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# GOURSAT PROBLEM FOR A MICRODIFFERENTIAL OPERATOR OF FUCHSIAN TYPE AND ITS APPLICATION

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## §0. INTRODUCTION.

The Goursat problem in the holomorphic (or the real analytic) category is treated by several authors and studied in depth. Moreover, C. Wagschal [W] extended the problem to the case of a system of integro-differential operators and obtained the Cauchy-Kovalevskaja type (*the unique solvability*) theorem. However, it seems that the study of the Goursat problem is not so satisfactory from the microlocal point of view. Therefore in this article, we treat a microdifferential operator of Fuchsian type with respect to several variables and consider the Goursat problem in the framework of holomorphic (or micro-) functions.

The notion of Fuchsian type (with respect to one variable) was introduced by M. S. Baouendi and C. Goulaouic [Ba-G] for a partial differential operator. This includes non characteristic type as a special case, and the Cauchy-Kovalevskaja type theorem was proved in [Ba-G]. Seeing this result, N. S. Madi [M] generalized Fuchsian type to several variables case by the name of "a Goursat operator of several Fuchsian variables" and obtained the Cauchy-Kovalevskaja type theorem for the Goursat problem in the framework of holomorphic functions. Note that Y. Laurent-T. Monterio Fernandes [La-MF] and Z. Szmydt and B. Ziemian [Sz-Zi] gave different definitions of Fuchsian type with respect to several variables respectively. On the other hand, succeeding to Baouendi-Goulaouic [Ba-G], many mathematicians have obtained almost sufficient results in Fuchsian type with respect to one variable. For example, H. Tahara [Ta] treated a Fuchsian system in the sense of Volevič and proved the Cauchy-Kovalevskaja type theorem in the complex domain. Further, as an application he obtained the existence and uniqueness theorem on an initial value problem for a Fuchsian hyperbolic system in the framework of hyperfunctions. Moreover, he proved the existence theorem on a homogeneous initial value problem for a Fuchsian microhyperbolic system of microdifferential operators in the framework of microfunctions. On the other hand, T. Ôaku proved the existence theorem on an inhomogeneous initial value problem for a Fuchsian hyperbolic microdifferential operator in [O1] and the uniqueness theorem under the F-mildness condition (but without the hyperbolicity assumption) in [O3] in the framework of microfunctions (cf. [O2]).

In this article, we define a matrix of microdifferential operators of Fuchsian type with respect to several variables as a natural generalization of one variable case due to Tahara [Ta] or non-microlocal case due to Madi

[M]. Moreover, we prove the Cauchy-Kovalevskaja type theorem for the Goursat problem in the space of holomorphic functions under the action of microdifferential operators due to J. M. Bony and P. Schapira [Bo-Sc]. As an application we solve the Goursat problem in the framework of micro-(or hyper-)functions; we prove the existence theorem for sufficiently "regular" initial data under suitable assumptions.

## §1. Statement of Main Theorem.

In this article, we use the following notation: $\mathbb{N}$ denotes the set of natural numbers (not containing 0) and $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. For a subset $D$ of some topological space, $[D]$ denotes the closure. For natural numbers $M, N \in \mathbb{N}$, and a linear space $L$ we denote by $\mathrm{Mat}(M \times N; L)$ the space of matrices of size $N \times N$ whose components are in $L$. Further set

$$\begin{cases} \mathrm{Mat}(N; L) := \mathrm{Mat}(N \times N; L), \\ \qquad L^N := \mathrm{Mat}(1 \times N; L), \\ \quad\ L^{\oplus N} := \mathrm{Mat}(N \times 1; L). \end{cases}$$

In addition, if $A$ has a norm $\| \ \|$, for $P = (P^{(\mu,\nu)})_{\mu,\nu=1}^N \in \mathrm{Mat}(M \times N; A)$ we set $\|P\| := \max\{\|P^{(\mu,\nu)}\|;\ 1 \leqslant \mu \leqslant M,\ 0 \leqslant \nu \leqslant N\}$. For natural numbers $d, n \in \mathbb{N}$, we use coordinates $\tau = (\tau_1, \dots, \tau_d) \in \mathbb{C}^d$ and $z = (z_1, \dots, z_n) \in \mathbb{C}^n$. Moreover for multi-indices $\gamma = (\gamma_1, \dots, \gamma_n)$ and

$\alpha = (\alpha_1, \ldots, \alpha_d)$, we set

$$\begin{cases} \partial_z{}^\gamma := \partial_{z_1}{}^{\gamma_1} \cdots \partial_{z_n}{}^{\gamma_n}, & \partial_\tau{}^\alpha := \partial_{\tau_1}{}^{\alpha_1} \cdots \partial_{\tau_d}{}^{\alpha_d}, \\ z^\gamma := z_1{}^{\gamma_1} \cdots z_n{}^{\gamma_n}, & \tau^\alpha := \tau_1{}^{\alpha_1} \cdots \tau_d{}^{\alpha_d}, \\ \gamma! := \gamma_1! \cdots \gamma_n!, & \alpha! := \alpha_1! \cdots \alpha_d!, \\ |\gamma| := \sum_{j=1}^n \gamma_j, & |\alpha| := \sum_{j=1}^d \alpha_j, \end{cases}$$

as usual. For vectors $R = (R_1, \ldots, R_d)$ and $R' = (R'_1, \ldots, R'_d) \in \mathbb{R}^d$, we define an order relation as follows:

$$R' \leqslant R \underset{\text{def.}}{\iff} R'_j \leqslant R_j \quad \text{for all } j,$$
$$R' < R \underset{\text{def.}}{\iff} R' \leqslant R \text{ and } R' \neq R,$$
$$R' \prec R \underset{\text{def.}}{\iff} R'_j < R_j \quad \text{for all } j\,.$$

For a vector $r = (r_1, \ldots, r_d) \in \mathbb{R}^d$, we set $[r]_+ := ([r_1]_+, \ldots, [r_d]_+)$, where $[r_j]_+ = \max\{r_j, 0\}$. We fix $m^{(\nu)} = (m_1^{(\nu)}, \ldots, m_d^{(N)})$ and $k^{(\nu)} = (k^{(1)}, \ldots, k^{(N)}) \in \mathbb{N}_0{}^d$ with $m^{(\nu)} \geqslant k^{(\nu)}$ $(1 \leqslant \nu \leqslant N)$ and set $m = (m^{(1)}, \ldots, m^{(N)})$ and $k = (k^{(1)}, \ldots, k^{(N)}) \in (\mathbb{N}_0{}^d)^N$. For any $N$-tuple of (generalized) functions $f(z, \tau) = {}^t(f_1(z, \tau), \ldots, f_N(z, \tau))$, we mean $f = O(\tau^{m-k})$ by

$$\partial_{\tau_j}{}^i f_\nu|_{\tau_j = 0} = 0 \qquad (1 \leqslant j \leqslant d,\ 1 \leqslant \nu \leqslant N,\ 0 \leqslant i \leqslant m_j^{(\nu)} - k_j^{(\nu)} - 1).$$

Set $\mathbf{1}_d := (1, \ldots, 1) \in \mathbb{N}^d$. For a vector $R = (R_1, \ldots, R_d) \in \mathbb{R}^d$ with $0 \prec R$, we set $B(R) := \{\tau \in \mathbb{C}^d;\ |\tau_j| < R_j\, (1 \leqslant j \leqslant d)\}$. Let $V \subset \mathbb{C}^n$ be a relatively compact open neighborhood of the origin and $h_0$ a positive number. We set

$$U = \{(z;\zeta) \in T^*\mathbb{C}^n\,;\, z \in V,\ \zeta_1 = 1,\ |\zeta_j| < h_0\,(2 \leqslant j \leqslant n)\}.$$

We denote the sheaf of rings of microdifferential operators of finite order (resp. of order at most $\nu$) by $\mathcal{E}$ (resp. $\mathcal{E}(\nu)$) as usual.

**1.1 Definition.** Let $P(z,\tau;\partial_z,\partial_\tau) = (P^{(\mu,\nu)}(z,\tau;\partial_z,\partial_\tau))_{\mu,\nu=1}^{N}$ be a matrix in $\mathrm{Mat}(N;\Gamma([U\times B(R)];\mathcal{E}_{\mathbb{C}^{n+d}}))$; that is, each $P^{(\mu,\nu)}$ is a microdifferential operator of finite order defined in some neighborhood of $[U\times B(R)]$. Then, $P$ is said to be *of Fuchsian type with weight* $(k,m)$ (*with respect* $\tau$*-variables*) if it has the following form:

$$P^{(\mu,\nu)}(z,\tau;\partial_z,\partial_\tau) = \sum_{0\leqslant\alpha\leqslant m^{(\nu)}} P_\alpha^{(\mu,\nu)}(z,\tau;\partial_z)\,{\partial_\tau}^{\alpha},$$

where each $P_\alpha^{(\mu,\nu)}$ is a microdifferential operator with holomorphic parameters $\tau$ and satisfies the following:

(1) The order $\mathrm{ord}\,P_\alpha^{(\mu,\nu)}$ of $P_\alpha^{(\mu,\nu)}$ is at most $|m^{(\nu)}| - |\alpha|$;

(2) There exist $P_\alpha^{1,(\mu,\nu)}(z,\tau;\partial_z)$ and $P_\alpha^{2,(\mu,\nu)}(z,\tau;\partial_z)$ $(0 \leqslant \alpha \leqslant m^{(\nu)})$ such that $\mathrm{ord}\,P_\alpha^{1,(\mu,\nu)} \leqslant 0$ and

$$\begin{aligned}P_\alpha^{(\mu,\nu)}(z,\tau;\partial_z) =&\,\tau^{[\alpha-m^{(\nu)}+k^{(\nu)}]_+}\,P_\alpha^{1,(\mu,\nu)}(z,\tau;\partial_z)\\ &+\tau^{[\alpha-m^{(\nu)}+k^{(\nu)}+1_d]_+}\,P_\alpha^{2,(\mu,\nu)}(z,\tau;\partial_z).\end{aligned}$$

**1.2 Remark.** (1) The Fuchsian property above is invariant under any coordinate change of $z$-variables, or more generally an arbitrary quantized contact transformation for $(z;\zeta)$-variables.

(2) The Fuchsian type defined in Definition 1.1 is a natural generalization of differential operators of Fuchsian type introduced by Madi [M]; that is, if $P$ is a differential operator of Fuchsian type in the sense of Definition 1.1, then $P$ is of Fuchsian type in the sense of Madi. Further if $d = N = 1$, a microdifferential operator of Fuchsian type is nothing but of Fuchsian type defined by Tahara (see [Ta]).

Let $T^{(\nu)} = (T_1^{(\nu)}, \dots, T_d^{(\nu)})$ $(1 \leqslant \nu \leqslant N)$ be indeterminates and set

$$\widetilde{T} := (T^{(1)}, \dots, T^{(N)}).$$

If $P$ is of Fuchsian type with weight $(k, m)$, we define *the indicial polynomial of $P$* by

$$\mathcal{I}_P(z;\zeta;\widetilde{T}) := \det\Big(\sum_{m^{(\nu)}-k^{(\nu)} \leqslant \alpha \leqslant m^{(\nu)}} \sigma_0(P_\alpha^{1,(\mu,\nu)})(z,0;\zeta)\,\mathcal{I}_\alpha(T^{(\nu)})\Big),$$

where $\mathcal{I}_\alpha(T^{(\nu)}) = \prod_{j=1}^{d} \mathcal{I}_{\alpha_j}(T_j^{(\nu)})$ with

$$\mathcal{I}_{\alpha_j}(T_j^{(\nu)}) := \begin{cases} T_j^{(\nu)}(T_j^{(\nu)} - 1)\cdots(T_j^{(\nu)} - \alpha_j + 1) & (\alpha_j \geqslant 1), \\ 1 & (\alpha_j = 0). \end{cases}$$

Let $A(z, \tau; \partial_z)$ be a microdifferential operator of finite order with holomorphic parameters $\tau$ defined in a neighborhood of $[U \times B(R)]$. Let

$c \in \mathbb{C}$ and set $\Sigma := \{z \in \mathbb{C}^n;\ z_1 = c\}$. Let $\Omega \subset V$ be an open convex set and assume that $\Omega$ is $h_0$-$\Sigma$-flat in the sense of Bony-Schapira; that is, if $z \in \Omega$, $w \in \Sigma$ and $h_0|z_j - w_j| \leqslant |z_1 - w_1|$ $(2 \leqslant j \leqslant n)$, then it follows that $w \in \Omega \cap \Sigma$. Let $f(z,\tau)$ be a holomorphic function defined on $\Omega \times B(R)$. If $p \in \mathbb{N}$, there exists a unique holomorphic function $g(z,\tau)$ on

$$\begin{cases} \partial_{z_1}{}^p g(z,\tau) = f(z,\tau), \\ \partial_{z_1}{}^j g|_{z_1=c} = 0 \qquad (0 \leqslant j \leqslant p-1). \end{cases}$$

Then, we define $(\partial_{z_1}{}^{-p})_\Sigma f(z,\tau) := g(z,\tau)$; that is,

$$(\partial_{z_1}{}^{-p})_\Sigma f(z,\tau) := \int_c^{z_1} \frac{(z_1 - w_1)^{p-1}}{(p-1)!} f(w_1, z')\, dw_1,$$

where $z' := (z_2, \ldots, z_n)$. We write formally

$$A(z,\tau;\partial_z) = \sum_{\gamma_1 \in \mathbb{Z},\ \gamma_2,\ldots,\gamma_n \in \mathbb{N}_0} A_\gamma(z,\tau)\, \partial_z{}^\gamma.$$

Then, applying the argument as in Bony-Schapira [Bo-Sc] regarding $\tau$ as holomorphic parameters, we find that

$$\begin{aligned} A_\Sigma f(z,\tau) := & \sum_{\gamma_1,\ldots,\gamma_n \in \mathbb{N}_0} A_\gamma(z,\tau)\, \partial_z{}^\gamma f(z,\tau) \\ & + \sum_{\gamma_1 < 0,\ \gamma_2,\ldots,\gamma_n \in \mathbb{N}_0} A_\gamma(z,\tau)\, (\partial_{z_1}{}^{\gamma_1})_\Sigma\, \partial_{z'}{}^{\gamma'} f(z,\tau) \end{aligned}$$

is holomorphic on $\Omega \times B(R)$. Let $s$ be a parameter with $0 < s < 1$. We fix a point $z_0 \in \Omega \cap \Sigma$ and set

$$\Omega_s := \{s(z - z_0) + z_0 \in \mathbb{C}^n;\ z \in \Omega\}$$

Consider the following condition:

**[A-1].** *There exist a positive constant $C > 0$ and a neighborhood $W$ of $[U]$ such that for any $(z;\zeta) \in [W]$ and $\beta = (\beta^{(1)}, \dots, \beta^{(N)}) \in (\mathbb{N}_0{}^d)^N$ with $\beta^{(\nu)} \geqslant m^{(\nu)} - k^{(\nu)}$ $(1 \leqslant \nu \leqslant N)$*

$$|\mathcal{I}_P(z;\zeta;\beta)| \geqslant C \prod_{\nu=1}^{N} (\beta^{(\nu)} + \mathbf{1}_d)^{m^{(\nu)}}.$$

Note that if $N = 1$, then [A-1] is a natural generalization of Madi's condition which is similar to the "Fuchsian ellipticity condition" due to Szmydt-Ziemian [Sz-Zi].

**1.3 Theorem.** *Let $P$ be a matrix of microdifferential operators defined in a neighborhood of $[U \times B(R)]$. Assume that $P$ is of Fuchsian type with weight $(k, m)$. and satisfies [A-1]. Then, there exist constants $r_0 > 0$ and $\overset{\circ}{R}$ with $0 \prec \overset{\circ}{R} \leqslant R$ such that the following hold:*

*Take arbitrary $h$ and $r$ with $0 < h < h_0$ and $0 < r < r_0$ respectively. Let $\Omega$ be any $h$-$\Sigma$-flat open convex subset of $V$ with $\operatorname{dia}\Omega \leqslant r$, where* dia *denotes the diameter. Then, there exists a constant $\delta$ such that for any $\widetilde{R}$ with $0 \prec \widetilde{R} \leqslant \overset{\circ}{R}$ it follows that for any holomorphic functions $f(z,\tau) = {}^t(f_1(z,\tau), \dots, f_N(z,\tau))$ and $g(z,\tau) = {}^t(g_1(z,\tau), \dots, g_N(z,\tau))$ on $\Omega \times B(\widetilde{R})$, there exists a unique holomorphic solution*

$$u(z,\tau) = {}^t(u_1(z,\tau), \dots, u_N(z,\tau))$$

*of the Goursat problem*

$$(G.P.) \qquad \begin{cases} P_\Sigma\, u = f, \\ u - g = O(\tau^{m-k}), \end{cases}$$

*and each $u_\nu(z,\tau)$ $(1 \leqslant \nu \leqslant N)$ is holomorphic on*

$$\bigcup_{0<s<1} \Big(\Omega_s \times \Big\{\tau \in B(\widetilde{R});\ \prod_{j=1}^{d} |\tau_j| < \delta\,(1-s)^{|m|}\Big\}\Big),$$

*where* $|m| := \sum_{\nu=1}^{N} \sum_{j=1}^{d} m_j^{(\nu)}$.

We can prove Theorem 1.3 by applying techniques of [O1] and [W].

**1.4 Remark.** Assume that $P$ is a differential operator. Then Theorem 1.3 is (essentially) obtained by Madi [M] (cf. [La-MF]).

## §2. Applications.

Let $M$ be $\mathbb{R}_x^n \times \mathbb{R}_t^d$ with its complexification $X := \mathbb{C}_z^n \times \mathbb{C}_\tau^d = Y \times \mathbb{C}^d$ and $\pi_M$ the canonical projection $T_M^*X \longrightarrow M$. Set $N := \mathbb{R}^n \cong M \cap \{t = 0\} \hookrightarrow M$, $L := X \cap \{\operatorname{Im} z = 0\} = \mathbb{R}^n \times \mathbb{C}^d$, $\widetilde{\Lambda} := T_L^*X \cong T_N^*Y \times \mathbb{C}^d$ and $\Lambda := T_M^*X \cap \widetilde{\Lambda}$. We denote the sheaf of microfunctions on $T_M^*X$ (resp. $T_N^*Y$) by $\mathcal{C}_M$ (resp. $\mathcal{C}_N$) as usual. Further, let $\mathcal{CO}_L$ be the sheaf of microfunctions with holomorphic parameters on $\widetilde{\Lambda}$; that is,

$$\mathcal{CO}_L := \mu_L(\mathcal{O}_X) \otimes or_{N/Y}[n],$$

where $\mu_L$ denotes Sato's microlocalization functor along $L$ and $or_{N/Y}$ denotes the relative orientation sheaf (see [K-Sc] and [S-K-K]). The sheaf $\mathcal{B}_M$ of hyperfunctions on $M$ and the sheaf $\mathcal{BO}_L$ of hyperfunctions with holomorphic parameters on $L$ are defined by $\mathcal{B}_M := \mathcal{C}_M|_M$ and $\mathcal{B}\mathcal{O}_L := \mathcal{C}\mathcal{O}_L|_L$ respectively. Let $\rho$ be a natural mapping

$$N \underset{M}{\times} T^*_M X \ni (x, 0; \sqrt{-1}\,(\langle \xi, dx\rangle + \langle \eta, dt\rangle)) \longmapsto (x\,; \sqrt{-1}\,\langle \xi, dx\rangle) \in T^*_N Y.$$

Then, we have the following canonical morphisms:

$$\mathcal{C}\mathcal{O}_L|_\Lambda \rightarrowtail \mathcal{C}_M|_\Lambda, \qquad \rho_!(\mathcal{C}_M|_{N \underset{M}{\times} T^*_M X}) \longrightarrow \mathcal{C}_N.$$

Set $p_0 := (0; \sqrt{-1}\,dx_1) \in T^*_N Y$ and assume that $P(x, t; \partial_x, \partial_t)$ is a matrix of microdifferential operators of Fuchsian type with weight $(k, m)$ defined in some neighborhood of $\rho^{-1}(p_0)$, then the following morphism is induced:

$$P\colon\ \rho_!(\mathcal{C}_M|_{N \underset{M}{\times} T^*_M X})_{p_0} \longrightarrow \rho_!\,(\mathcal{C}_M|_{N \underset{M}{\times} T^*_M X})_{p_0},$$

where $\rho_!\,(\mathcal{C}_M|_{N \underset{M}{\times} T^*_M X})_{p_0}$ denotes the stalk at $p_0$.

Consider the following condition:

**[A-2].** $\det(\sigma_{|m^{(\nu)}|}(P^{(\mu,\nu)})(z, \tau; \zeta, \eta)) = \tau^{\widetilde{k}}\,\widetilde{P}(z, \tau; \zeta, \eta)$ for a function $\widetilde{P}$ ($\widetilde{k} := \sum_{\nu=1}^{N} k^{(\nu)} \in {\mathbb{N}_0}^d$) which satisfies the following condition:

There exist positive constants $h_0$, $M$ and $\nu_i$ with $\nu_i \geqslant 1$ $(1 \leqslant i \leqslant d)$ such that $\widetilde{P}(z, t; \zeta, \eta)$ never vanishes on the set

$$\Big\{(z, t; \zeta, \eta) \in \mathbb{C}^n \times \mathbb{R}^d \times \mathbb{C}^n \times \mathbb{C}^d;\ |z|,\ |t| < h_0,$$
$$|\zeta_j| < h_0|\zeta_1|\ (2 \leqslant j \leqslant n),\ |\mathrm{Im}(\eta_i/\zeta_1)| = \nu_i\,\lambda\ (1 \leqslant i \leqslant d)$$
$$\text{for } \exists\lambda > M\Big(\sum_{j=1}^{n} |\mathrm{Im}\, z_j| + \sum_{j=2}^{n} |\mathrm{Im}(\zeta_j/\zeta_1)|\Big)\Big\}.$$

**2.1 Remark.** Condition [A-2] is satisfied if

**[A-3].** $\widetilde{P}(x,t;\xi,\eta) = \prod_{j=1}^{d} P_j(x,t;\xi,\eta_j)$ and each $P_j$ is of degree $\sum_{i=1}^{N} m_j^{(i)}$ and hyperbolic with respect to the direction $t_j$ (cf. Kashiwara-Kawai [K-K]).

**2.2 Theorem.** *Assume that $P$ satisfies* [A-1] *and* [A-2]. *Then, for any microfunctions with holomorphic parameters*

$$f(x,t),\ g(x,t) \in \rho_* \left(\mathcal{C}\mathcal{O}_L|_{N \underset{M}{\times} T^*_M X}\right)_{p_0}^{\oplus N},$$

*there exists a microfunction*

$$u(x,t) \in \rho_! \left(\mathcal{C}_M|_{N \underset{M}{\times} T^*_M X}\right)_{p_0}^{\oplus N}$$

*such that $u$ is a solution of the Goursat problem*

$$(G.P.) \qquad \begin{cases} P(x,t;\partial_x,\partial_t)\, u(x,t) = f(x,t), \\ \qquad u(x,t) - g(x,t) = O(t^{m-k}). \end{cases}$$

Outline of Proof of Theorem 2.2 is as follows: First, choosing suitable defining functions, we can solve $(G.P.)$ in a complex open set by using Theorem 1.3. Next, we can apply the holomorphic continuation method due to Kashiwara-Kawai [K-K] by assumption [A-1].

**2.3 Remark.** The author does not know how to prove the uniqueness of $u(x,t)$ in Theorem 2.2 (cf. [O2] and [O3]).

**2.4 Corollary.** *Let $P$ be a matrix of an analytic differential operators of Fuchsian type defined on a neighborhood of $(x,t) = (0,0)$. Assume that* [A-1] *and* [A-3]*. Then, for any holomorphic hyperfunctions with holomorphic parameters*

$$f(x,t),\ g(x,t) \in (\mathcal{B}\mathcal{O}_L|_M)_0^{\oplus N},$$

*there exists a hyperfunction*

$$u(x,t) \in (\mathcal{B}_M)_0^{\oplus N}$$

*such that $u$ has $t$ as real analytic parameters and is a solution of the Goursat problem*

$$(G.P.) \qquad \begin{cases} P(x,t;\partial_x,\partial_t)\,u(x,t) = f(x,t), \\ \qquad u(x,t) - g(x,t) = O(t^{m-k}). \end{cases}$$

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