_{q,p,s}x$, and $A^{+}H_{q,p,s}x$ are in $\mathbf{D}_{q,p,s}$ for all x in $\mathbf{D}_{q,p,s}$. Actually, the subspace $\mathcal{D}_{q,p,s}$ is a core for $\mathcal{L}_{q,p,s}$.

For every $A \in \mathbf{X}_c(H_{q,p,s})$, we denote the Heisenberg operator of A by $A(t)$ in the Liouville space $\mathbf{X}_c(H_{q,p,s})$, i.e.,

$$A(t) \stackrel{\text{def}}{=} e^{i\mathcal{L}_{q,p,s}t}A.$$

And we define the canonical autocorrelation function of A by

$$R_A(t) \stackrel{\text{def}}{=} R_A(0,t) \equiv < A(0); A(t) > .$$

Remark 2.1: The time evolution $A(t)$ coincides with the Heisenberg picture $e^{iH_{q,p,s}t}Ae^{-iH_{q,p,s}t}$ for every operator A in $\mathcal{D}_{q,p,s}$ and $t \in \mathbf{R}$ (see Proposition 3.13 in Ref.[13]).

So, we denote the canonical autocorrelation function of the momentum operator p by $R_p(t)$. We define here a function $[R_p](z)$ ($z \in \mathbf{C}$ with $\mathrm{Im}z > 0$) by the Fourier-Laplace transform as

$$[R_p](z) \stackrel{\text{def}}{=} \int_0^\infty dte^{itz}R_p(t).$$

Here, we have the properties concerning poles of $[R_p](z)$:

The spectra of $\mathcal{L}_{q,p,s}$ is given by the closure of the set of all $\lambda_m - \lambda_n$'s:

$$\sigma(\mathcal{L}_{q,p,s}) = \overline{\{\lambda_m - \lambda_n \,|\, m,n \in \mathbf{N}^*\}}^{\text{closure}}, \tag{8}$$

which is proved in Lemma 3.1 in [19].

There exist non-negative constants $A_{m,n}$ ($m,n \in \mathbf{N}^*$) such that

$$R_p(t) = \sum_{m,n\in\mathbf{N}^*} A_{m,n}e^{it(\lambda_m-\lambda_n)}, \tag{9}$$

whose proof is given by Lemma 3.2 in [19].

We denote the set of all positive poles of $[R_p](z)$ by $\mathbf{P}_+^R$, and the set of all negative poles of $[R_p](z)$ by $\mathbf{P}_-^R$. Then, by (9) and the following assumption, each poles of $[R_p](z)$ agree with differences of two λ_n's.

(**A.2**) For $\mathbf{P}_+^R = \{\varepsilon_k \,|\, k = 0,1,\cdots\}$, $\inf_{k\in\mathbf{N}^*}(\varepsilon_{k+1} - \varepsilon_k) > 0$.
Moreover, for $\mathbf{P}_-^R = \{\eta_k \,|\, k = 0,1,\cdots\}$, $\inf_{k\in\mathbf{N}^*}(\eta_k - \eta_{k+1}) > 0$.

We set the last two conditions: Because we consider a system governed by the Hamiltonian $H_{q,p,s} \equiv p^2/2m + V(x) + H_{q,s} + H_{\text{int}}$ with (**A.1**), the condition that $p \in \mathbf{T}(H_{q,p,s})$ is natural assumption.

(**A.3**) $p \in \mathbf{T}(H_{q,s,p})$. Furthermore, $\sum_{k=0}^{\infty}\left(\lim_{z\to\varepsilon_k}\frac{1}{i}(z-\varepsilon_k)[R_p](z)\right)\varepsilon_k^2 < \infty$, and $\sum_{k=0}^{\infty}\left(\lim_{z\to\eta_k}\frac{1}{i}(z-\eta_k)[R_p](z)\right)(-\eta_k)^2<\infty.$

(A.4) $\lim_{z\to 0;z\in\mathbf{C}^+} z[R_p](z)=0,$ where $\mathbf{C}^+ \stackrel{\text{def}}{=} \{z\in\mathbf{C}\,|\,\mathrm{Im}z>0\}.$

Here we introduce the symmetrized autocorrelation function $S_p(t)$ by using well-known relation in Theorem 3 in Ref.[20]. For $R_A(t)$ $(A\in\mathbf{X}_c(H_{\mathrm{q,p,s}}))$, since $R_A(t)$ is continuous and positive-definite, there exists a unique measure Δ_A^{can} such that

$$R_A(t)=\int_{-\infty}^{\infty}e^{it\omega}\Delta_A^{can}(d\omega)$$

according to Bochner's theorem. Then, we define the symmetrized autocorrelation function $S_A(t)$ for $A\in\mathbf{X}_c(H_{\mathrm{q,p,s}})$ by

$$S_A(t)\stackrel{\text{def}}{=}\int_{-\infty}^{\infty}e^{it\omega}\beta E_\beta(\omega)\Delta_A^{can}(d\omega), \tag{10}$$

where $E_\beta(\omega)$ is the average energy of the harmonic oscillator with the frequency ω at temperature $T=1/k\beta$,

$$E_\beta(\omega)=\frac{\hbar\omega}{2}\coth\frac{\beta\hbar\omega}{2}. \tag{11}$$

(We note here that we set $\hbar=1$ in this paper.)

For $A\in\mathbf{X}_c(H_{\mathrm{q,p,s}})$, we define the response function $P_A(t)$ by

$$P_A(t)\stackrel{\text{def}}{=}-\beta\frac{d}{dt}<A;\,A(t)>. \tag{12}$$

We have another Liouville space $\mathbf{X}_\beta(H_{\mathrm{q,p,s}})$ by completion of $\mathbf{T}(H_{\mathrm{q,p,s}})$ by the following inner product[21]: For $A,B\in\mathbf{T}(H_{\mathrm{q,p,s}})$, we set

$$<A|B>\stackrel{\text{def}}{=}Z(\beta)^{-1}\mathrm{tr}\left(\left\{\left(Ae^{-\beta H_{\mathrm{q,p,s}}/2}\right)^-\right\}^*\left\{\left(Be^{-\beta H_{\mathrm{q,p,s}}/2}\right)^-\right\}\right). \tag{13}$$

Then, we can define the Liouville operator $\mathcal{L}^{\mathrm{q,p,s}}$ with certain dense domain in $\mathbf{X}_\beta(H_{\mathrm{q,p,s}})$ (see §II and §III in Ref.[21]) in the same way as $\mathcal{L}_{\mathrm{q,p,s}}$. So we can get the Heisenberg operator $e^{i\mathcal{L}^{\mathrm{q,p,s}}t}A$ for $A\in\mathbf{X}_\beta(H_{q,p,s})$, which denotes

$$A[t]\stackrel{\text{def}}{=}e^{i\mathcal{L}^{\mathrm{q,p,s}}t}A\in\mathbf{X}_\beta(H_{\mathrm{q,p,s}}) \tag{14}$$

in order to distinguish it from $A(t)\in\mathbf{X}_c(H_{\mathrm{q,p,s}})$.

We denote $Z(\beta)^{-1}\mathrm{tr}\left(Oe^{-\beta H_{\mathrm{q,p,s}}}\right)$ by $<O>$. Then, of course, the well-known relation (see Theorem 3 in Ref.[20]) means the following proposition in our Liouville's spaces: If A is a symmetric operator acting in $\mathcal{F}_{\mathrm{q,p,s}}$ with $A\in\mathbf{X}_c(H_{\mathrm{q,p,s}})$ and $A\in\mathbf{X}_\beta(H_{\mathrm{q,p,s}})$, then

$$S_A(t)=\frac{1}{2}<AA[t]+A[t]A>. \tag{15}$$

We will prove this relation in Proposition 3.3 in [19].

Furthermore, concerning the response function, of course a well-known fact in our version holds: If A is a symmetric operator acting in $\mathcal{F}_{\mathrm{q,p,s}}$ with $A \in \mathbf{X}_c(H_{\mathrm{q,p,s}})$ and $A \in \mathbf{X}_\beta(H_{\mathrm{q,p,s}})$, then

$$P_A(t) = -i < [\, A \ , \ A\,[t]\,] >, \tag{16}$$

where, of course, $[\, A \ , \ A\,[t]\,] = AA\,[t] - A\,[t]\,A$. We will also prove this relation in Proposition 3.4 in [19].

Now, we can state our main theorem:

Theorem: *Suppose that total Hamiltonian $H_{\mathrm{q,p,s}}$ of the system of the quantum particle with the quantum system, and the momentum operator p of the quantum particle, satisfy assumptions* (**A.1**), (**A.2**), (**A.3**) *and* (**A.4**). *Then the function $[R_p]\,(z)$ can be extended to a meromorphic function on the complex plane, and the set $\{\omega_j\}_{j=1}^{\infty}$ of all positive zero points of $[R_p]$ is counted in such a way that*

$$\omega_j \in (\varepsilon_{j-1}, \varepsilon_j), \qquad \textit{with} \quad \varepsilon_j > 0, \qquad j \in \mathbf{N}.$$

Give the mass m_j of the particle of the quantum system by

$$m_j = \frac{2mR_p(0)}{\omega_j^2 i\,[R_p]'\,(\omega_j)}, \qquad \textit{where } [R_p]'\,(z) \equiv d\,[R_p]\,(z)/dz.$$

Let $\mu(t)$ be the memory function of H_{IO} with frequency ω_j and mass m_j above, i.e.,

$$\mu(t) = \sum_{j=1}^{\infty} m_j \omega_j^2 \cos(\omega_j t)\, \theta(t).$$

Then, there exist a memory function $\kappa_\tau(t)$ and quantum fluctuation $I(t)$ such that the Heisenberg operator $p(t) \equiv e^{i\mathcal{L}_{\mathrm{q,p,s}}t}p$ of the momentum operator satisfies the following quantum Langevin equation :

$$\frac{d}{dt}p(t) + \lim_{\tau \uparrow t} \int_{-\infty}^{t} ds \kappa_\tau(t-s)\frac{p(s)}{m} = I(t) \tag{17}$$

on the Liouville space $\mathbf{X}_c(H_{\mathrm{q,p,s}})$ with

$$\lim_{\tau \to \infty} \kappa_\tau(t) = \mu(t), \qquad t > 0, \tag{18}$$

a fluctuation-dissipation relation :

$$\frac{R_p(0)}{m}\mu(t) =< I(0)\ ;\ I(t) >, \quad t > 0, \tag{19}$$

with

$$< p\,;\ I(t) >= 0, \qquad t \in \mathbf{R},$$

and

$$[R_p](z) = R_p(0)\frac{1}{-iz + [\mu](z)/m}, \qquad z \in \mathbf{C} \setminus \{\omega_j\}_{j=1}^{\infty}. \tag{20}$$

Furthermore, the fluctuation-dissipation relation (19) *implies that the symmetrized autocorrelation function* $S_I(t)$ *of* $I(t)$ *is*

$$S_I(t) = \frac{1}{2kT}\sum_{j=1}^{\infty} m_j\omega_j^3 \coth\left(\frac{\omega_j}{2kT}\right)\cos(\omega_j t), \tag{21}$$

and response function $P_I(t)$ *is*

$$P_I(t) = \frac{R_p(0)}{mkT}\sum_{j=1}^{\infty} m_j\omega_j^3 \sin(\omega_j t). \tag{22}$$

(*We note here we set* $\hbar = 1$ *now.*)

Remark 2.2: $I(t)$ may be decomposed into a summation of $V'(x)$ and a certain quantum force $F(t)$. However, information in the theorem is not enough to decompose $I(t)$ in such a way. As a matter of fact, the fluctuation $I(t)$ is Mori's fluctuation on the Liouville space $\mathbf{X}_c(H_{\mathrm{q,p,s}})$, and then $\mu(t)$ agrees with Mori's memory function multiplied by the mass m for $t \geq 0$.

The proof of our theorem is in [19].

参考文献

[1] G. W. Ford and M. Kac, J. Stat. Phys. **46**, 803 (1987)

[2] V. Hakim and V. Ambegaokar, Phys. Rev. A **32**, 423 (1985)

[3] H. Grabert, P. Schramm and G.-L. Ingold, Phys. Rev. Lett. **58**, 1285 (1987)

[4] G. W. Ford, J. T. Lewis and R. F. O'Connell Phys. Rev. **A**, **37**, 4419 (1988)

[5] X. L. Li, G. W. Ford and R. F. O'Connell Am. J. Phys. **61**, 924 (1993)

[6] J. -P. Antoine and M. Mathot, Ann. Inst. H. Poincaré, **46**, 299 (1987)

[7] J. -P. Antoine, A. Inoue and C. Trapani, Publ. RIMS, Kyoto Univ., **26**, 359 (1990)

[8] G. Epifanio and C. Trapani, J. Math. Phys. **29**, 536 (1988)

[9] M. Hirokawa, Ann. Phys. **229**, 354 (1994); ERRATUM, Ann. Phys. **235**, 240 (1994)

[10] Y. Okabe, Hokkaido Mathematical Journal **15**, 163 (1986)

[11] H. Mori, Prog. Theo. Phys. **33**, 423 (1965)

[12] H. Mori, Prog. Theo. Phys. **34**, 399 (1965)

[13] M. Hirokawa, Ann. Phys. **224**, 301 (1993)

[14] H. Araki and M. Yanase, Phys. Rev. **120**, 622 (1960)

[15] M. Ozawa, Phys. Rev. Lett. **67**, 1956 (1991)

[16] H. Ezawa, in *Quantum Control and Measurement*, edited by H. Ezawa and Y. Murayama, (North-Holland, Amsterdam, 1993)

[17] O. Bratteli and D. W. Robinson, *Operator algebras and Quantum Statistical Mechanics II*, (Springer-Verlag, New York, 1981)

[18] J. Naudts, A. Verbeure and R. Weder, Comm. Math. Phys. **44**, 87 (1975)

[19] M. Hirokawa, J. Math. Phys., **37**, 121 (1996)

[20] R. Kubo, J. Phys. Soc. Japan **12**, 570 (1957)

[21] M. Hirokawa, Ann. Phys. **223**, 1 (1993)

[22] J. Glimm and A. Jaffe, Ann. Math., **91**, 362 (1970)

[23] M. Reed and B. Simon, *Methods of Modern Mathematical Physics Vol.II: Fourier Analysis, Self-Adjointness*, (Academic Press, New York, 1975)

[24] J. Glimm and A. Jaffe, Phys. Rev., **176**, 1945 (1968)

[25] M. Reed and B. Simon, *Methods of Modern Mathematical Physics Vol.I: Functional Analysis*, (Academic Press, New York, 1980)

[26] M. Reed and B. Simon, *Methods of Modern Mathematical Physics Vol.IV: Analysis of Operators*, (Academic Press, New York, 1978)

[27] J. Glimm, Comm. Pure Appl. Math., **22**, 401 (1969)