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EXISTENCE AND OPTIMALITY OF EXTENDED LINDAHL EQUILIBRIA IN A LARGE PUBLIC GOODS ECONOMY WITH CONGESTION

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ABSTRACT

In this paper, we consider a public goods economy where congestion is present. We assume that the set of consumers is non-atomic, so that each consumer's individualistic change of utilization of the public goods will not affect the degree of congestion. We formulate a market mechanism where each consumer is charged a Lindahl personalized share for constructing the public goods, together with the common tax rate for the personal utilization of the public goods and the personalized subsidy for allowing some level of congestion. The total amount of this subsidy comes from the taxes that each individual pays for his utilization described above. We will prove that there exist such competitive equilibria under the standard conditions and they are always weakly Pareto optimal.

1. INTRODUCTION

In this paper, we consider a public goods economy where congestion is present. There are various phenomena arising from congestion that are becoming more important these days. Some of the classical examples, such as freeway congestion, are now quite severe problems for public administrative policy. A relatively new, but still important part of this problem is informational communication networks such as the Internet.

We assume that the set of consumers is non-atomic, so that each consumer's individualistic change of utilization of the public goods will not affect the degree of congestion, since the measure of one single consumer is zero. In such economies, we will propose a new concept for a market mechanism by extending the Lindahl equilibrium concept for the pure public goods case, and even with congestion, we prove that it will be possible to guarantee the existence and Pareto optimality of the equilibrium under the perfectly competitive market.

The price system adopted here is a mixture of standard prices which are common to all consumers and Lindahl-type personalized price systems. Like the Lindahl mechanism, each consumer will be imposed the personalized Lindahl share for constructing the public goods. The producer will maximize its profit by selling the public goods at the price of the total Lindahl share and buying the private goods as an input at the standard prices which are common to everybody.

In order to attain Pareto optimality, we treat the level of congestion as a type of external diseconomy. Each consumer will receive subsidies by accepting the congestion according to the personalized subsidy rate.

This subsidy comes only from the payments by consumers for utilizing and occupying the public goods. This balancedness of the budget of the government will assure Walras's Law in this economy.

In the literature, there a number of proofs of the existence of competitive equilibrium. For the Walrasian economy without public goods, there are many contributions including Debreu (1969), Shafer and Sonnenshein (1975), and Khan and Vohra (1984). Khan and Vohra (1984) prove the existence of the Walras equilibrium with a measure space of agents which is directly related to this paper.

In the pure public goods case, Foley (1970) and Milleron (1972) proved the existence of the Lindahl equilibrium with a finite number of consumers. Roberts (1973) is the first who proved the existence of Lindahl equilibrium with a measure space of agents which allows an infinite number of consumers. He found a way to reduce the problem to a finite dimensional case. More recently, Emmons (1984) proved the existence by using non-standard analysis. Khan and Vohra (1985) used a more direct approach with the fixed point theorem in infinite dimensional spaces. Our proof of the existence theorem is along the line of this proof by Khan and Vohra (1985).

On the other hand, the proof of the first fundamental theorem in this paper uses the standard argument by contradiction.

In Section 2, I will present the formal model and the results. Sections 3 and 4 will be devoted to the proofs. Mathematical tools used in this paper will be found in many mathematics textbooks including Aliprantis and Border (1994), Dunford and Shwartz (1954), and Edwards (1965).

2. MODEL AND RESULTS

Consider an economy with m private goods and l public goods which are denoted by x and y. The public goods will be produced from the private goods using the production set $G \subset \Re^{m+l}$. The set of consumers will be assumed to be $T = [0,1]$ together with the standard Lesbegue measure. Each agent $t \in T$ is concerned not only about his consumption level of private goods $x(t)$ and the total supply of the public goods y, but his actual level of utilization of public goods $y(t)$ $(y(t) \leq y)$ and the level of congestion which is represented by $\int_T y(s)ds$. His preference relation $\succ_t$ is defined on his consumption set $V(t) \equiv X(t) \times Y \times Y \times Y \subset \Re^m \times \left(\Re^l\right)^3$ whose typical element will be denoted by

$$v(t) \equiv (v_{x(t)}(t), v_{y(t)}(t), v_{\int y}(t), v_y(t)) \equiv (x(t), y(t), \int_T y(s)ds, y).$$

Hence $X(t)$ will be interpreted as the consumption set of the private goods and Y is the consumption set of the public goods which is common among all consumers. Moreover, consumer t is assumed to have his initial endowments vector $e(t) \in X(t)$ of private goods, and his profit share $\theta(t)$ of the firm producing the public goods, which is an integrable function from T to $\Re_+$ with $\int_T \theta(s)ds = 1$.

We will consider the following six assumptions:

Assumption 1. G is convex and compact[1], and contains the origin.

Assumption 2. For all $t \in T$, $V(t) = X(t) \times Y \times Y \times Y \subset \Re^m_+ \times \left(\Re^l_+\right)^3$, $0 \in V(t)$, is closed and convex, and contains the origin.

[1]The compactness assumption of the production set is made only for the sake of simplicity. One can relax this assumption using the standard method, such as introducing the concept of asymptotic cone. For details, see Debreu (1969).

Assumption 3. $X(t)$ is a measurable map, i.e., the graph of X is a measurable set in the product space.

Assumption 4. $\succ_t$ is irreflexive, convex, continuous, strictly increasing with respect to the level consumption of private goods $x(t)$, the level of utilization of public goods $y(t)$, and the level of total supply of public goods y, and strictly decreasing with respect to the level of congestion $\int_T y(s)ds$.

Assumption 5. $\succ_t$ is a measurable map, i.e., the graph of $\succ$ is a measurable set in the product space.

Assumption 6. $e(\cdot)$ is integrable and $e(t) \gg 0$ for almost all $t \in T$.

Note that the above assumptions are all standard in the literature.

Let us define a competitive equilibrium in this model with a personalized price system.

Definition 1. $(p, q, r) \in \Delta^{m+2l-1}$,[2] $q(t), r(t) \in \mathfrak{L}_1(T, \Re^l_+)$ with $\int_T q(s)ds = q$ and $\int_T r(s)ds = r$, and $(x(t), y(t), z, y)$ $(t \in T)$ is called an *extended Lindahl equilibrium* if

(1) (Individual Feasibility) $(x(t), y(t), \int_T y(s)ds, y) \in V(t)$ for almost all $t \in T$ and $(z, y) \in G$.

(2) (Profit Maximization) $(z, y) \in \text{argmax}(p, r)G$, i.e.,

$$(p, r)(z, y) \geq (p, r)g \qquad \text{for all } g \in G.$$

[2] $\Delta^{m+2l-1} \equiv \{s \in \Re^{m+2l} : \sum s_i = 1 \quad s_i \geq 0\}$

(3) (Budget Constraint)

$$px(t) + qy(t) - q(t)\int_T y(s)ds + r(t)y \leq pe(t) + \theta(t)\max(p,r)G$$

and

$$y(t) \leq y.$$

(4) (Preference Maximization) for all $v \in V(t)$,

$$v(t) \succ_t (x(t), y(t), \int_T y(s)ds, y)$$

implies

$$(p, q, -q(t), r(t))v > pe(t) + \theta(t)\max(p,r)G \quad \text{or} \quad v_{y(t)} \not\leq v_y.$$

(5) (Market Clearing)

$$\int_T x(s)ds = \int_T e(s)ds + z$$

The corresponding allocation is called an *extended Lindahl equilibrium allocation.*

In this definition, each consumer reports the optimal level of his consumption of private goods, the optimal level of his own utilization of public goods, his optimal allowance of congestion level, and his optimal level of capacity of total public goods supply. The market mechanism will adjust all consumers' reported level of congestion and the level of total public goods supply so that they will be equal among consumers at the equilibrium. This is a reason why we consider the "personalized price system" in these two.

The Lindahl share $r(t)$ will be paid to the producer for constructing the public goods. This is the reason why the equation $\int_T r(s)ds = r$ holds. On the other hand, the equation $\int_T q(s)ds = q$ means that the subsidy $q(t)$ to each consumer t for allowing congestion comes from the payment q for individual utilization of the public goods.

The following two definitions will introduce the concept of the weak Pareto optimality:

Definition 2. An allocation $(x(t), y(t), z, y)$ $(t \in T)$ is called *feasible* if

(1) $(x(t), y(t), \int_T y(s)ds, y) \in V(t)$ for almost all $t \in T$ and $(z, y) \in G$.

(2) $y(t) \leq y$ for almost all $t \in T$.

(3) $\int_T x(s)ds = \int_T e(s)ds + z$.

Definition 3. A feasible allocation $(x(t), y(t), z, y)$ $(t \in T)$ is called *weakly Pareto optimal* if there is no feasible allocation which is strictly better for almost all $t \in T$.

Now we can assert the following two theorems:

Theorem 1. *Under Assumptions 1 – 6, there exists an extended Lindahl equilibrium.*

Theorem 2. *Any extended Lindahl equilibrium allocation is weakly Pareto optimal.*

Note that we do not need any of the above assumptions from 1 to 6 in the second theorem.

The proofs of Theorems 1 and 2 are in the following two sections.

3. PROOF OF THEOREM 1

Extend the production set as

$$\hat{G} = \{v_f = (x_f, y_f, y_{of}, \bar{y}_f) : \quad (x_f, \bar{y}_f) \in G, \quad y_f = y_{of}, \quad y_{of} \leq \bar{y}_f, \quad y_{of} \in Y\}.$$

For any natural number k, define the following truncated consumption set and the set of Lindahl shares:

$$V^k(t) = V(t) \cap k\left[(e(t), 0, 0, 0) + (2, \ldots, 2) + \Re_-^{m+3l}\right]$$

$$\mathfrak{D}^k = \left\{\rho : \quad \int_T \rho = 1, \quad 0 \leq \rho(t) \leq 2^k \quad \text{a.e. in } T\right\}$$

Let us denote $(\mathfrak{D}^k)^l$ be l-fold of $\mathfrak{D}^k$.

For each $t \in T$, $\phi \equiv (p, q, r) \in \Delta^{m+2l-1}$ and σ, $\delta \in (\mathfrak{D}^k)^l$, define:

$$w(t, \phi) = pe(t) + \theta(t) \cdot \max\{(p, r)G\}$$

$$B^k(t, \phi, \sigma, \delta) =$$

$$\{v(t) \in V^k(t) : \quad (p, q, -\sigma(t)q, \delta(t)r)v(t) \leq w(t, \phi), \quad v_{y(t)}(t) \leq v_y(t)\}$$

$$E^k(t, \phi, \sigma, \delta) = \{v(t) \in B^k(t, \phi, \sigma, \delta) :$$

$$v' \in V^k(t), \ v'_{y(t)} \leq v'_y, \ v' \succ_t v(t) \quad \Longrightarrow \quad (p, q, -\sigma(t)q, \delta(t)r)v' > w(t, \phi)\}$$

$$F(\phi) = \operatorname{argmax}\left\{(p, q, -q, r)\hat{G}\right\}$$

$$Z = \int_T V^k(t)dt - \hat{G} - (\int_T e(t)dt, 0, 0, 0).$$

Now define the following correspondences:

$$\zeta(\phi, \sigma, \delta) = \left\{v \in \mathfrak{L}_1(T, V^k) : \quad v(t) \in E^k(t, \phi, \sigma, \delta) \quad \text{a.e. in } T\right\}$$

$$\beta(\phi,\sigma,\delta,v) = \left(\int_T v_{x(t)}(s)ds, \int_T v_{y(t)}(s)ds, \int_T \sigma(s)v_{\int y}(s)ds, \int_T \delta(s)v_y(s)ds\right) \\ - F(\phi) - \left(\int_T e, 0, 0, 0\right),$$

where the vector multiplication is interpreted as component-wise.

$$\gamma(n) = \{\phi \equiv (p,q,r) \in \Delta^{m+2l-1} : \\ (p,q,-q,r)n \geq (p',q',-q',r')n \qquad \forall \phi' \equiv (p',q',r') \in \Delta^{m+2l-1}\}$$

$$\xi_i = \text{argmin}\left\{\int_T \sigma_i(s)v_{\int y_i}(s) : \quad \sigma_i \in \mathfrak{D}^k\right\} \qquad (i = 1,\ldots,l)$$

$$\psi_i = \text{argmax}\left\{\int_T \delta_i(s)v_{y_i}(s) : \quad \delta_i \in \mathfrak{D}^k\right\} \qquad (i = 1,\ldots,l)$$

Then

$$\zeta : \Delta^{m+2l-1} \times (\mathfrak{D}^k)^l \times (\mathfrak{D}^k)^l \twoheadrightarrow \mathfrak{L}_1(T, V^k)$$

$$\beta : \Delta^{m+2l-1} \times (\mathfrak{D}^k)^l \times (\mathfrak{D}^k)^l \times \mathfrak{L}_1(T, V^k) \twoheadrightarrow Z$$

$$\gamma : Z \twoheadrightarrow \Delta^{m+2l-1}$$

$$\xi : \mathfrak{L}_1(T, V^k) \twoheadrightarrow (\mathfrak{D}^k)^l$$

$$\psi : \mathfrak{L}_1(T, V^k) \twoheadrightarrow (\mathfrak{D}^k)^l.$$

Finally, define a correspondence α from $\Delta^{m+2l-1} \times (\mathfrak{D}^k)^l \times (\mathfrak{D}^k)^l \times \mathfrak{L}_1(T, V^k) \times Z$ into itself as:

$$\alpha \equiv \zeta \times \beta \times \gamma \times \xi \times \psi.$$

Step 1. *The correspondence α has a fixed point:*

$$(\phi^k, \sigma^k, \psi^k, v^k, n^k) \equiv ((p^k, q^k, r^k), \sigma^k, \psi^k, v^k, (\int_T v^k(t)dt - v_f^k - (\int_T e(t)dt, 0, 0, 0))) \\ \in \alpha(\phi^k, \sigma^k, \psi^k, v^k, n^k)$$

where

$$v^k \equiv (x^k(t), y^k(t), y_o^k(t), \bar{y}^k(t)) \in E^k(t, \phi, \sigma, \delta) \subset V^k(t)$$

and

$$v_f^k \equiv (x_f^k, y_f^k, y_{of}^k, \bar{y}_f^k) \in F(\phi) \subset \hat{G}.$$

Proof. Since α is a nonempty-valued, convex-valued and weakly upper hemi-continuous correspondence from a nonempty, convex and weakly compact set into itself, apply Fan-Glicksberg's fixed point theorem to the correspondence α.

Q.E.D.

Step 2.

$$\int_T x^k(t)dt \leq x_f^k + \int_T e(t)dt$$
$$\int_T y^k(t)dt \leq \int_T \sigma^k(t) y_o^k(t)dt$$
$$\int_T \delta^k(t)\bar{y}^k(t)dt \leq \bar{y}_f^k$$

Proof. This follows from Walras's Law, i.e., by integrating the budget constraint:

$$(p^k, q^k, -\sigma^k(t)q^k, \delta^k(t)r^k)v^k(t) = w(t, \phi^k) = p^k e(t) - \theta(t)[p^k x_f^k + r^k \bar{y}_f^k]$$

and the fact that

$$(p^k, q^k, -q^k, r^k)[(\int_T x^k(t)dt, \int_T y^k(t)dt, \int_T \sigma^k(t)y_o^k(t)dt, \int_T \delta^k(t)\bar{y}^k(t)dt$$
$$- (x_f^k, y_f^k, y_{of}^k, \bar{y}_f^k) - (\int_T e(t)dt, 0, 0, 0)]$$

$$\geq (p', q', -q', r')[(\int_T x^k(t)dt, \int_T y^k(t)dt, \int_T \sigma^k(t)y_o^k(t)dt, \int_T \delta^k(t)\bar{y}^k(t)dt$$
$$- (x_f^k, y_f^k, y_{of}^k, \bar{y}_f^k) - (\int_T e(t)dt, 0, 0, 0)]$$

for all $(p', q', r') \in \Delta^{m+2l-1}$ and

$$y_f^k = y_{of}^k.$$

Q.E.D.

Step 3. *There exists S_k, $\lambda(S_k) \leq \frac{1}{2^k}$ such that for all $t \notin S_k$,*

$$y_o^k(t) \geq \int_T \sigma^k(s)y_o^k(s)ds$$

$$\bar{y}^k(t) \leq \int_T \delta^k(s)\bar{y}^k(s)ds$$

Proof. Suppose, for example, that for some i, there exists W, $\lambda(W) > \frac{1}{2^k}$ such that

$$\bar{y}_i^k(t) > \int_T \delta_i^k(s)\bar{y}_i^k(s)ds \qquad \text{for all } t \in W.$$

Choose δ' as

$$\delta_i'(t) = \begin{cases} 1/\lambda(W) & \text{if } t \in W \\ 0 & \text{if } t \notin W. \end{cases}$$

Then

$$\bar{y}_i^k(t) > \int_T \delta_i^k(s)\bar{y}_i^k(s)ds \geq \int_T \delta_i'(s)\bar{y}_i^k(s)ds = \frac{1}{\lambda(W)}\int_W \bar{y}_i^k(s)ds$$

for all $t \in W$, which is a contradiction.

Q.E.D.

Step 4. *By taking appropriate subsequences, there are $\phi^*, v_f^*, x_u, y_u, y_{ou}, \bar{y}_u$ such that*

$$\phi^k \to \phi^*$$

$$v_f^k \to v_f^*$$

$$\int_T \sigma^k(t)dt \to u \equiv (1, \ldots, 1)$$

$$\int_T \psi^k(t)dt \to u$$

$$\int_T x^k(t)dt \to x_u$$

$$\int_T y^k(t)dt \to y_u$$

$$\int_T \sigma^k(t)y_o^k \to y_{ou}$$

$$\int_T \delta^k(t)\bar{y}^k(t)dt \to \bar{y}_u$$

Proof. It follows from the fact that the above sequences are bounded.

Q.E.D.

Step 5. *There exists*

$$(\sigma^*(t), \psi^*(t), x^*(t), y^*(t), \sigma^*(t)y_o^*(t), \delta^*(t)\bar{y}^*(t))$$
$$\in Ls(\sigma^k(t), \psi^k(t), x^k(t), y^k(t), \sigma^k(t)y_o^k(t), \delta^k(t)\bar{y}^k(t))$$

such that

$$(\int_T \sigma^*(t)dt, \int_T \psi^*(t)dt, \int_T x^*(t)dt, \int_T y^*(t)dt, \int_T \sigma^*(t)y_o^*(t)dt, \int_T \delta^*(t)\bar{y}^*(t)dt)$$
$$\leq (u, u, x_u, y_u, y_{ou}, \bar{y}_u)$$

Proof. This is a direct conclusion from Fatou's Lemma (See Hildenbrand (1974 page 69, Lemma 3).

Q.E.D.

Step 6. *Almost all $t \in T$,*

$$\bar{y}^*(t) \leq \bar{y}_u.$$

Proof. Suppose not. Then there are i and S with $\lambda(S) > 0$ such that, for some i,

$$\bar{y}_i^*(t) > \bar{y}_{ui} \qquad \text{for all } t \in S.$$

Pick ϵ such that $0 < \epsilon < \frac{1}{2}$. Then there is a sufficiently large $\bar{k}$ such that $(\frac{1}{2^k}) \leq \epsilon\lambda(S)$.

Since for all k, by Step 3, there is S_k with $\lambda(S_k) \leq (\frac{1}{2^k})$ such that

$$\bar{y}_i^k(t) \leq \int_T \delta_i^k(s)\bar{y}_i^k(s)ds \qquad \forall t \notin S_k.$$

Moreover,

$$\lambda(S) > 2\epsilon\lambda(S) > 2\frac{1}{2^k} \geq \lambda(\cup_{k\geq\bar{k}} S_k).$$

Hence $\lambda(S \setminus \cup_{k\geq\bar{k}} S_k) > 0$ and almost all $t \in S \setminus \cup_{k\geq\bar{k}} S_k$,

$$\bar{y}_i^k(t) \leq \int_T \delta_i^k(s)\bar{y}_i^k(s)ds \to \bar{y}_{ui}$$

Therefore

$$y \in Ls(\bar{y}_i^k(t)) \quad \text{implies} \quad y \leq \bar{y}_{ui},$$

which is a contradiction.

Q.E.D.

Step 7. *Almost all $t \in T$,*

$$y^*_o(t) - \int_T y^*(s)ds \geq y_{ou} - y_u = 0.$$

Proof. Suppose that the first inequality is not true. Then there are i and S with $\lambda(S) > 0$ such that, for some i,

$$y^*_{oi}(t) - \int_T y^*_i(s)ds < y_{oui} - y_{ui} \qquad \text{for all } t \in S.$$

Pick ϵ such that $0 < \epsilon < \frac{1}{2}$. Then there is a sufficiently large $\bar{k}$ such that $(\frac{1}{2^{\bar{k}}}) \leq \epsilon\lambda(S)$.

Since for all k, by Step 3, there is S_k with $\lambda(S_k) \leq (\frac{1}{2^k})$ such that

$$y^k_{oi}(t) \geq \int_T \sigma^k_i(s) y^k_{oi}(s)ds \qquad \forall t \notin S_k.$$

Moreover,

$$\lambda(S) > 2\epsilon\lambda(S) > 2\frac{1}{2^{\bar{k}}} \geq \lambda(\cup_{k \geq \bar{k}} S_k).$$

Hence $\lambda(S \setminus \cup_{k\geq\bar{k}} S_k) > 0$ and almost all $t \in S \setminus \cup_{k\geq\bar{k}} S_k$,

$$y^k_{oi}(t) - \int_T y^k_i(s)ds \geq \int_T \sigma^k_i(s) y^k_{oi}(s)ds - \int_T y^k_i(s)ds \to y_{oui} - y_{ui}$$

Therefore

$$z \in Ls(y^k_{oi}(t)) - \lim \int_T y^k_i(s)ds \quad \text{implies} \quad y \geq y_{oui} - y_{ui},$$

which is a contradiction.

In order to prove the second equality, note from the monotonicity of preferences that all the prices are strictly positive at the limit, hence for sufficiently large k, the assertion of Step 2 holds with equality. Hence by Step 4 $y_{ou} = y_u$.

Q.E.D.

Step 8.

$$\int_T x^*(t)dt = x_u$$
$$\int_T y^*(t)dt = y_u$$
$$y_o^*(t) = \int_T y^*(s)ds \qquad \textit{for almost all } t \in T$$
$$\bar{y}^*(t) = \bar{y}_u \qquad \textit{for almost all } t \in T.$$

Proof. By integrating the budget constraint before and after taking the limit, one can get

$$(p^*, q^*, -q^*, r^*)(\int_T x^*(t)dt - x_u, \int_T y^*(t)dt - y_u,$$
$$\int_T \sigma(t)y_o^*(t)dt - y_{ou}, \int_T \delta(t)\bar{y}(t)dt - \bar{y}_u)$$
$$= (p^*, q^*, r^*)(\int_T x^*(t)dt - x_u, \int_T y^*(t)dt - \int_T \sigma(t)y_o^*(t)dt, \int_T \delta(t)\bar{y}(t)dt - \bar{y}_u)$$
$$= 0.$$

Since all the prices are strictly positive by monotonicity of preferences, by inequalities of Steps 5 and 7,

$$\int_T x^*(t)dt = x_u$$
$$\int_T y^*(t)dt = \int_T \sigma(t)y_o^*(t)dt$$
$$\int_T \delta(t)\bar{y}(t) = \bar{y}_u.$$

By Steps 6 and 7,

$$\bar{y}^*(t) = \bar{y}_u \qquad \text{for almost all } t \in T$$
$$y_o^*(t) = \int y*(s)ds \qquad \text{for almost all } t \in T.$$

Q.E.D.

Step 9.

$$\int_T x^*(t)dt = x_f^* + \int_T e(t)dt$$

Proof. It is straightforward by taking the limit in Step 2 and applying Walras's Law with the strict monotonicity of preferences.

Q.E.D.

Now the proofs of profit maximization and preference maximizations are straightforward.

Q.E.D.

4. PROOF OF THEOREM 2

Proof is by contradiction. Suppose, on the contrary to the assertion of Theorem 2, that there exists an extended Lindahl equilibrium allocation $(x^*(t), y^*(t), z^*, y^*)$ $t \in T$, which is not weakly Pareto optimal. Then there is an alternative feasible allocation $(x(t), y(t), z, y)$ $t \in T$, which is strictly better for almost all $t \in T$.

Hence by utility maximization, for almost all $t \in T$,

$$\begin{aligned} px(t) + qy(t) - q(t) \int_T y(s)ds + r(t)y &> pe(t) + \theta(t) \max(p, r)G \\ &\geq pe(t) + \theta(t)(p, r)(z, y). \end{aligned}$$

By integrating this inequality,

$$p(\int_T x(s)ds - \int_T e(s)ds - z) > 0.$$

Since the allocation $(x(t), y(t), z, y)$ $t \in T$ is feasible,

$$\int_T x(s)ds - \int_T e(s)ds - z = 0,$$

which is a contradiction.

Q.E.D.

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